2007 Mathematical Methods (CAS) GA 3: Examination 2

GENERAL COMMENTS

There were 1530 students who sat the Mathematical Methods (CAS) Examination 2 in 2007. Marks ranged from 2 to 78 out of a maximum possible score of 80. Student responses showed that the paper was accessible and that it provided an opportunity for students to demonstrate what they knew.

There is some evidence to suggest that, as with the Mathematical Methods cohort, students found the 2007 paper more difficult than the 2006 paper; however, the cohort was much larger, therefore possibly allowing for a larger spread in ability. Of the whole cohort, 13% of students scored 78% or more of the available marks, and 44% scored 54% or more of the available marks. The mean score for the paper was 39.5, with a mean of 14 out of 22 for the multiple-choice section and a mean of 25.8 out of 58 for the extended answer section. The median score for the paper was 41 marks.

Only five of the multiple-choice questions were different to those on the Mathematical Methods paper: Questions 3, 5, 10, 17 and 20. Except for Question 3, the other questions were answered correctly by less than 53% of students. The Mathematical Methods (CAS) students performed as least as well as, and generally better than, the Mathematical Methods students on all 17 common multiple-choice questions.

In the extended answer section, students sometimes did not give answers in exact form. If a numerical approximation is not asked for in the question, an exact answer must be given.

Some students lost marks because they did not give all the answers required in the question. This emphasises the importance of rereading questions before starting the next question. This occurred in Questions 2c., 2g., 3c. and 5fi.

Students should take care when drawing graphs – they need to use appropriate scales, make them clearly visible, use the correct domain and use a ruler for linear graphs.

It was pleasing to see that students showed their working for questions worth more than one mark. In some cases the formulation of the required computation is the appropriate working; specific examples are given under Section 2 below. An important exception to this is for 'show that' questions, where detailed working must be given.

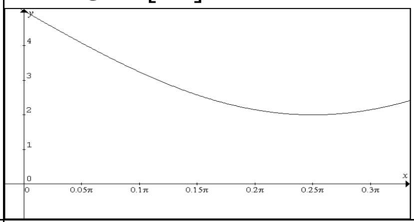
When students present working and develop their solutions, they are expected to use conventional mathematical expressions, symbols, notation and terminology. This was generally well done. However, students must be careful not to write down a CAS output as their final answer if it is not in simplest form.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	69	3	7	1	20	0	
2	8	1	2	86	3	0	
3	10	7	78	2	3	0	
4	1	6	71	13	9	0	
5	18	24	36	10	10	2	$mx+12y=24$ $3x+my=m$ There will be no solution if $\begin{vmatrix} m & 12 \\ 3 & m \end{vmatrix} = 0$, or equivalent. A unique solution exists for $m \in R \setminus \{-6,\ 6\}$.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
6	10	39	12	36	3	0	The minimum value of f occurs at the turning point, $x=\frac{\pi}{4}$ and $f\left(\frac{\pi}{4}\right)=2$, not the end point. The maximum value occurs at $x = 0$. $f(0)=3\lvert\sin(0)-1\rvert+2=5$ The range is $[2, 5]$. 
7	78	9	5	5	3	0	
8	13	52	11	9	15	0	The graph of the derivative is not continuous at $x=-2$ as f is not differentiable at $x=-2$.
9	7	3	12	3	75	0	
10	5	52	8	31	3	1	At the point(s) of intersection, $x^2+8x=kx-3$ or $x^2+(8-k)x+3=0$ there will be two distinct solutions when the discriminant is greater than zero. $b^2-4ac>0$ $(8-k)^2-4\times1\times3>0$ $(8-k)^2-12>0$ Solving for k gives the result $k<8-2\sqrt{3}$ or $k>8+2\sqrt{3}$
11	3	3	5	6	82	0	
12	7	16	69	5	3	0	
13	5	12	11	68	4	1	
14	72	12	11	3	2	0	
15	2	95	1	1	0	0	
16	8	16	67	4	5	0	
17	10	8	17	17	47	1	$f(f(x))=x$ for $f(f(x))=x$ Define $f(x)$ and evaluate $f(f(x))$. Algebraically, $f(f(x))=\dfrac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$ $=\dfrac{x+1+x-1}{x+1-x+1}$ $=\dfrac{2x}{2}$ $=x$
18	3	12	11	71	3	0	
19	84	3	2	3	8	0	

