The Mathematical Association of Victoria

MATHEMATICAL METHODS and MATHEMATICAL METHODS (CAS)

Trial written examination 1

2008

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2008 Mathematical Methods and Mathematical Methods (CAS) Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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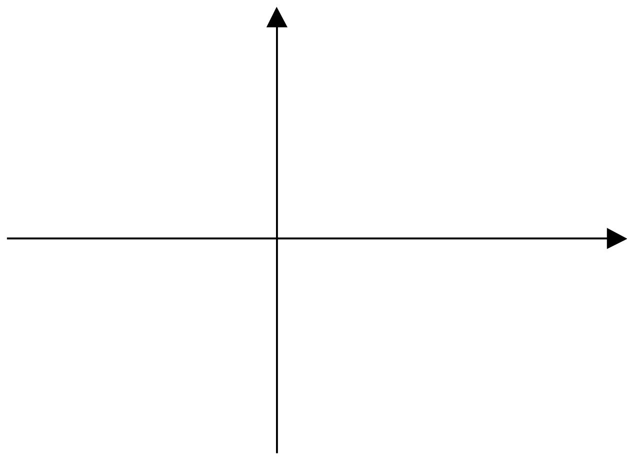
Working space

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. If $f : R \to R$, where $f(x) = |x - 2| + 1$, sketch the graph of f on the set of axes below, clearly labelling key features with their coordinates.



2 marks

b. Solve $f(x) = 2$ for x.

2 marks

c. Explain why f^{-1} does not exist.

1 mark

Question 2

a. Solve the equation $3\log_2(2x) - 2\log_2(x) = 1$.

2 marks

b. The unhealed area, A cm, of a particular wound, t days after it was sustained, is given by the function $A(t) = 8e^{-\frac{t}{10}}$, $t \geq 0$. According to this model, what is the time required for the area of unhealed wound to be halved?

2 marks

Question 3

a. Let $y = \dfrac{\tan(x)}{x}$. Find $\dfrac{dy}{dx}$.

2 marks

b. If $f(x) = \sin(x)$ and $g(x) = \log_e(x)$, find the value of $\dfrac{d}{dx}\left(f\left(g\left(1\right)\right)\right)$.

2 marks

Question 4

Consider the function $f : [0, a] \rightarrow R, f(x) = 2\cos(\pi x) - 1$.

a. If the domain of f covers exactly **one period** of the graph of $y = f(x)$, show that $a = 2$.

1 mark

b. Find $\{x : f(2x) = 0, x \in [0, 2]\}$.

2 marks

Question 5

It is known that $\frac{3}{4}$ of the students who sat a particular mathematics exam used a calculator manufactured by the Tasmania Implements company. A random sample of three students was selected.

a. What is the probability that exactly two of the students used a Tasmania Implements calculator in the exam?

2 marks

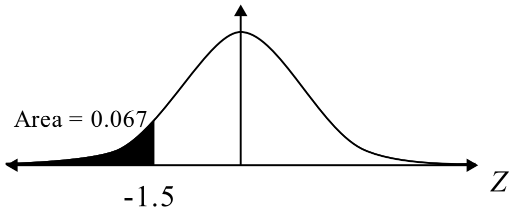
b. Given that at least one of the students in the sample used a Tasmania Implements calculator, what is the probability that exactly two students used a Tasmania Implements calculator?

2 marks

Question 6

a. When used in a Casino Notepad calculator, the life of Neveready batteries is a normally distributed random variable, X, with a mean of 240 hours and a standard deviation of 20 hours.

The area under the curve of the standard normal distribution for $Z < -1.5$, is shown. Correct to three decimal places, this area is equal to 0.067.



Find, correct to three decimal places, $\Pr(240 < X < 270)$.

2 marks

b. The probability density function for a continuous random variable, X, is given by

$$f(x) = \begin{cases} x^{\frac{4}{3}} & \text{for } 0 \le x \le 1 \\ x & \text{for } 1 < x \le a \\ 0 & \text{otherwise} \end{cases}$$

Find the value of a, where a is a real constant.

4 marks

Question 7

A pile of sand at a quarry forms a right circular cone. Sand is being added at a constant rate of $2\text{m}^3/\text{s}$.
The growing pile is shaped such that the radius of its base is equal to half of its height.

a. What is the volume of the pile when the height of the pile is 6 m?

1 mark

b. At what rate is the height of the pile increasing when the pile is 6 m high?

3 marks

Question 8

Consider the function with the rule $f(x) = \sin^3(x)$.

a. Find $f'(x)$.

2 marks

b. **Hence** find:

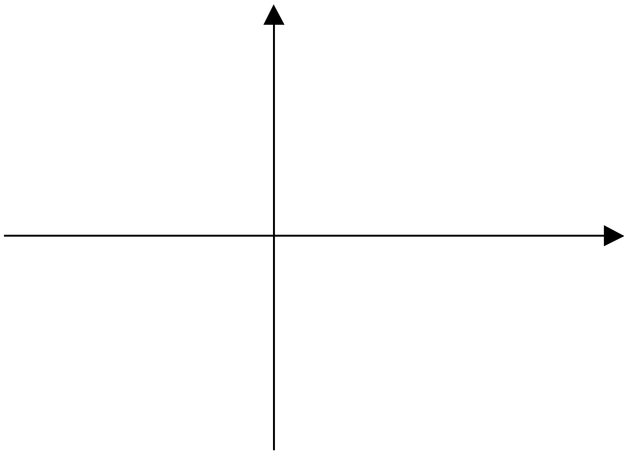
i. $\int \left(\cos(x)\sin^2(x)\right)dx$

ii. the gradient of the normal to the graph of $y = f(x)$ at $x = \frac{\pi}{4}$.

1 + 2 = 3 marks

Question 9

a. Sketch the graph of $f : [0, 4] \to R$, where $f(x) = -3\left(\frac{1}{2}x - 1\right)^5 + 3$ on the axes below. Label the axes intercepts with their coordinates.



2 marks

b. Find the area bounded by the graph of f and the coordinate axes.

3 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS and MATHEMATICAL METHODS (CAS)

Trial written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time This formula sheet is provided for your reference

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a\ \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a\ \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		**mean**	**variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET