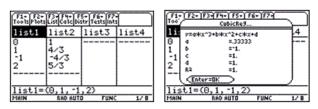
The Mathematical Association of Victoria MATHEMATICAL METHODS (CAS) 2008 Trial written examination 2 – Solutions – Multiple choice

SECTION 1 Solutions

1. D	2. E	3. A	4. C	5. A	6. B
7. E	8. C	9. E	10. B	11. B	12. A
13. D	14. E	15. B	16. B	17. A	18. C
19. A	20. B	21. B	22. C		

Question 1

Enter data into the calculator and choose cubic regression.



The exact answer is required.

$$y = \frac{1}{3}x^3 - x^2 + x + 1$$

$$3y = x^3 - 3x^2 + 3x + 3$$

Question 2

h(x) has factors x, x,(x - b) and (x - c). The coefficient of x^4 is negative.

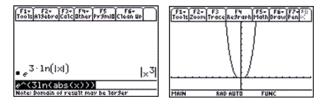
Since a > 0, the rule is $h(x) = -ax^2(x-b)(x-c)$.

Question 3

$$f(g(x)) = e^{3\log_e |x|} = e^{\log_e |x^3|} = |x^3|$$

The domain of f(g(x)) is the same as the domain of $g(x) : R \setminus \{0\}$.

The range is R^+ .



Answer D

Answer E

Answer A

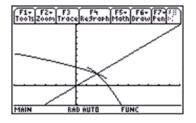
 $f: (-\infty, 3] \to R$ where $f(x) = (x - 3)^{\frac{2}{3}} + 1$ Let $y = (x - 3)^{\frac{2}{3}} + 1$ Inverse swap x and y $x = (y - 3)^{\frac{2}{3}} + 1$ $(y - 3)^{\frac{2}{3}} = x - 1$ $y - 3 = -(x - 1)^{\frac{3}{2}}$, take the negative square root $y = 3 - (x - 1)^{\frac{3}{2}}$

OR

Solve
$$x = (y - 3)^{\frac{2}{3}} + 1$$
 for *y* on the CAS

F1+ F2+ F3+ ToolsAl9ebraCalc	Ether Pr3mID C	F6+ Iean Up
<pre>solve(x = (</pre>	y - 3) ^{2/3} +	1,9)
y=3-(x-	1) ^{3/2} and	×≥1 ▶
MAIN DEG	AUTO FUNC	1/30

$$f: [1, \infty) \to R$$
, where $f^{-1}(x) = 3 - (x - 1)^{\frac{3}{2}}$



Answer C

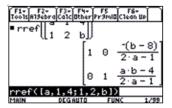
Solve the matrix equation to obtain the result

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{8-b}{2a-1} \\ \frac{ab-4}{2a-1} \end{bmatrix}$$

OR

Determinant = $2a - 1 \neq 0$

Hence there will be a unique solution for $a \in R \setminus \left\{\frac{1}{2}\right\}$ and $b \in R$



Note also that if:

 $a = \frac{1}{2}$ and b = 8, there are infinite solutions (the solutions are identical) $a = \frac{1}{2}$ and $b \in R \setminus \{8\}$, there is no solution (the solutions are inconsistent)

Question 6

Answer B

Answer A

$$-h(2x-4) = -h(2(x-2))$$

The transformation from h(x) to -h(2(x-2)) involves

A reflection in the x-axis, a dilation by a scale factor of $\frac{1}{2}$ from the y-axis and a translation

3

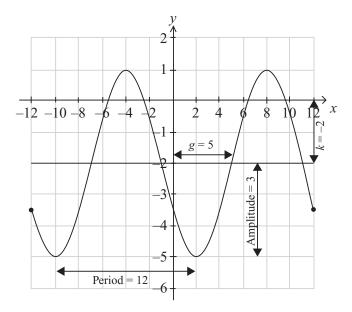
2units right.

The rule will be of the form $f(x) = a\sin(n(x-\varepsilon)) + k$.

Amplitude = 3, therefore a = 3. Translation down 2 units, therefore k = -2. Phase shift, $\varepsilon = 5$.

