MAV Trial Examination Papers 2009
Mathematical Methods Examination 2
SOLUTIONS

SECTION 1 - ANSWERS

1. E	**2. D**	**3. A**	**4. B**	**5. C**	**6. B**
7. E	**8. A**	**9. E**	**10. A**	**11. C**	**12. E**
13. C	**14. A**	**15. A**	**16. B**	**17. A**	**18. B**
19. B	**20. D**	**21. D**	**22. A**		

SECTION 1 - SOLUTIONS

Question 1 **Answer E**

$f:[-4,2)\to R, f(x)=(x-1)^2$

The local minimum occurs at the turning point
(1, 0) and the endpoint maximum occurs at $(-4, f(-4))=(-4,25)$.
Hence the range is [0, 25].

Question 2 **Answer D**

The graph of g is a transformation of the graph of $y=|x|$, as follows.

Reflection in the x-axis: $y=-|x|$

Translation 2 units right: $y=-|x-2|$

Translation 3 units up: $y=-|x-2|+3$

The rule is $g(x)=-|x-2|+3$

Question 3 **Answer A**

$$\frac{2x-1}{x+3}=\frac{2(x+3)-7}{x+3}$$
$$=\frac{2(x+3)}{x+3}-\frac{7}{x+3}$$
$$=2-\frac{7}{x+3}$$
$$=2-7f(x+3)$$

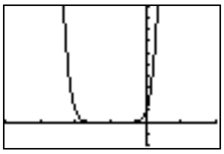
Alternatively, use the division algorithm.

Question 4 **Answer B**

From the graph of the quartic function the coefficient of x^4 is negative and the single roots are $x=a$ and $x=c$. The corresponding factors are $(x-a)$ and $(x-c)$. The double root is $(x-b)$ (notwithstanding the fact that a and b are negative numbers).
A possible rule is
$f(x)=-(x-a)(x-c)(x-b)^2$

Question 5 **Answer C**

From the graph of $f(x)=(2x+1)^8=\left(2\left(x+\frac{1}{2}\right)\right)^8$, note that *f* will be a one-to-one function for $x \in \left[-\frac{1}{2},\infty\right)$.



Therefore, *f* will have an inverse function when $m \geq -\frac{1}{2}$.

Question 6 **Answer B**

The domain of *f* is $R\setminus\{0\}$ and the domain of *g* is *R*. Hence, the domain of *f* − *g* is where the dom *f* ∩ dom *g* which is $R\setminus\{0\}$.

Question 7 **Answer E**

$\cos(3x)=-\frac{1}{2}$

$3x=\frac{2\pi}{3},\frac{4\pi}{3},...$

$x=\frac{2\pi}{9},\frac{4\pi}{9},\frac{8\pi}{9}$ as the period is $\frac{2\pi}{3}$.

The sum is $\frac{14\pi}{9}$.

Question 8 **Answer A**

$y=2\sin(3x-1)=2\sin\left(3\left(x-\frac{1}{3}\right)\right)$

There is a dilation of a factor of 2 from the *x*-axis, a dilation of a factor of $\frac{1}{3}$ from the *y*-axis and a translation of $\frac{1}{3}$ of a unit to the right.

Question 9 **Answer E**

$\log_e(2x)-\sqrt{x^3}=\log_e(2x)-x^{\frac{3}{2}}$

$$\frac{d}{dx}\left(\log_e(2x)-x^{\frac{3}{2}}\right)=\frac{2}{2x}-\frac{3x^{\frac{1}{2}}}{2}$$

$$=\frac{1}{x}-\frac{3\sqrt{x}}{2}$$

Question 10 **Answer A**

Using the chain rule,

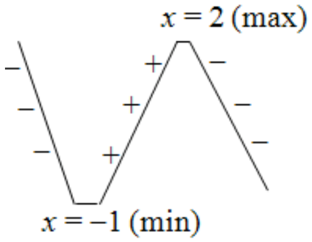
$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$, where $y=g(x)$ and $u=f(x)$

$$\frac{d\left(g(x)\right)}{dx}=\frac{d\left(-\cos\left(f(x)\right)\right)}{d\left(f(x)\right)}\times\frac{d\left(f(x)\right)}{dx}$$

$$=\sin\left(f(x)\right)\times f'(x)$$

Question 11 **Answer C**

Consider the sign of the gradient.



Local min. at $x = -1$ and max. at $x = 2$.

Question 12 **Answer E**

The derivative of f is not defined at $x = 1$ as there is a cusp at $x = 1$.

f' exists for $x \in R \setminus \{1\}$.

Question 13 **Answer C**

The area is given by

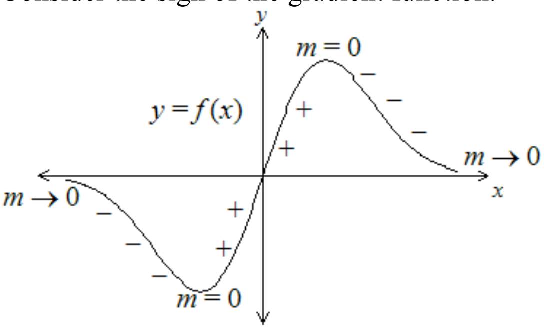
$$\int_{-2}^{0} f(x)\,dx - \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$$

Note that $\int_{1}^{0} f(x)\,dx = -\int_{0}^{1} f(x)\,dx$

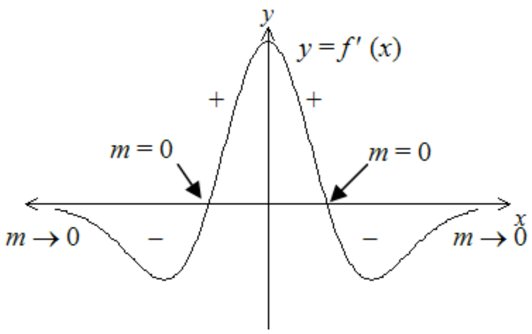
The equivalent option is $\int_{-2}^{0} f(x)\,dx + \int_{1}^{0} f(x)\,dx + \int_{1}^{2} f(x)\,dx$

Question 14 **Answer A**

Consider the sign of the gradient function.



Option **A** shows the graph of the gradient function with this sign profile.



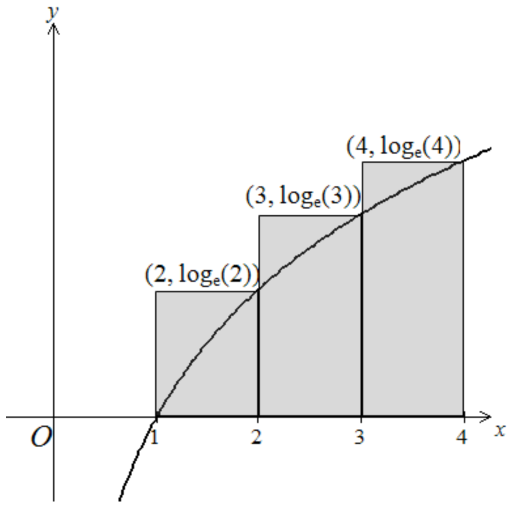
Question 15 **Answer A**

$$\int_{-2}^{1}\left(2x-\frac{u(x)}{2}\right)dx=\int_{-2}^{1}(2x)\,dx-\frac{1}{2}\int_{-2}^{1}u(x)\,dx$$
$$=\left[x^2\right]_{-2}^{1}-\frac{1}{2}\times 8$$
$$=[1-4]-4$$
$$=-7$$

Question 16 **Answer B**

The area of each rectangle = length × width

$$\text{Area }=\log_e(2)\times 1+\log_e(3)\times 1+\log_e(4)\times 1$$
$$=\log_e(2\times 3\times 4)$$
$$=\log_e(24)$$



Question 17 **Answer A**

The average rate of change is given by the average gradient of the graph of h.

$$\text{av. rate}=\frac{h(\pi)-h\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$$
$$=\frac{(\sin(\pi)+\cos(\pi))-(\sin(\pi/2)+\cos(\pi/2))}{\pi/2}$$
$$=\frac{-2}{\pi/2}$$
$$=\frac{-4}{\pi}$$

Question 18 **Answer B**

Let R represent the probability it will rain.

