2008 VCAA Mathematical Methods Exam 2 Solutions

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SECTION 1: Multiple-choice questions

1	2	3	4	5	6	7	8	9	10	11
E	C	D	A	E	A	D	D	D	C	D

12	13	14	15	16	17	18	19	20	21	22
B	C	D	E	C	B	E	B	B	D	C

Q1 The quotient rule:

$$\frac{d}{dx}\left(\frac{\log_e(2x)}{2x}\right)=\frac{(2x)\left(\frac{1}{x}\right)-2\log_e(2x)}{4x^2}=\frac{1-\log_e(2x)}{2x^2}$$ E

Q2 Translate $y=|x|$ to the left by 2 units and down by 2 units to obtain $y=|x+2|-2$. C

Q3 $3\log_e(2x-3)=6$, $\log_e(2x-3)=2$, $2x-3=e^2$, $x=\frac{1}{2}\left(e^2+3\right)$. D

Q4 $\int_1^3(2f(x)-3)dx=2\int_1^3 f(x)dx-\int_1^3 3dx=2\times5-[3x]_1^3=4$ A

Q5 $\mu=np=1.2$, $\sigma^2=np(1-p)$, $0.72=1.2(1-p)$, $p=0.4$ and $n=3$. E

Q6 $\int\left(e^{3(x-2)}+\frac{2}{2-x}\right)dx=\frac{1}{3}e^{3(x-2)}-2\log_e|2-x|+c$

$=\frac{1}{3}e^{3(x-2)}-2\log_e|x-2|+c$ A

Q7 $y=\frac{1}{\sqrt{x}}-3$, inverse is $x=\frac{1}{\sqrt{y}}-3$, $\sqrt{y}=\frac{1}{x+3}$,

$y=\frac{1}{(x+3)^2}$, $f^{-1}(x)=\frac{1}{(x+3)^2}$. D

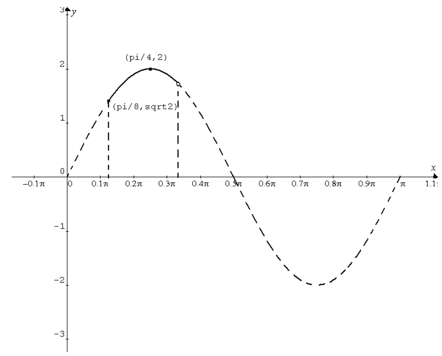
Q8 $f(x)=\frac{x-3}{2-x}=-\left(\frac{x-3}{x-2}\right)=-\left(\frac{x-2-1}{x-2}\right)=-\left(1-\frac{1}{x-2}\right)$

$=\frac{1}{x-2}-1$. Asymptotes: $x=2$, $y=-1$. D

Q9 $\int_1^3\left(6x^2+\frac{3a}{x^2}\right)dx=\left[2x^3-\frac{3a}{x}\right]_1^3=(54-a)-(2-3a)$

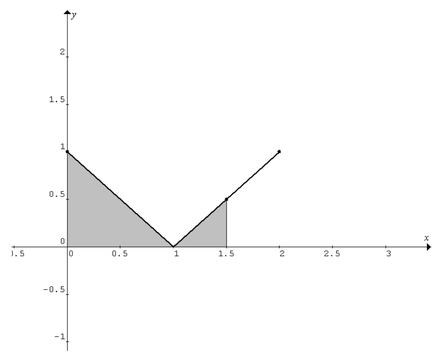
$=52+2a$. D

Q10



The range is $\left[\sqrt{2},2\right]$, C

Q11



$\Pr(X<1.5)=\frac{1}{2}\times1\times1+\frac{1}{2}\times0.5\times0.5=0.625$ D

Q12 The quotient rule:

$f'(x)=\frac{2\pi g(x)\cos(2\pi x)-\sin(2\pi x)g'(x)}{[g(x)]^2}$ B

Q13 Binomial distribution: $n=10$, $p=0.30$.

$\Pr(X\ge7)=1-\Pr(X\le6)=1-binomcdf(10,0.30,6)=0.0106$ C

Q14 $\Pr(allheads)<0.0005$, $0.5^n<0.0005$,

$n>\frac{\log_e 0.0005}{\log_e 0.5}=10.97$, $\therefore$ minimum n is 11. D

Note: < is changed to > because $\log_e 0.5$ has a negative value.