Question	% A	% B	% C	% D	% E	% No Answer	Comments
20	51	11	12	11	15	0	The average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$ $= \frac{1}{\frac{\pi}{8}}\int_0^{\frac{\pi}{8}} \tan(2x)\,dx$ $= \frac{2}{\pi}\log_e(2)$
21	27	11	20	27	15	1	$\cos^2(x) + 2\cos(x) = 0$ $\cos(x)(\cos(x)+2) = 0$ $\{x : \cos(x) = 0\}$ is the only solution set since $\cos(x) + 2 > 0$
22	19	16	19	8	37	0	$f(g(0)) = f(0) = 0$ The point (0, 0) belongs to the curve. $g(x) < 0$ elsewhere $f(x) > 0$ for $x < 0$ Hence, $f(g(x)) > 0$ elsewhere

Section 2

Question 1a.

Marks	0	1	Average
%	24	76	**0.8**

$h = \frac{V}{\pi r^2}$

This question was well done. The formula $V = \pi r^2 h$ was given on the formula sheet. Some students found *r*, not *h* as required.

Question 1b.

Marks	0	1	2	Average
%	35	4	60	**1.3**

$A = 2\pi rh + 2\pi r^2 = 2\pi r \times \frac{V}{\pi r^2} + 2\pi r^2 = \frac{2V}{r} + 2\pi r^2$

This question was generally answered well; however, some students did not know, or were unable to simply obtain the expression for, the total surface area of a **closed** cylinder. The formula for the curved surface area was given on the formula sheet. Some students did not show the substitution of *h* as required.

Question 1c.

Marks	0	1	2	Average
%	59	8	33	**0.8**

Solve $\frac{dA}{dr} = 0$ for r, $r = \sqrt[3]{\frac{V}{2\pi}}$

To obtain the method mark it was sufficient to write down 'Solve $\frac{dA}{dr} = 0$ for *r*', or similar, and then use CAS to get the answer.

Question 1d.

Marks	0	1	2	Average
%	96	1	3	**0.1**

$$A\left(\frac{V^{\frac{1}{3}}}{10}\right)=\left(\frac{\pi}{50}+20\right)V^{\frac{2}{3}}$$

This question was quite challenging and was not handled well. Many students found the minimum surface area instead of the maximum. Some students chose the wrong endpoint. To get the method mark, it was sufficient to write down

$$A\left(\frac{V^{\frac{1}{3}}}{10}\right)$$

Question 2a.

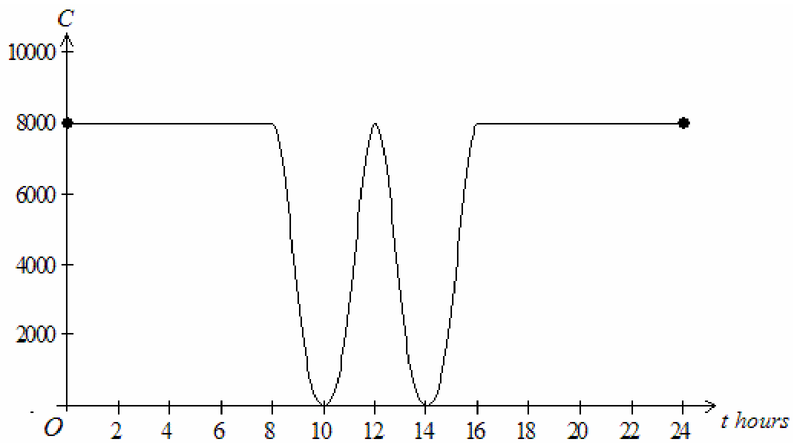
Marks	0	1	Average
%	56	44	**0.5**

$m = 8000$

This question was not done well. Many students did not know to substitute $t = 8$ or $t = 16$ into the equation $C(8)=C(16)=8000$. It was stated in the introduction that the concentration of insects in the gorge was a continuous function of time.

Question 2b.