The probability it will rain on Wednesday $=RRR+RR'R$

$=0.35\times 0.35+0.65\times 0.22$

$= 0.2655$

Question 19 **Answer B**

The median $=\frac{3+5}{2}=4$

Question 20 **Answer D**

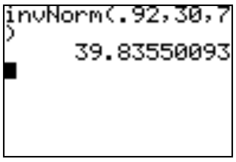
$$\Pr(X=6\mid X\geq 3)=\frac{\Pr(X=6)}{\Pr(X\geq 3)}$$
$$=\frac{0.15}{0.6}$$
$$=0.25$$

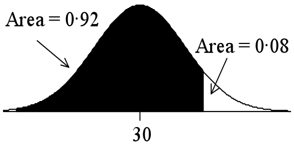
Question 21 **Answer D**

Let X represent the Exam Result

$X \sim N(30, 49)$

Using the inverse normal distribution, 40 is the cut off score.



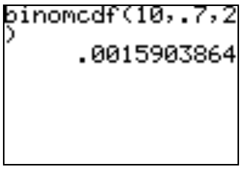


Question 22 **Answer A**

Let X represent being well

$X \sim \text{Bi}(10, 0.7)$

$\Pr(X \le 2) = 0.0016$ correct to 4 decimal places



END OF SECTION 1 SOLUTIONS

SECTION 2 - SOLUTIONS

Question 1

a. i A is the amplitude

$A = 10$ cm **[1A]**

B is the vertical translation

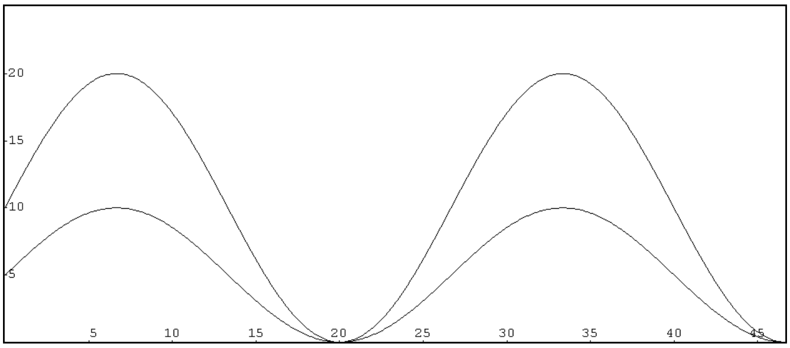
$B = 10$ cm **[1A]**

ii. $P = \frac{2\pi}{n} = \frac{4}{3} \times 20 = \frac{80}{3}$ **[1M]**

$n = \frac{6\pi}{80}$

$= \frac{3\pi}{40}$ as required

b. i.



Correct shape **[1A]**

Coordinates labelled **[1A]**

$(0, 5), \left(\frac{20}{3}, 10\right), (20, 0), \left(\frac{100}{3}, 10\right), \left(\frac{140}{3}, 0\right)$

ii. $d_b = 5\sin\left(\frac{3\pi}{40}x\right) + 5$ **[1A]**

iii. The areas are the same. **[1A]**

iv. $\frac{3}{2}\int_0^{\frac{140}{3}} d\ dx$

Must have dx **[1A]**

Could use $\frac{3}{2}d$ or $3(d - d_b)$ or $3d_b$.

