THE SCHOOL FOR EXCELLENCE
UNIT 3 & 4 MATHEMATICAL METHODS 2009
COMPLIMENTARY WRITTEN EXAMINATION 1 - SOLUTIONS

PRINTING SPECIFICATIONS

Please ensure that the paper size on your printer is selected as **A4** and that you select “**None**” under “**Page Scaling**”.

ERRORS AND UPDATES

Please report errors by email (admin@tsfx.com.au).

Errors and updates relating to this examination paper will be posted at www.tsfx.com.au/vic/examupdates

MARKING SCHEME

- ($A4\times\frac{1}{2}\downarrow$) means four answer half-marks rounded **down** to the next integer.
  Rounding occurs at the end of a part of a question.
- M1 = 1 **M**ethod mark.
- **A**1 = 1 **A**nswer mark (it **must** be this or its equivalent).
- **H**1 = 1 consequential mark (**H**is/**H**er mark…correct answer from incorrect statement or slip).

QUESTION 1

(a) $y = x^2 e^{-3x}$

Using the Product Rule: $\frac{dy}{dx} = x^2 \times \frac{d}{dx}(e^{-3x}) + e^{-3x} \times \frac{d}{dx}(x^2)$ **M1**

$= \left(x^2 \times -3e^{-3x}\right) + \left(e^{-3x} \times 2x\right)$ **A1**

$= e^{-3x}(-3x^2 + 2x)$

$= \frac{-3x^2 + 2x}{e^{3x}}$ and so $a = -3$ and $b = 2$ **A(** $2 \times \frac{1}{2} \downarrow$ **)**

(b) $f(x) = \frac{x^2 + 1}{x^2 - 1}$

Using the Quotient Rule: $f'(x) = \frac{(x^2 - 1)2x - (x^2 + 1)2x}{(x^2 - 1)^2}$ **M1**

$= \frac{2x(x^2 - 1 - x^2 - 1)}{(x^2 - 1)^2}$

$= \frac{-4x}{(x^2 - 1)^2}$ **A1**

Therefore, $f'(2) = -\frac{8}{9}$ **H1**

Total = 6 marks

QUESTION 2

(a) $P(x) = x^4 + x^3 + ax^2 - x + b$

P (1) = 0 and so 1 + 1 + *a* – 1 + *b* = 0.

Therefore *a* + *b* = – 1 (i) **A1**

P (– 2) = 12 and so 16 – 8 + 4*a* + 2 + *b* = 12 .

Therefore 4*a* + *b* = 2 (ii) **A1**

Subtracting (i) from (ii) gives 3*a* = 3 and so *a* = 1 as required.

Substituting *a* = 1 into (i) gives 1 + *b* = – 1 and so *b* = – 2 as required.

(b) Knowing that $(x-1)(x+1)$ are factors of $P(x)$ then dividing $P(x)$ by x^2-1 gives

$$\begin{array}{r l}
 & x^2+x+2 \\
x^2-1 & \overline{)x^4+x^3+x^2-x-2} \\
 & \underline{x^4+0x^3-x^2} \\
 & x^3+2x^2-x \\
 & \underline{x^3+0x^2-x} \\
 & 2x^2-2 \\
 & \underline{2x^2-2} \\
 & 0
\end{array}$$ **M1**

The factors of $P(x)$ are $(x-1)(x+1)\,(x^2+x+2)$ **A1**

Total = 4 marks

QUESTION 3

$\log_4(x)-\log_4(x-4)=1$

$\log_4\left(\dfrac{x}{x-4}\right)=\log_4 4$ **M1**

$\dfrac{x}{x-4}=4$ **A1**

$4x-16=x$

$x=\dfrac{16}{3}$

$a=16$ and $b=3$ **A1**

Total = 3 marks

QUESTION 4

(a) $2\pi \div \left(\dfrac{\pi}{8}\right)=16$ hours **A1**

(b) 25° C **A1**

(c) 35° C when t = 8 (8.00 am) **A1**

Total = 3 marks

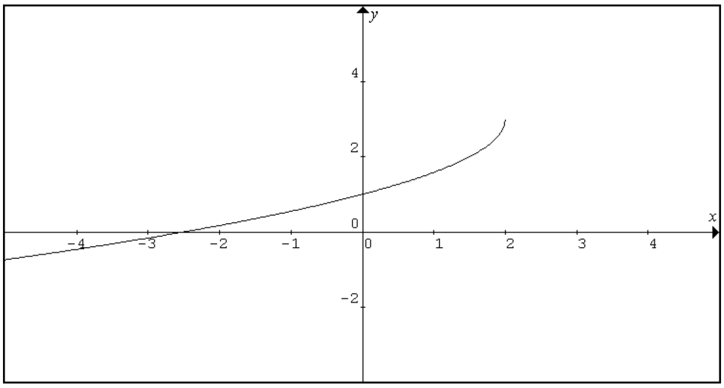
QUESTION 5

(a) Dilation by a factor of either √2 from the *x*-axis **or** by a factor 0.5 from the *y*-axis; Reflection in the *x*-axis; Translations 2 to the left and 3 up

(Order is important with the translations coming last). **A2 (** $4\times\frac{1}{2}\downarrow$ **)**

-1 if incorrect order

(b)



Vertex clearly at (2, 3); *y*-intercept (0, 1); *x*-intercept (−2.5, 0); Shape.

