Neap:

Trial Examination 2007

VCE Mathematical Methods Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
11	11	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

Question and answer booklet of 10 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this book during reading time.
Write your **name** and **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2007 VCE Mathematical Methods Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Given $f(x) = (x-1)^2$ and $g(x) = \sqrt{x} + 1$, find $g(f(x))$.

1 mark

Question 2

For the function $f: (-\infty, a) \to R,\ f(x) = x^2 + 2x$

a. find the maximum value of a such that the inverse function f^{-1} exists.

1 mark

b. find the rule for f^{-1}.

3 marks
Total 4 marks

Question 3

Consider the function $g : [0, 12] \to R,\ g(x) = 1 - 2\sin\left(\frac{\pi}{6}(x-2)\right)$.

a. **i.** Find the period of g.

_____

_____

ii. Find the range of g.

_____

_____

1 + 1 = 2 marks

b. Find the coordinates of the minimum of g.

_____

_____

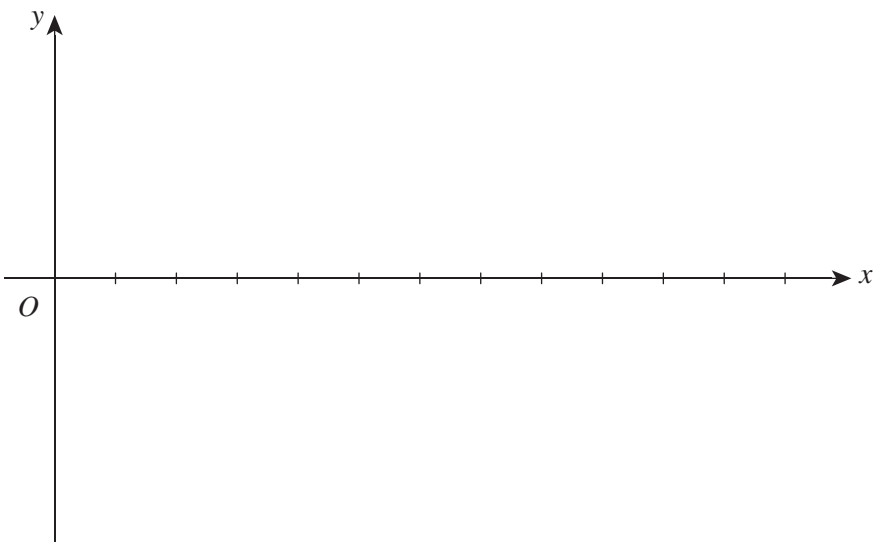
_____

_____

_____

2 marks

c. Sketch the graph of $y = g(x)$ on the set of axes below. Label end points and maximum and minimum points with their exact coordinates.



3 marks

Total 7 marks

Question 4

Let f be the function whose rule is given by $f(x) = \log_e\left(\frac{x}{x-1}\right)$. Show that $f'(x) = \frac{1}{x(1-x)}$ and hence state the gradient of the normal to the graph of f at $x = -\frac{1}{2}$.

4 marks

Question 5

Given that $\int_1^5 h(x)dx = 4$, find the value of $\int_5^1 2(h(x)-1)dx$.

2 marks

Question 6

The probability density function, $f(x)$, of the continuous random variable X, is defined by

$$f(x) = \begin{cases} \frac{k}{x^2}, & 1 \leq x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

a. Find the value of k.