$= \frac{3}{2}\int_0^{\frac{140}{3}} \left(10\sin\left(\frac{3\pi}{40}x\right) + 10\right)dx$

$= 763.66$ cm^3 correct to 2 decimal places **[1A]**

c. $d_b = 5\sin\left(\frac{3\pi}{40}\left(\frac{20}{3} - 2\right)\right) + 5$ **[1H]**

The height is 9.46 cm correct to two decimal places. **[1A]**

Question 2

Consider the function $f: R \to R, f(x) = 2xe^{-\frac{x}{2}}$.

a. Using the product rule **[1M]**

$$f'(x) = 2e^{-\frac{x}{2}} - xe^{-\frac{x}{2}}$$

$$= (2-x)e^{-\frac{x}{2}}$$ **[1A]**

b. For a stationary point, $f'(x) = 0$.

$$(2-x)e^{-\frac{x}{2}} = 0$$ **[1M]**

Using the null factor law, noting that $e^{-\frac{x}{2}} > 0$ for all values of *x*,

$$(2-x) = 0$$

$$x = 2$$ **[1A]**

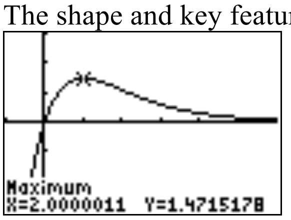
$$f(2) = 2 \times 2e^{-\frac{2}{2}}$$

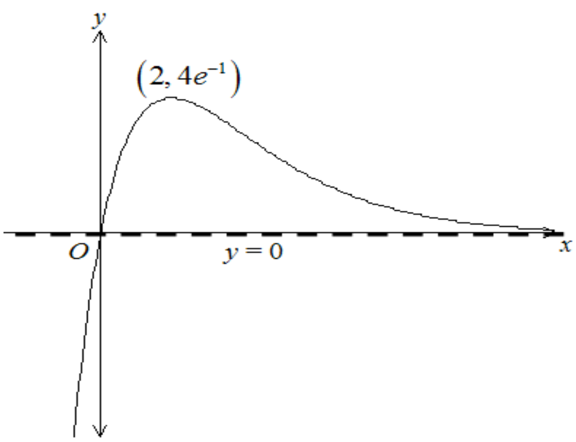
$$= 4e^{-1} = \frac{4}{e}$$

Turning point at $(2, 4e^{-1}) = \left(2, \frac{4}{e}\right)$ **[1A]**

The height is 9.46 cm correct to two decimal places. **[1A]**

c. The shape and key features of the graph may be obtained using a graphical calculator.





Correct shape, crossing at the origin: **[1A]**
Asymptote and turning point labelled: **[1A]**

d. **i.** $f'(1) = (2-1)e^{-\frac{1}{2}}$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

The gradient at $x=1$ is $e^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{e}}$. **[1A]**

ii. $m=\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

$$\frac{f(a)-0}{a-0}=e^{-\frac{1}{2}}$$

$$\frac{2ae^{-\frac{a}{2}}-0}{a-0}=e^{-\frac{1}{2}}$$ **[1M]**

$$2e^{-\frac{a}{2}}=e^{-\frac{1}{2}}$$

$$e^{-\frac{a}{2}}=\frac{e^{-\frac{1}{2}}}{2}$$

$$-\frac{a}{2}=\log_e\left(\frac{e^{-\frac{1}{2}}}{2}\right)$$ **[1M]**

$$-\frac{a}{2}=\log_e\left(e^{-\frac{1}{2}}\right)-\log_e(2)$$

$$-\frac{a}{2}=-\frac{1}{2}-\log_e(2)$$

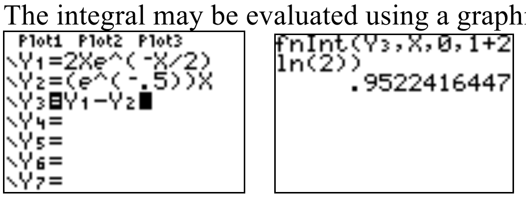
$a=1+2\log_e(2)$, as required **[1A]**

iii. The equation of the line PQ is $y=\left(e^{-\frac{1}{2}}\right)x$

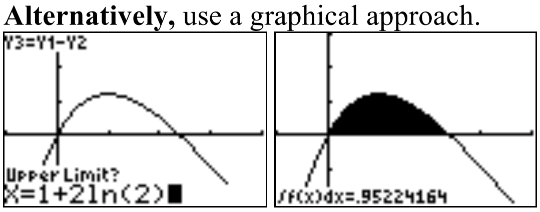
$$\text{Area } = \int_0^{1+2\log_e(2)}\left(\left(2xe^{-\frac{x}{2}}\right)-\left(\left(e^{-\frac{1}{2}}\right)x\right)\right)$$ **[1M]**

$$=0.9522$$ **[1A]**

The integral may be evaluated using a graphical calculator.