Marks	0	1	2	3	Average
%	25	21	17	37	**1.7**



Students needed to show two correct minimum turning points, one correct maximum turning point and two correct lines over the restricted domain. The curve had to be continuous and should have been smooth at $t = 8$ and $t = 16$. Some students ignored m and drew $C(t)=1000\left(\cos\left(\frac{\pi(t-8)}{2}\right)+2\right)^2-1000$ for $0 \le t \le 24$. Others drew cusps instead of turning points.

Question 2c.

Marks	0	1	2	Average
%	31	21	48	**1.2**

The minimum concentration was 0 at $t = 10$ and $t = 14$.

This question was generally well handled. Some students forgot to state that the minimum concentration was zero.

Question 2d.

Marks	0	1	Average
%	59	41	**0.4**

Solve $1000\left(\cos\left(\frac{\pi(t-8)}{2}\right)+2\right)^2-1000=1250$ for t, $t=9\frac{1}{3}$, safety level starts at 9:20 am

Many students did not give an exact answer for this question. Students must remember that an exact answer is required unless a numerical answer is asked for in the question. A common incorrect answer was 9.33 hours after midnight.

Question 2e.

Marks	0	1	2	Average
%	50	15	35	**0.9**

$\frac{32}{3}-\frac{28}{3}+\frac{44}{3}-\frac{40}{3}=\frac{8}{3}$ hours = 2 hours and 40 minutes

This question could easily be done using CAS to solve $1000\left(\cos\left(\frac{\pi(t-8)}{2}\right)+2\right)^2-1000=1250$ over a restricted domain $8\le t\le 24$. A common incorrect answer was 2.67 hours.

Question 2fi.

Marks	0	1	2	Average
%	44	1	55	**1.2**

$5=p(q-1), 12.5=p(q^2-1), q=1.5, p=10$

Many students who could not do the previous parts to this question were able to successfully complete this part. Some students correctly set up the equations but were not able to get the answers from their CAS. Students should be familiar with the multiplication conventions required by their CAS for correct evaluation.

Question 2fii.

Marks	0	1	Average
%	51	49	**0.5**

$T=10(1.5^4-1)=40.625$ min = 40 minutes and 37.5 seconds

Some students gave the incorrect answer of 40 minutes and 38 seconds; otherwise, this question was generally well handled by the students who were successful in Question 2fi.

Question 2g.

Marks	0	1	2	3	Average
%	54	22	11	14	**0.9**

$40.625+\frac{40.625}{2}+19=79.9375,\ 80-79.9375=0.0625$ minutes $\left(=\frac{1}{16}\text{ minutes = 3.75 seconds}\right)$

Many students rounded too soon and some used 80.33 instead of 80. Others used 160, both safe time periods, instead of 80. Some students did not complete their answer – a discussion was given about subtracting from 80 but no final answer was stated.

Question 3a.

Marks	0	1	2	Average
%	25	25	49	1.3

Solve $g'(x) = 0$ for x, $x = \frac{6-2\sqrt{3}}{3}$

This question was quite well done. Some students gave the incorrect answer of 0.282, while others gave $x = \frac{6+2\sqrt{3}}{3}$ which is the x value for which $g(x)$ is a minimum. Some students showed full algebraic working by using the quadratic formula but this was not necessary to obtain full marks. It was sufficient to give something like 'solve $g'(x) = 0$ for x' for the method mark.

Question 3bi.

Marks	0	1	2	Average
%	25	22	53	1.3

$$2\int_0^1 \left(g(x) - f(x)\right)dx + 2\int_1^2 \left(f(x) - g(x)\right)dx$$

or

$$\int_0^1 \left(g(x) - f(x)\right)dx + \int_1^2 \left(f(x) - g(x)\right)dx + \int_2^3 \left(g(x) - f(x)\right)dx + \int_3^4 \left(f(x) - g(x)\right)dx$$

This question was done quite well. Most students gave the second answer and most had 'dx' as part of the definite integral. The most common error was to give $\int_2^3 \left(f(x) - g(x)\right)dx + \int_3^4 \left(g(x) - f(x)\right)dx$ for the area of the shaded region below the x-axis.

Question 3bii.

Marks	0	1	2	Average
%	35	18	48	1.2

$$\frac{3(3\pi - 8)}{\pi} + \frac{24 - 7\pi}{\pi} = 2 \text{ units}^2$$

As this question was worth two marks, it was pleasing to see that most students showed suitable working.