Period =
$$\frac{2\pi}{n}$$

 $12 = \frac{2\pi}{n}$
 $n = \frac{2\pi}{12}$
 $n = \frac{\pi}{6}$



Question 8

Answer C

 $cos(x + \pi) = -cos(x)$ (this can be seen from a unit circle or the graph of y = cos(x)).

Hence $f(x + \pi) = -f(x)$

This can be verified using a CAS.



Let x' and y' be the images of x and y, respectively, under T. The transformation of $y = e^x$ to $y = e^{-2(x+1)}$ involves:

x = -2(x' + 1) and y = y' $x' + 1 = -\frac{1}{2}x$ $x' = -\frac{1}{2}x - 1$ and y' = y

Question 10

 $e^{2x} + be^{x} + 1 = 0$ Let $e^{x} = p$ $p^{2} + bp + 1 = 0$ $\Delta < 0$ for no real solutions $b^{2} - 4 = 0$

Hence $\{b : -2 < b < 2\}$

Also there is no solution at b = 2 as $e^{2x} + 2e^x + 1 = (e^x + 1)^2 = 0$ $e^x \neq -1$ Thus $\{b: -2 < b \le 2\}$

Question 11

 $f(x) = \begin{cases} x^2 + 4x + a & \text{for } x \ge 0\\ bx + c & \text{for } x < 0 \end{cases}$

For f(x) to be differentiable at x = 0, f(x) has to be continuous at x = 0.

$$x^{2} + 4x + a = bx + c$$

Hence at $x = 0$, $a = c$
Also $\lim_{x \to 0^{-}} f'(x) = \lim_{x \to 0^{+}} f'(x)$
 $2x + 4 = b$
At $x = 0$ $b = 4$

Question 12

$$f(x+h) \approx hf'(x) + f(x)$$

$$f(x+h) - f(x) \approx hf'(x)$$

$$\approx -0.1 \times \frac{2}{2x+1}$$

$$\approx -0.1 \times \frac{2}{3} = -\frac{1}{15}$$

Answer E

Answer B

Answer B

Answer A

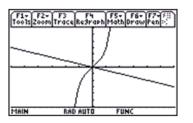
 ${\color{blue}{\textbf{SOLUTIONS}}-\text{continued}}$

$$y = x^5 + 2x$$

 $\frac{dy}{dx} = 5x^4 + 2 = 2$ at $x = 0$.

The equation of the normal is

$$y = -\frac{1}{2}x.$$



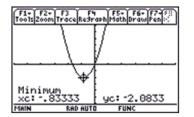
Question 14

$$\frac{x^2}{y} = 1, y = x^2$$

Substitute $y = x^2$ into p
 $p = 5x + 3y = 5x + 3x^2$
$$\frac{dp}{dx} = 5 + 6x = 0 \text{ or } x = -\frac{b}{2a} = -\frac{5}{6}$$

 $x = -\frac{5}{6}$

p has a minimum value when $x = -\frac{5}{6}$



Answer E

$$x = \sqrt{(1-t)}$$

$$v = \frac{dx}{dt} = -\frac{1}{2\sqrt{(1-t)}}$$

$$a = \frac{dv}{dt} = -\frac{1}{4(1-t)^{\frac{3}{2}}}$$

$$a = -\frac{1}{4x^{3}}$$

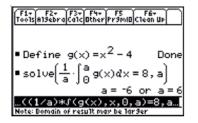
 $\left| \begin{array}{c} f_{1}^{1} \\ f_{0}^{1} \\ f_{0}^{1$

Question 16

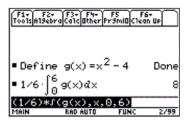
average value $= \frac{1}{b-a} \int_{a}^{b} g(x) dx$ Apply the formula to this case $8 = \frac{1}{a-0} \int_{0}^{a} (x^2 - 4) dx$ Solving for *a* gives the result

a = 6

Reject the negative solution because the domain of g requires that a > 0.