Alternatively, use a graphical approach.



e. The gradient of the tangent : $m_T=e^{-\frac{1}{2}}$.

Gradient of normal: $m_N=-\dfrac{1}{m_T}=-e^{\frac{1}{2}}$

Normal at $(1,f(1))=\left(1,2e^{-\frac{1}{2}}\right)$ **[1M]**

The equation of the normal is

$$y-2e^{-\frac{1}{2}}=-e^{\frac{1}{2}}\left(x-1\right)$$

$$y=-e^{\frac{1}{2}}x+e^{\frac{1}{2}}+2e^{-\frac{1}{2}}$$ **[1A]**

Alternatively

$$y=-\sqrt{e}\,x+\sqrt{e}+\frac{2}{\sqrt{e}}$$

Question 3

a. Using Pythagoras' theorem,

$$BD=\sqrt{x^2+6^2}=\sqrt{x^2+36}$$ **[1A]**

b.

i. $\text{time}=\dfrac{\text{distance}}{\text{speed}}$ **[1M]**

Time from A to B = $\dfrac{14-x}{20}$

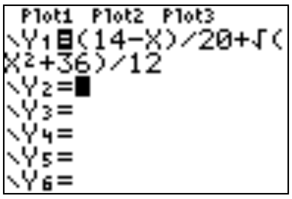
Time from B to D = $\dfrac{\sqrt{x^2+36}}{20}$ **[1M]**

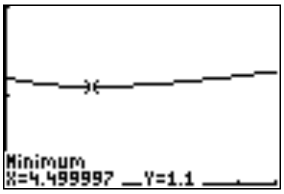
$$T\left(x\right)=\frac{14-x}{20}+\frac{\sqrt{x^2+36}}{12},\ 0\le x\le 14$$ **[1A]**

Using technology to find the coordinates of the local minimum of the graph of T,

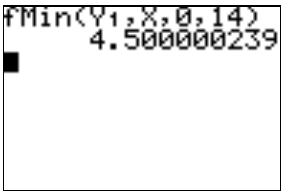
ii. Minimum time occurs when x = 4.5 km. **[1A]**

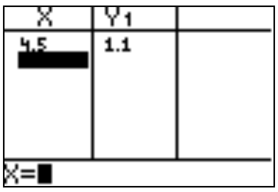
iii. The minimum time is 1.1 hours **[1A]**





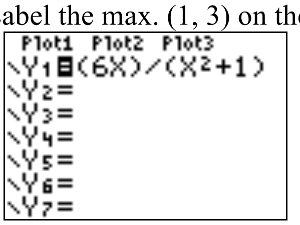
Alternatively

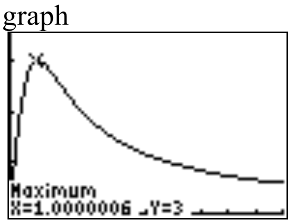




c. **i.** The maximum concentration is 3 units/cm^3 and it occurs 1 hour after the dose is administered. **[2A]**