Question 3c.

Marks	0	1	2	Average
%	53	14	33	0.9

The maximum value of 1.08 occurs when $x = 0.38$ or $x = 3.62$

Given its level of difficulty, this question was not well done. This was likely due to problems with interpreting the absolute value sign in the expression. Some students gave the values of x but not the maximum value of the expression. Others gave the values of x for the four turning points.

Question 3di.

Marks	0	1	2	Average
%	51	36	13	0.7

A translation of 3 units to the left and a reflection in the x-axis or vice versa

or

A reflection in the *y*-axis followed by a translation 1 unit to the right

or

A translation of 1 unit to the left followed by a reflection in the *y*-axis

Some students did not seem to know the meaning of the word 'transformation', as they gave instructions to differentiate. Others did not consider the domains of *h* and *f*.

Question 3dii.

Marks	0	1	2	Average
%	77	10	13	**0.4**

$y = -2(x+3)(x+1)(x-1)$

or

$y = 2(x+3)(x+1)(x-1)$

This was a more challenging question and it was not attempted by many students. Students could have used their transformations from Question 3di. to obtain the answer. Other methods were also acceptable, such as translating the graph of *g* 3 units to the left or using $y = A(x+3)(x+1)(x-1)$, where *A* is a real constant, and substituting in an appropriate point.

Question 4a.

Marks	0	1	2	Average
%	35	38	26	**0.9**

a = 1, (from *y* = 1) and *b* = – 1 (goes through origin)

Most students were able to explain why *a* = 1 by discussing the asymptote; however, many of them were unable to explain why *b* = – 1. The most common error occurred when students tried to discuss reflections and translations but failed to indicate which graph they were transforming.

Question 4b.

Marks	0	1	2	Average
%	27	12	60	**1.4**

range = $\left[0, 1-e^{-2}\right]$

This question was generally well done. Some students were unable to obtain full marks because they rounded up $1-e^{-2}$ to 0.8647. Others used round brackets instead of square brackets.

Question 4ci.

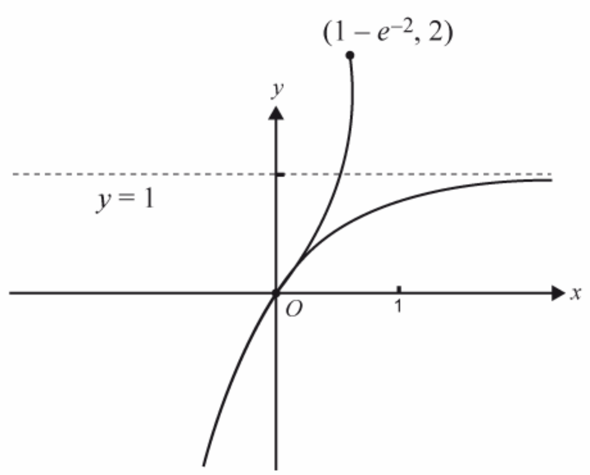
Marks	0	1	Average
%	35	65	**0.7**

$h^{-1}: \left[0, 1-e^{-2}\right] \to R$, where $h^{-1}(x) = -\log_e(1-x)$

When the inverse function is asked for, the domain must be given. Students will be penalised in the future if the domain is left out. If only the **rule** for the inverse function is asked for, the domain does not have to be given. Some students gave the incorrect answer $h^{-1}(x) = -\log_e|1-x|$.

Question 4cii.

Marks	0	1	2	Average
%	35	50	15	**0.8**



Many students drew the graph of $h^{-1}:[-\infty,1)\to R,$ where $h^{-1}(x)=-\log_e(1-x)$. Only 15 percent of students restricted the domain correctly.

Question 4di.