OR

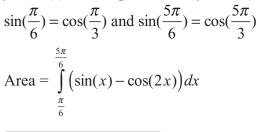


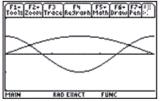
Answer B



7

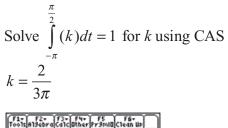
 $y = \sin(x)$ is the top curve and $y = \cos(2x)$ is the bottom curve.

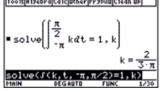




Question 18

For a probability density function, $\int_{0}^{\infty} f(t) dt = 1$.





OR

$$0 + \int_{-\pi}^{\pi/2} k \, dt = 1$$
$$\left[kt\right]_{-\pi}^{\pi/2} = 1$$
$$\left[\frac{\pi}{2}k - (-\pi k)\right] = 1$$
$$\frac{3\pi}{2}k = 1$$
$$k = \frac{2}{3\pi}$$

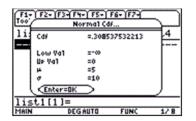
Answer C

Let *X* be the percentage increase in the value of stocks.

 $X \sim N(\mu = 5, \sigma^2 = 10^2)$

Stocks decrease in value if X < 0.

$$\Pr(X \le 0) \approx 0.31$$



Question 20

Answer B

Answer A

This is a problem of selection without replacement. Let *S* denote dialling Stewart's phone number. $Pr(S', S', S) = \frac{7}{8} \times \frac{6}{7} \times \frac{1}{6}$

Note from this pattern that the probability that the first, second, third, ..., eighth number

dialled is Stewart's number, is $\frac{1}{8}$ each time.

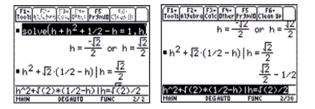
Answer B

Question 21

 $\sum_{k=1}^{n} p(x) = 1$ $h + h^{2} + \frac{1}{2} - h = 1$ $h^{2} = \frac{1}{2}$ $h = \frac{1}{\sqrt{2}} \quad \text{(Reject negative solution as } h \in [0,1]\text{))}$ $\mu = \sum_{k=1}^{n} x p(x)$ $= 0 + \left(1 \times \left(\frac{1}{\sqrt{2}}\right)^{2}\right) + \left(\sqrt{2} \times \left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right)\right)$ $= \frac{1}{2} + \frac{\sqrt{2}}{2} - 1$ $= \frac{\sqrt{2}}{2} - \frac{1}{2}$ $= \frac{\sqrt{2} - 1}{2}$

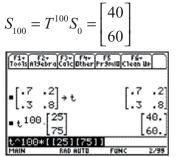
OR

Use CAS



Method 1

Consider, say, the 100th state.



Method 2

The cardinal number of elements in this case is 100. The steady state matrix will therefore be of $\begin{bmatrix} 100 - r \end{bmatrix}$

the form $\begin{bmatrix} 100 - x \\ x \end{bmatrix}$. When the steady-state has been reached, $T \times \begin{bmatrix} 100 - x \\ x \end{bmatrix} = \begin{bmatrix} 100 - x \\ x \end{bmatrix}$.

Solving the matrix equation for x gives the result x = 60, 100 - x = 40

F1+ F2+ F3+ F4+ ToolsAl9ebraCalcather Pr	FS F6+ 9millClean UP
■ [.7 .2] → t	[.7 .2]
[.3 .8] ∎[^{100 - ×}]→s	[100 - ×]
■ solve(t·s = s, x)	[×]
Solve(t*s=s,x) MAIN RAD AUTO	FUNC 4/99

Method 3

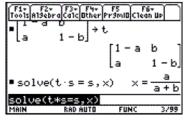
If Method 2 above is applied to a general

transition matrix, $T = \begin{bmatrix} 1-a & b \\ a & 1-b \end{bmatrix}$ and the steady state matrix is written as the proportions

 $\begin{bmatrix} 1-x \\ x \end{bmatrix}$, the solution is $x = \frac{a}{a+b} = \frac{0.3}{0.3+0.2} = 0.6.$

As a **proportion**, the steady-state matrix is $\begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$. However, there are 100 elements.