Q15 $\Pr(\{1,2\}\cap\{2,4,6\})=\Pr(\{2\})=\frac{1}{6}$,

$\Pr(\{1,2\})\Pr(\{2,4,6\})=\frac{2}{6}\times\frac{3}{6}=\frac{1}{6}$.

$\therefore \{1,2\}$ and $\{2,4,6\}$ are independent. E

Q16 $V=\pi r^2h=\pi2^2h=4\pi h$, $\frac{dV}{dt}=4\pi\frac{dh}{dt}$,

$\therefore\frac{dh}{dt}=\frac{1}{4\pi}\times\frac{dV}{dt}=\frac{1}{4\pi}\times2=\frac{1}{2\pi}$. C

Q17 $e^{2x}-2=e^x$, $e^{2x}-e^x-2=0$, $\left(e^x\right)^2-e^x-2=0$, $\left(e^x-2\right)\left(e^x+1\right)=0$.

Since $e^x+1>0$, $\therefore e^x-2=0$, $x=\log_e 2$. B

Q18 $\sin(4x)+1 \rightarrow -(\sin(4x)+1) \rightarrow -\left(\sin 4\left(\frac{x}{4}\right)+1\right)=-\sin x-1$.

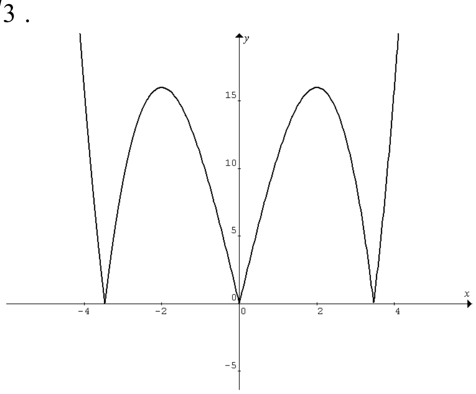
The domain of the transformed function is $\left[0,\frac{\pi}{2}\times 4\right]$,

i.e. $[0,2\pi]$. E

Q19 The gradient of the antiderivative graph B is the graph of function *f*. B

Q20 Domain $B=\left(\frac{1}{2},\infty\right)$ makes *f* a one-to-one function for it to have an inverse function. B

Q21 $y=x^3-12x=x\left(x^2-12\right)=x\left(x-2\sqrt{3}\right)\left(x+2\sqrt{3}\right)$,
$x=0,\pm 2\sqrt{3}$.



Positive gradient: $x\in\left(-2\sqrt{3},-2\right)\cup(0,2)\cup\left(2\sqrt{3},\infty\right)$ D

Q22 For $x=2$, $f(x)\neq 0$. C

SECTION 2:

Q1ai $\Pr(SSSSSSSS)=(\Pr(S))^8=0.80^8\approx 0.1678$

Q1aii Binomial: $n=8$, $p=0.80$,
$\Pr(X=6)=binompdf(8,0.80,6)=0.2936$.

Q1aiii Conditional probability:
Let A be the event that the first 4 are successful, and B the event that exactly 6 of the first 8 are successful.
$\Pr(A\mid B)=\frac{\Pr(A\cap B)}{\Pr(B)}=\frac{0.8^4\times binompdf(4,0.8,2)}{0.2936}\approx 0.214$.

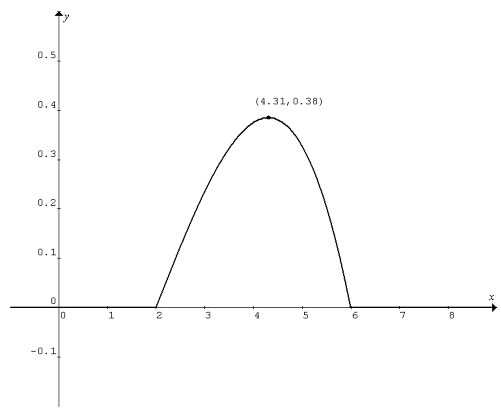
Q1bi
$S\xrightarrow{0.84}S$, $S\xrightarrow{0.16}S'$, $S'\xrightarrow{0.64}S$, $S'\xrightarrow{0.36}S'$.

$\Pr(SSSSSSS)=0.84^7=0.2951$.