Marks	0	1	2	Average
%	53	21	26	**0.8**

$$\begin{aligned} f(u)\,f(v) &= \left(1-e^{-u}\right)\left(1-e^{-v}\right) \\ &= 1-e^{-v}-e^{-u}+e^{-u}e^{-v} \\ &= f(u)+f(v)-1+e^{-u}e^{-v} \\ &= f(u)+f(v)-\left(1-e^{-(u+v)}\right) \\ &= f(u)+f(v)-f(u+v) \end{aligned}$$

Many students did not know how to approach this question, probably because it was unfamiliar to them. As it was a ‘show that’ question, full working needed to be shown. Some students assumed that the left hand side equalled the right hand side at the start. This will be penalised in the future. Some students let *u* and *v* equal an integer value, which is not acceptable as it only establishes (or not) a result for that particular set of values.

Question 4dii.

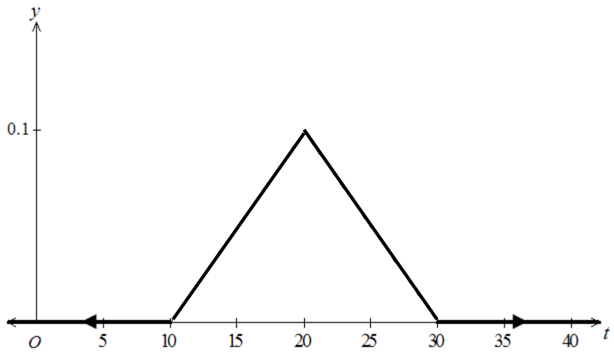
Marks	0	1	Average
%	71	29	**0.3**

$$\begin{aligned} f(u)\,f(-u) &= \left(1-e^{-u}\right)\left(1-e^{u}\right) \\ &= 1-e^{u}-e^{-u}+1 \\ &= f(u)+\left(1-e^{-(-u)}\right) \\ &= f(u)+f(-u) \end{aligned}$$

Many students were able to show the correct substitution but unable to complete the proof.

Question 5a.

Marks	0	1	2	Average
%	32	46	22	**1.0**



Students need to take care when sketching graphs. A common error was that (20, 1) were given as the coordinates of the maximum, not (20, 0.1). The two oblique lines needed to be drawn to scale along the *t*-axis and they had to be straight. Some students did not show the lines along the *t*-axis, while others only showed the left hand one.

Question 5b.

Marks	0	1	2	Average
%	35	12	53	**1.3**

$$\Pr\left(f(t) < 25\right) = 1 - \frac{1}{100}\int_{25}^{30}(30-t)\,dt = \frac{7}{8}$$

This question was generally well done. More students could have accessed method marks for these types of questions if they shaded the appropriate area on a graph.

Question 5c.

Marks	0	1	2	Average
%	46	11	43	**1.0**

$$\frac{\Pr(T \le 15) \cap \Pr(T \le 25)}{\Pr(T \le 25)} = \frac{\frac{1}{8}}{\frac{7}{8}} = \frac{1}{7}$$

This question was generally done, given that it was a conditional probability question.

Question 5d.

Marks	0	1	2	Average
%	42	22	37	**1.0**

$$X \sim \text{Bi}\left(6, \frac{7}{8}\right), \ \Pr(X > 4) \approx 0.9709$$

More students could have accessed method marks for these types of questions if they gave the appropriate distribution with its parameters. Of those who attempted the question, it was pleasing to see that most students used correct mathematical notation, as calculator syntax is not acceptable.

Many students did not attempt Questions 5e. and 5f., which were more demanding questions.

Question 5e.

Marks	0	1	2	Average
%	70	5	25	**0.6**

$$\binom{6}{3}p^3(1-p)^3+\binom{6}{4}p^4(1-p)^2=5p^3(1-p)^2(4-p)$$

Most students who attempted this question were able to complete it.

5fi.

Marks	0	1	2	Average
%	78	7	15	**0.4**

Solve $Q'(p)=0$ for $p,\ p=2-\sqrt{2},\ Q=20\left(17-12\sqrt{2}\right)$

Some students did not give the Q value. There were many different CAS outputs for Q which were acceptable, although the given answer is in simplest form. Some students gave $p=2+\sqrt{2}$, which is greater than one.

Question 5fii.

Marks	0	1	2	Average
%	87	4	9	**0.2**

$$\frac{1}{100}\int_b^{30}(30-t)dt=1-\left(2-\sqrt{2}\right),\ b\approx 20.9$$

Some students incorrectly used Q instead of f to solve the equation. More students could access method marks for these types of questions if they shaded the appropriate area on a graph.