Using the cardinal numbers, the steady-state matrix is $\begin{vmatrix} 40 \\ 60 \end{vmatrix}$.



The Mathematical Association of Victoria MATHEMATICAL METHODS (CAS) 2008 Trial written examination 2: Solutions – Extended Answer Questions

SECTION 2 Question 1 a. Solve $h^2 + \left(\frac{b}{2}\right)^2 = b^2$ for h $h = \frac{\sqrt{3}b}{2}$ [1M] SA = area of the 2 triangles + 3 rectanglesbh + 3lb = 120 $\frac{\sqrt{3}b^2}{2} + 3lb = 120$ [1M] $l = \frac{240 - \sqrt{3}b^2}{6b}, \ 3 \le b \le 9$ F1+ F2+ F3+ F4+ F5 F6+ Toolslangebra Calc Other PromiDiclean UP ∎ solve • solve $\left(h^2 + \left(\frac{b}{2}\right)^2 = b^2, h\right)$ olve(J **b.** V = area of the triangle \times length [1M] $=\frac{b}{2}\times\frac{\sqrt{3}b}{2}\times\frac{240-\sqrt{3}b^2}{6b}$ [1M] $=\frac{b(80\sqrt{3}-b^2)}{8}, 3 \le b \le 9$ as required F1+ F2+ F3+ F4+ F5 F6+ noiseisebra Calcather Promit Clean Up -(b²·Ј3 - 240) 80.13 2*5(3)b/2* c. (6.80, 78.48) 60 (9, 64.76) (3, 48.59) 40 20 ģ 10 11 Correct shape [1A]

Correct turning point (6.80, 78.48) [1A]

Correct end points with closed circles (3, 48.59) and (9, 64.76) [1A]

d. The maximum occurs at the turning point not an end point.

$$\frac{dV}{db} = 10\sqrt{3} - \frac{3b^2}{8}$$

Solve $10\sqrt{3} - \frac{3b^2}{8} = 0$ for b [1M]

$$b = \frac{4\sqrt{5}}{\sqrt[4]{3}} \text{ as requ}$$
[1A]

$$h = \frac{\sqrt{3}}{2}b = 2\sqrt{5}\sqrt[4]{3}\,\mathrm{cm}$$
 [1A]

$$l = \frac{240 - \sqrt{3}b^2}{6b} = \frac{4\sqrt{5}}{3^{\frac{3}{4}}} \text{ cm}$$
 [1A]

$$\begin{array}{c} \hline \begin{array}{c} \hline f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} & f_{5}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} & f_{5}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} & f_{5}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} & f_{5}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} \\ \hline f_{oot} \begin{bmatrix} f_{1}^{*} & f_{2}^{*} & f_{3}^{*} & f_{4}^{*} \\ \hline f_{0}^{*} & f_{1}^{*} & f_{2}^{*} \\ \hline f_{0}^{*} & f_{1}^{*} & f_{1}^{*} & f_{1}^{*} \\ \hline f_{0}^{*} & f_{1}^{*} & f_{1}^{*}$$

...(b^2-80*1(3))/8,b,3,9)

e.
$$V = \frac{b(80\sqrt{3} - b^2)}{8} = \frac{80\sqrt{5}}{3^{\frac{3}{4}}} \text{ cm}^3$$
 [1A]

$$\int \frac{f_{0}}{f_{0}} \frac{f$$

a.
$$N(t) = 200e^{-mt}$$

Solve for m , $N(1) = 140$
 $140 = 200 e^{-m}$
 $m = -\log_e \left(\frac{7}{10}\right)$
 $m = \log_e \left(\frac{10}{7}\right)$, as required. [1M]

Checking with CAS

$$\begin{array}{c} \hline \textbf{F1} & \textbf{F2} & \textbf{F3} & \textbf{F4} & \textbf{F5} \\ \hline \textbf{F0} & \textbf{F3} & \textbf{F4} & \textbf{F5} & \textbf{F6} & \textbf{F6} \\ \hline \textbf{F0} & \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} & \textbf{F1} & \textbf{F1} \\ \hline \textbf{F1} & \textbf{F1} \\ \hline$$

b. percentage
$$=\frac{n(0.595)}{200} \times \frac{100}{1}$$
 [1M]

$$= 100 \times e^{-\log_e \left(\frac{10}{7}\right) \times 0.595}$$

= 81% [1A]