Q1bii $\Pr(2 of next 3)=\Pr(SSSS')+\Pr(SSS'S)+\Pr(SS'SS)$
$=0.84\times0.84\times0.16+0.84\times0.16\times0.64+0.16\times0.64\times0.84=0.2849$

Q1ci $y=f(x)=\begin{cases}\frac{1}{64}(6-x)(x-2)(x+2) & if\quad 2\le x\le 6\\ 0 & elsewhere\end{cases}$

Local maximum $(4.31,0.38)$, graphics calculator.



Q1cii $\Pr(X<3)=\int_2^3 f(x)dx=0.1211$, graphics calculator.

Q1ciii Mean time $=\int_2^6 xf(x)dx=4.1333$, graphics calculator.

Q2ai $f(1)=7$, $f(a)=\frac{7}{a}$.

Gradient of $CA=\frac{\frac{7}{a}-7}{a-1}=\frac{7(1-a)}{a(a-1)}=-\frac{7}{a}$.

Q2aii Gradient of tangent $=f'(x)=-\frac{7}{x^2}=-\frac{7}{a}$ for $x>0$.

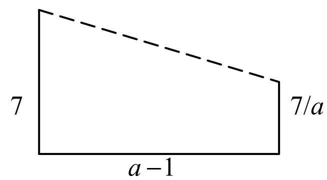
$\therefore x^2=a$, $x=\sqrt{a}$.

Q2bi $\int_1^e \frac{7}{x}dx=\left[7\log_e x\right]_1^e=7\log_e e-7\log_e 1=7$

Q2bii $\int_b^1 \frac{7}{x}dx=\left[7\log_e x\right]_b^1=7\log_e 1-7\log_e b=-7\log_e b=7$.

$\therefore \log_e b=-1$, $b=e^{-1}=\frac{1}{e}$.

Q2ci



Area of trapezium $= \frac{1}{2}\left(7+\frac{7}{a}\right)(a-1) = \frac{7}{2}\left(1+\frac{1}{a}\right)(a-1)$.

Q2cii $\frac{7}{2}\left(1+\frac{1}{a}\right)(a-1) = 7$, $\left(1+\frac{1}{a}\right)(a-1) = 2$.

Expand and simplify to $a^2 - 2a - 1 = 0$, where $a > 1$.
Use the quadratic formula to find $a = 1+\sqrt{2}$.

Q2ciii $\int_1^a f(x)dx < 7$, i.e. less than the area of the trapezium, because the function $f(x)$ is below the line CA between $x = 1$ and $x = 1+\sqrt{2}$.

From Q2bi, $\int_1^e f(x)dx = 7$, $\int_1^a f(x)dx < \int_1^e f(x)dx$, $\therefore a < e$.

Q3a $50\log_e(1+2t) < 100$, $\log_e(1+2t) < 2$, $1+2t < e^2$,

$t < \frac{1}{2}\left(e^2 - 1\right) \approx 3.19453$ hours

or 3hours and 12 minutes (to the nearest minute as required by the question). Tasmania would be killed by then.

Q3b Time required $= \frac{18}{5} = 3.6$ hours > 3.19453 hours.

Q3c $NY = XM = \sqrt{3^2 + x^2} = \sqrt{9+x^2}$.

$T = \frac{2\sqrt{9+x^2}}{5} + \frac{18-2x}{13} = 2\left(\frac{\sqrt{9+x^2}}{5} + \frac{9-x}{13}\right)$.

Q3d $\frac{dT}{dx} = 2\left(\frac{x}{5\sqrt{9+x^2}} - \frac{1}{13}\right) = 0$ for $x > 0$,

$\frac{x}{\sqrt{9+x^2}} = \frac{5}{13}$, $\frac{x^2}{9+x^2} = \frac{25}{169}$, $144x^2 = 225$, $\therefore x = \frac{5}{4}$.

Q3e Minimum time $= 2\left(\frac{\sqrt{9+\left(\frac{25}{16}\right)^2}}{5} + \frac{9-\frac{5}{4}}{13}\right) = 2.4923 < 3.19453$

Q3f Curve AB: $z = \frac{16}{d+1}$.
Point A: $d = 0$, $z = 16$, $(0,16)$.
Point B: $d = 1$, $z = 8$.
Point C: $d = 1$, $z = 8+16 = 24$, $(1,24)$.