ii. Label the max. (1, 3) on the graph **[1A]**

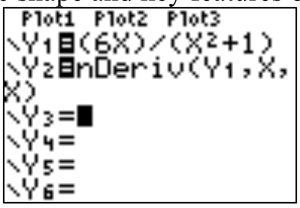


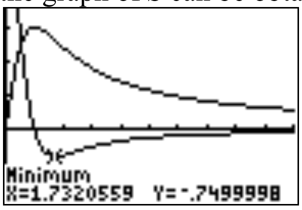


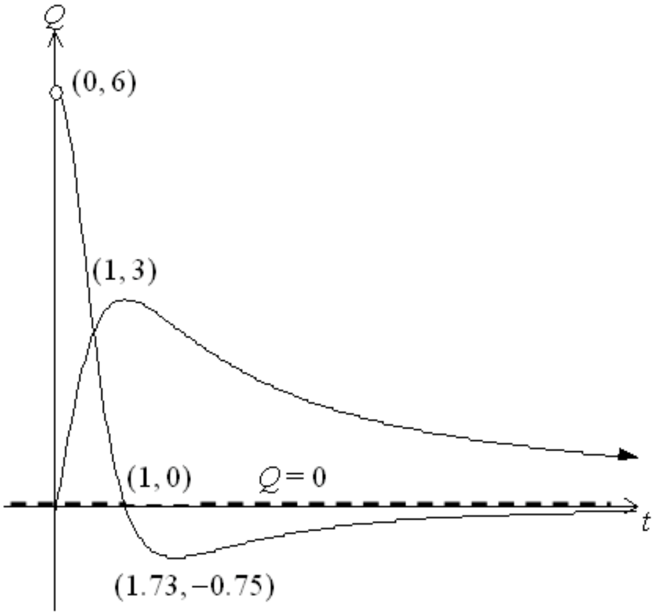
d. $Q(t) > 1.25$ when $0.22 < t < 4.58$. **[1M]**

Pain relief: 4.58 − 0.22 = 4.36 hours **[1A]**

e. The shape and key features of the graph of S can be obtained using a graphical calculator.







Correct shape **[1A]**

Local min. and x-intercept labelled **[1A]**

Asymptote labelled and (0, 6) shown as an "open circle" **[1A]**

Question 4

a. **i.** $\Pr(3\text{ red}) = \frac{10}{13} \times \frac{9}{12} \times \frac{8}{11} = \frac{60}{143}$ **[1A]**

ii. $\Pr(2\text{ green}) = \frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$ **[1A]**

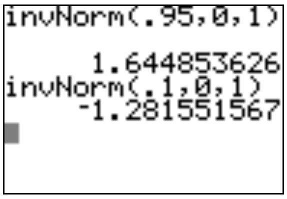
b. **i.** Pr(tea) = 0.1 **[1A]**

ii. Let coffee be c, tea t and nothing n.

Pr(ct) + Pr(tt) + Pr(nt) $= 0.7 \times 0.1 + 0.1 \times 0.3 + 0.2 \times 0.4$ **[1M]**

$= 0.18$ **[1A]**

c. Use the inverse normal distribution, with $\mu = 0$ and $\sigma = 1$.



$\frac{30-\mu}{\sigma} = 1.64485 \ldots(1)$ **[1M]**

$\frac{15-\mu}{\sigma} = -1.28155 \ldots(2)$

$30-\mu = 1.64485\sigma \ldots(1)$

$15-\mu = -1.28155\sigma \ldots(2)$

$(1) - (2)$

$15 = 2.9264\sigma$

$\sigma = 5.1 \text{ g}$ **[1A]**

$30-\mu = 1.64485 \times 5.12575$

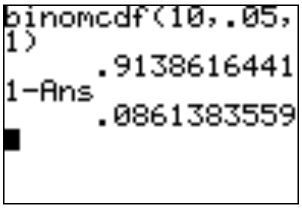
$\mu = 21.6 \text{ g}$ **[1A]**

d. 0.85^4 **[1M]**

≈ 0.5220 **[1A]**

e. Let $Y \sim Bi(10, 0.05)$ **[1M]**

$\Pr(Y \geq 2) \approx 0.0861$ **[1A]**



f. Let $W \sim Bi(n, 0.05)$

$\Pr(W \geq 1) > 0.95$

$1 - \Pr(W = 0) > 0.95$ **[1M]**

$\Pr(W = 0) < 0.05$

$0.95^n < 0.05$ **[1M]**

$n > \frac{\log_{10}(0.05)}{\log_{10}(0.95)} \approx 58.404$

$n = 59$ **[1A]**

END OF SECTION 2 SOLUTIONS