_____

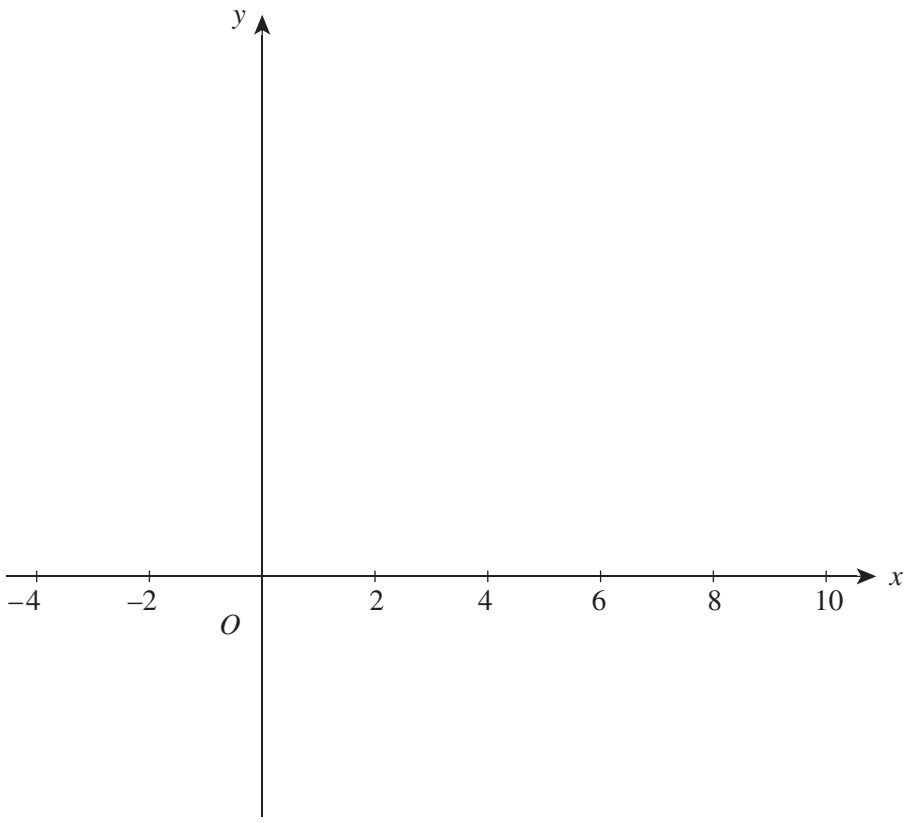
_____

_____

_____

2 marks

b. Sketch the graph of $y = f(x)$ on the axes provided.

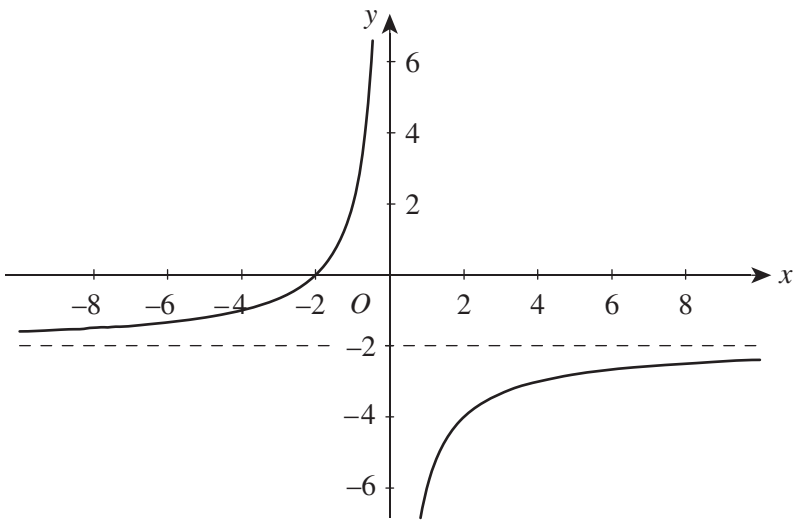


2 marks

Total 4 marks

Question 7

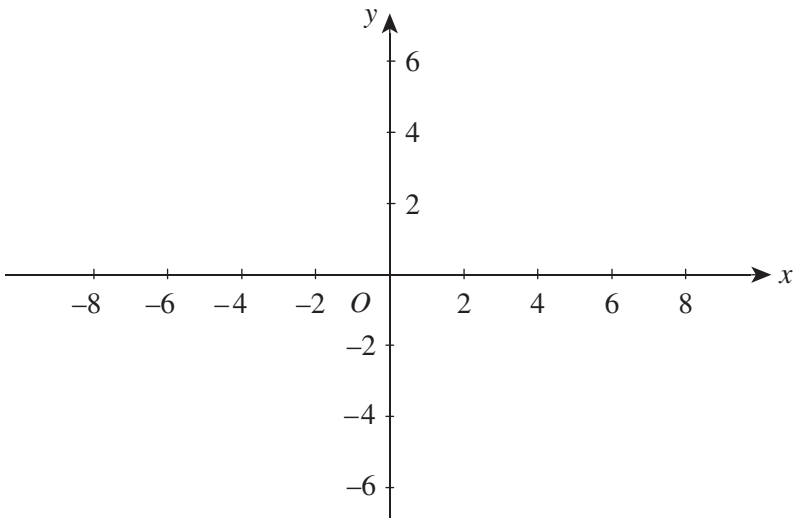
The graph of $f: R\backslash\{0\} \to R,\ f(x) = \dfrac{-2(x+2)}{x}$ is as shown below.



a. On the set of axes above, sketch the graph of $y = |f(x)|$.