F1- F Tools A19	2+ F3+ F4+ ebraCalcOtherP	FS F6 r9mi0(Clear	ν.up
■ 4 i = 2i	00·e ^{-m·t} 1	m = -1n(7/10) Done
<u>n(.5</u>	95) 9	80	.8785
n(.595 MBIN	5)/200*100 Red auto	FUNC	2/99

=

[1M]

[1A]

c.
$$N(t) = 100e^{-mt}$$

To find m, solve $N\left(\frac{1}{2}\right) = 80$
 $80 = 100e^{-\frac{1}{2}m}$
 $m = 2\log_e\left(\frac{5}{4}\right)$

Define $n(t) = 100 \cdot e^{-m \cdot t}$
 $n = 2 \cdot 1n(5 \times 4)$

Solve($n(1/2) = 80, m$)
 $m = 2 \cdot 1n(5 \times 4)$

Solve($n(1/2) = 80, m$)
 $m = 2 \cdot 1n(5 \times 4)$

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 $m = 2 \cdot 1n(5 \times 4)$
Solve($n(1/2) = 80, m$)
 $m = 2 \cdot 1n(5 \times 4)$
Solve($n(1/2) = 80, m$)

To find when 75% of words are replaced,

2/99

$$25 = 100e^{-2\log_e\left(\frac{5}{4}\right) \times t}$$

$$t = \frac{\log_e\left(2\right)}{\log_e\left(\frac{5}{4}\right)}$$
[1M]

t = 3.106

It will take 3106 years for 75% replacement.

F1+ F2+ F3+ F4+ F5 ToolsAlgebraCalcOtherPrgmiOClean Up Dor solve(n(t) = 25, t) solve(n(t) = 25, 062 solve(n(t)=25,t)

16

d. To find
$$a, B(0) = 15$$

$$\frac{a}{1+3e^{0}} = 15$$

$$a = 15 \times 4$$

$$B(11) = 22$$

$$\frac{60}{1+3e^{-k \times 11}} = 22$$
Solve for k

$$k = -\frac{1}{11}\log_{e}\left(\frac{19}{33}\right) = \frac{1}{11}\log_{e}\left(\frac{33}{19}\right)$$
[1M]
$$\frac{f(x)=h(x) = f(x) = f(x) = 1}{1+3 \cdot e^{-k \cdot t}}$$

$$= solve(b(11) = 22, b)$$

$$k = -\frac{1n(19 \cdot 53)}{11}$$
Solve for $t, B(t) = 40$

$$t \approx 35.70$$
The buzz word count achieved at 36 months.
[1A]

e. For a probability density function,

FUN

7009

1/99

solve(b(t) = 40, t)

AUTO

solve(b(t)=40,t)

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

Solve for k , $\int_{-\infty}^{\infty} k e^{-\frac{t}{200}} dt = 1$
 $k = 0.005$, as⁰ required
$$\int_{0}^{\frac{1}{1001s[n136bra]ColeDither[pr3mit](16an lup]}} \int_{0}^{\infty} \left(\frac{-t}{k \cdot e^{-200}} \right)_{at} 200 \cdot k}$$

= solve(200 · k = 1, k)
= solve(200 · k = 1, k)
= ken auto Func 2795

[1M]

f. $\mu = \int_{-\infty}^{\infty} x f(x) dx$ for a probability density function

$$\mu = 0.005 \int_{0}^{\infty} \left(t \, e^{-\frac{t}{200}} \right) dt$$

$$= 200$$
[1M]