Q3g Curve CD: Curve AB is translated to the right by 1 unit and upwards by 8 units. $z = \frac{16}{(d-1)+1} + 8$,

i.e. $z = \frac{16}{d} + 8$, where $d \in [1,2)$.

Q3h

Day	z
1	16 to 8
2	24 to 16
3	32 to 24
4	40 to 32
5	48 to 40
6	56 to 48

$\therefore 6$ days.

Q4ai $f(x) = \tan\left(\frac{x}{2}\right)$, $f'(x) = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$,

$f'\left(\frac{\pi}{2}\right) = \frac{1}{2}\sec^2\left(\frac{\pi}{4}\right) = 1$.

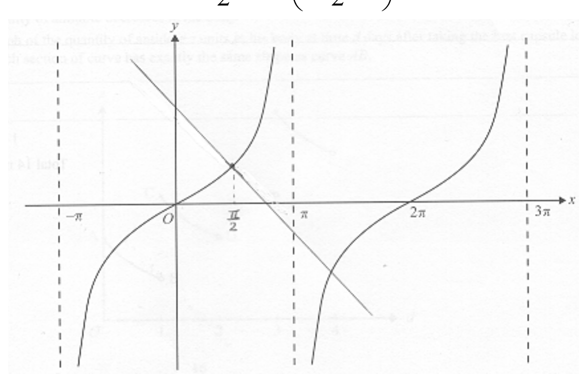
Q4aii At $x = \frac{\pi}{2}$, $y = \tan\left(\frac{\pi}{4}\right) = 1$, $\left(\frac{\pi}{2},1\right)$.

Gradient of the normal $= -1$.

Equation: $y - 1 = -1\left(x - \frac{\pi}{2}\right)$, $y = -x + \frac{\pi}{2} + 1$.

Q4aiii x-intercepts: $y = 0$, $x = \frac{\pi}{2} + 1$, $\left(\frac{\pi}{2}+1, 0\right)$.

y-intercepts: $x = 0$, $y = \frac{\pi}{2} + 1$, $\left(0, \frac{\pi}{2}+1\right)$.



Q4b $f'(x) = f'\left(\frac{\pi}{2}\right)$ when $x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$.

Q4c $-1 < a < 1$, $0 < \frac{1-a}{2} < 1$ and

$g(1) = f(1-a) = \tan\left(\frac{1-a}{2}\right) = 1$.

$\therefore \frac{1-a}{2} = \frac{\pi}{4}$, $a = 1 - \frac{\pi}{2}$.

Q4di $h(x) = \sin\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right) + 2$,

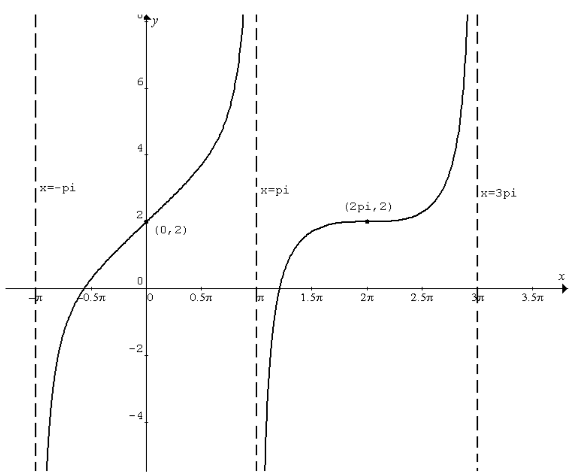
$h'(x) = \frac{1}{2}\left(\cos\left(\frac{x}{2}\right) + \sec^2\left(\frac{x}{2}\right)\right)$.

Q4dii $h'(x) = \frac{1}{2}\left(\cos\left(\frac{x}{2}\right) + \sec^2\left(\frac{x}{2}\right)\right) = 0$,

$\cos\left(\frac{x}{2}\right) + \sec^2\left(\frac{x}{2}\right) = 0$, $\cos\left(\frac{x}{2}\right) + \frac{1}{\cos^2\left(\frac{x}{2}\right)} = 0$,

$\cos^3\left(\frac{x}{2}\right) = -1$, $\therefore \cos\left(\frac{x}{2}\right) = -1$, $\frac{x}{2} = \pi$, $x = 2\pi$.

Q4e



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