1 mark

b. One the set of axes provided below, sketch the graph of $y = |f(-x)|$.



1 mark

c. State the range of the function whose graph you have sketched in part **b.** above.

_____

1 mark

Total 3 marks

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Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

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MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\,,\ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{Var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

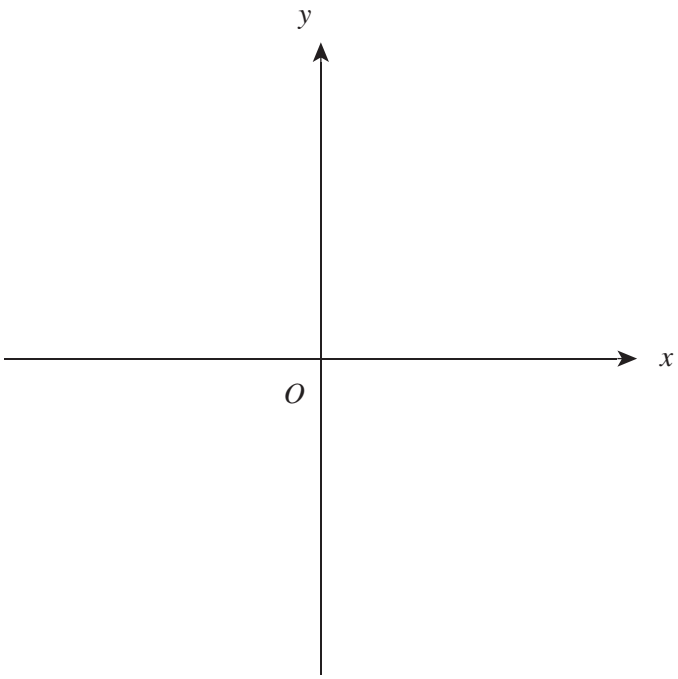
probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\ dx$	$\mu = \int_{-\infty}^{\infty} x f(x)\ dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx$

END OF FORMULA SHEET

Question 8

Let $f(x) = 12 - x^2$ for $x \geq 0$ and $f(x) \geq 0$.

a. Sketch the graph of $y = f(x)$ on the axes below.



1 mark

b. The tangent to the graph of $y = f(x)$ at the point where $x = p$ intersects the x-axis at $(4, 0)$.
Find p.

4 marks
Total 5 marks

Question 9

The radius of a sphere, r cm, is increasing at a constant rate of $\frac{1}{25}$ cm/s.

a. At the time when the radius of the sphere is 10 cm, show that the volume is increasing at the rate of 16π cm^3/s.

_____

_____

_____

_____

_____

_____

2 marks

b. At the instant when the rate of increase of the area of a cross-section through the centre of the sphere is $\frac{6\pi}{25}$ cm^2/s, calculate the volume of the sphere.

_____

_____

_____

_____

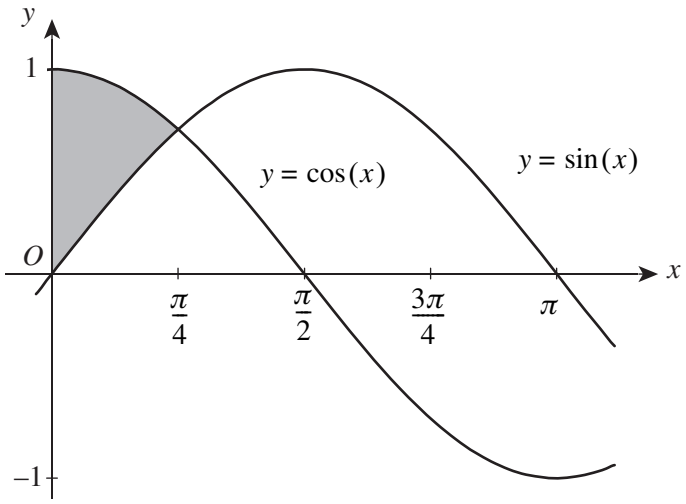
_____

_____

2 marks

Total 4 marks

Question 10



The region enclosed by the y-axis and the graphs of $y = \sin(x)$ and $y = \cos(x)$ is shaded in the graph above. Find the area of this region.

3 marks

Question 11

Fred catches a bus to school every day. If he is on time for school one day, the probability that he will be on time the next day is 0.6. If he is late for school, the probability that he will be on time the next day is 0.8.

Suppose that Fred has been late to school on Monday, Tuesday and Wednesday. What is the probability that he will be late on Friday?

3 marks

END OF QUESTION AND ANSWER BOOKLET