F1+ F2+ F3+ F4+ F5 ToolsAlgebra(calc)Other Pr3miD(clean Up	
Define $f(t) = .005 \cdot e^{\frac{-t}{200}}$	
Done	
$= \int_0^\infty (t \cdot f(t)) dt \qquad 200.$	
∫(t*f(t),t,0,∞) MAIN RADAUTO FUNC 2/99	

g.
$$\Pr(T > 300) = 0.005 \int_{0}^{\infty} e^{-\frac{t}{200}} dt$$
 [1M]
 $\Pr(T > 300) = 0.223$ ³⁰⁰ [1A]

$$\Pr(T > 300) = 0.223^{300}$$

F1+ F2+ F3+ F4+ F5 ToolsAl9ebraCalcOtherPr9r	niBCTean UP
Define f(t)=.005	-t 1.e ⁻²⁰⁰
- Der Ine 1(0) - 1000	Done
■∫ ₃₀₀ f(t)at	.22313
∫(f(t),t,300,∞) MAIN RAD AUTO	FUNC 2/99

h. To find the median, *m*, solve

$$0.005 \int_{0}^{m} e^{-\frac{t}{200}} dx = \frac{1}{2}$$
 [1M]

$$m = 200 \log_{0}(2)$$

F1- F2- F3- F4- F5 F6- UP Tools Algebra Calc Other Primil Clean UP 200 ■Define f(t) .005 Done ■solve(∫₈ f(t)dt = 1/2, m m = 200 · 1n(..lue(f(t),t,0,m)=1/2,m)

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Question 3

a.
$$X \sim N(\mu = 712, \sigma^2 = 20^2)$$

Pr(X < 700) = 0.274 [1M]

[1A]

The proportion of cartons less than 700 g is 0.274.



b.
$$X \sim N(\mu = 712, \sigma^2 = 20^2)$$

Area = 0.4
Area = 0.6
m 712

.274253 ,695,

.7207

7

$$Pr(X > m) = 0.6$$

$$Pr(X \le m) = 0.4$$
(1M]
$$m = 706.9$$
60% of cartons exceed 706.9 grams.
[1A]

F1+ F2+ F3+F4+ F5+ F6+ F7+ se Norn 11 inverse =706.533 Ared # =.4 =712 =20 t2[1]= RAD AUTO FUNC 2/6

at.normcdf(

...df(-**,695,712,20)/ans(1) Main RAD AUTO FUNC 2/

27425306456975

c.
$$\Pr(X < 695 | X < 700)$$

$$= \frac{\Pr(X < 695 \cup X < 700)}{\Pr(X < 700)}$$

$$= \frac{\Pr(X < 695)}{\Pr(X < 700)}$$
[1M]
 $\Pr(X < 695 | X < 700) = 0.721$
[1A]

$$\frac{\Pr(X < 695 | X < 700) = 0.721}{\Pr(X < 695 | X < 700) = 0.721}$$

$Y \sim Bi(8, 0.274)$	[1M]
(The probability of a carton being underweight was found in part a . to be 0.274)	
$\Pr(Y \ge 3) = 0.383$	[1A]
F1+ F2+ F3+ F4+ F5+ F6+ F7+ Too Binomial Cdf	
11: Cdf =.382636	

(Enter=OK)

e. The transition matrix is as follows:

	Large today	Medium today
Large Tomorrow	0.45	0.75
Medium Tomorrow	0.55	0.25

The probability that Jamie orders medium-sized today and large-sized tomorrow is 0.75. [1A]

f. i.
$$Pr(L,L,M) = 0.75 \times 0.45 \times 0.55$$

 $Pr(L,L,M) = 0.1856$ [1A]

ii. The *n*th state is given by $S_n = T^n \times S_0$

This Tuesday,
$$S_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \leftarrow \text{Large}$$

 $\leftarrow \text{Medium}$

Next Tuesday, $S_7 = T^7 \times S_0$

$$S_7 = \begin{bmatrix} 0.5770\\ 0.4230 \end{bmatrix}$$
[1M]

The probability of large-sized is 0.5770.

 $\begin{array}{c|c} \hline f_{1}^{1} & f_{2}^{2} & f_{3}^{2} & f_{4}^{1} & f_{5}^{5} & f_{6}^{1} & f_{5}^{5} \\ \hline f_{0} & f_{1}^{1} & f_{2}^{2} & f_{3}^{2} & f_{4}^{1} & f_{5}^{5} & f_{6}^{1} \\ \hline \vdots & 55 & .25 \end{bmatrix} \xrightarrow{\uparrow} \underbrace{\vdots & 55 & .25} \\ \hline \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \xrightarrow{\uparrow} s & \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \\ \hline t^{7} \cdot s & \begin{bmatrix} .577049 \\ .422951 \\ \hline t^{7} \ast s \\ \hline \\ \hline MAIN & RAD AUTD & FUNC & 3/99 \end{array}$

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$2\sin(x)\cos(x) = \frac{1}{2}$ Using CAS	
$x = \frac{(12n+5)\pi}{12}$	
$x = \frac{(12n+1)\pi}{12} \text{ where}$	$n \in \mathbb{Z}$
F1+ F2+ F2+ F4+ F5 ToolsAlgebraCalcOtherPr3niOClean Up	F1+ F2- F3- F4+ F5 Tools Aldebra Calc Other PromiD Clean Up
• solve(2 · sin(x) · cos(x) = 1/3) $x = \frac{(12 \cdot 0n2 + 5) \cdot \pi}{12} \text{ or } x = \frac{1}{2}$ Solve(2*sin(x)*cos(x)=1/2)	solve(2*sin(x)*cos(x)=1/2
Bolve(2*sin(x)*cos(x)=1/2 MAIN RAD AUTO FUNC 1/30	solve(2*sin(x)*cos(x)=1/2 MAIN RAD AUTO FUNC 1/30

OR

$$2\sin(x)\cos(x) = \frac{1}{2}$$

$$\sin(2x) = \frac{1}{2}$$

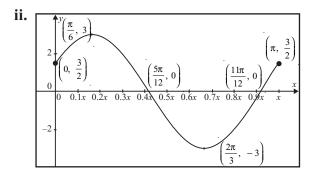
$$x = \dots, \frac{\pi}{12}, \frac{5\pi}{12}, \dots$$

$$x = \frac{\pi}{12} + n\pi \text{ or } x = \frac{5\pi}{12} + n\pi, \text{ where } n \in \mathbb{Z}$$

$$\boxed{\frac{f_{12}}{f_{12}} + \frac{f_{22}}{f_{12}} + \frac{f_{22}}{f_{12}}$$

[1A]

b. i. A dilation of a factor of 3 from the *x*-axis and a translation of $\frac{\pi}{12}$ units to the left. (The order does not matter.)



Correct shape

Correct coordinates for the *x*-intercepts

$$\left(\frac{5\pi}{12}, 0\right)$$
 and $\left(\frac{11\pi}{12}, 0\right)$ [1A]

Correct end points with closed circles

$$\left(0, \frac{3}{2}\right)$$
 and $\left(\pi, \frac{3}{2}\right)$ [1A]

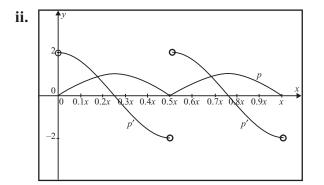
Correct turning points

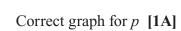
$$\left(\frac{\pi}{6}, 3\right)$$
 and $\left(\frac{2\pi}{3}, -3\right)$ [1A]

[1A] [1A]

c. i.
$$p'(x) = \begin{cases} 2\cos(2x) \text{ if } \sin(2x) \ge 0\\ -2\cos(2x) \text{ if } \sin(2x) < 0 \end{cases}$$

F1+ F2+ F3+ F4+ F5 ToolsAl3ebraCalcOtherPr3miDClean UP [|sin(2·×)|] d× a(abs(sin(2x)),x)





Correct graph for p' [1A]

x-intercepts of p':
$$\frac{\pi}{4}, \frac{3\pi}{4}$$

Open circles [1A]

iii.
$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin(2x) - 2\cos(2x)) dx$$
 [1M]

$$= 1.5 \text{ unit}^2$$

π

F1+ F2+ F3+ F4+ F5 F6+ ToolsAlgebraCalcOtherPrgmIDClean U π 2 π . (sin(2 $\cos(2 \cdot x))d$ 2 d S(sin(2x)-2cos