A2 ($4\times\frac{1}{2}\downarrow$ **)**

(c) $f(x) = 3 - \sqrt{(4-2x)}$ **M1**

Interchanging x for y and vice-versa gives $x = 3 - \sqrt{(4-2y)}$

$x - 3 = -\sqrt{(4-2y)}$

Squaring both sides: $4 - 2y = (x-3)^2$ and so $2y = 4 - (x-3)^2$

$f^{-1}(x) = 2 - \frac{1}{2}(x-3)^2$ for $x \in (-\infty, 3]$ **A1 + A1**

Total = 7 marks

QUESTION 6

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 16\pi$ and $\frac{dV}{dr} = 4\pi r^2$ **M1**

$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$

$= 16\pi \div 4\pi r^2$ **H1**

= $\frac{16\pi}{4\pi r^2}$

= $\frac{4}{400}$ at $r = 20$

Answer: 0.01 cm/minute **A1**

Total = 3 marks

QUESTION 7

$E(aX + b) = aE(X) + b$ **M1**

$E(Y) = E(4X - 5) = 4E(X) - 5$
= $4 \times 10 - 5$
$= 35$ **A1**

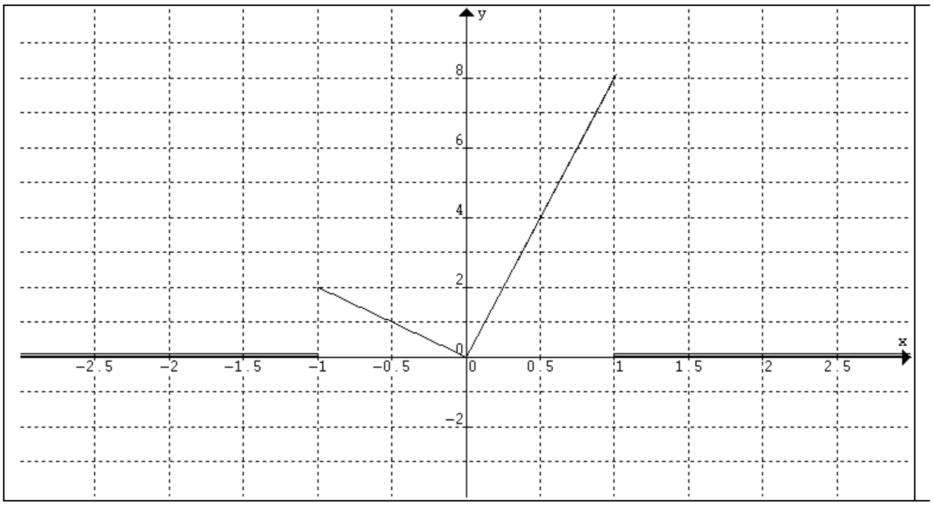
Variance $(aX + b) = a^2$ Variance (X)

Variance (Y) = Variance $(4X - 5)$
= 4^2 Variance(X)
$= 16 \times 2$
$= 32$ **A1**

Total = 3 marks

QUESTION 8

(a)



A1 The points (-1,2) and (1,8) should be filled in and the points (-1,0) and (1,0) empty.

A1 Horizontal lines.

A1 Oblique lines.

(b) The area under the curve must be 1 for a probability density function. There are two triangles whose area needs to be found: $\left(\frac{1}{2}\times 1\times 2k\right)+\left(\frac{1}{2}\times 1\times 8k\right)$ = k + 4k = 5k

Hence 5k = 1 and so k = 0.2 **A1**

(c) The equation of the straight line for the interval [– 1, 0) is y = – 0.4x and the equation of the straight line for the interval [0 , 1] is y = 1.6x. The mean is given by $\int x\times g(x)dx$.

Mean = $\int_{-1}^{0} -0.4x^2dx+\int_{0}^{1} 1.6x^2dx$ **M1 for two "bits"**

$= -\left[\frac{0.4}{3}x^3\right]_{-1}^{0}+\left[\frac{1.6}{3}x^3\right]_{0}^{1}$

$= \left(0+\frac{0.4}{3}\right)+\left(\frac{1.6}{3}-0\right)$

$= \frac{2}{3}$ **A1**

Total = 6 marks

QUESTION 9

(a) If $f(x)=\sqrt{(x+2)}$ then $f'(x)=\dfrac{1}{2\sqrt{(x+2)}}$ **A1**

At $x=-1$, $y=1$ and $f'(-1)=\dfrac{1}{2\sqrt{(-1+2)}}=\dfrac{1}{2}$ **H1**

Equation of tangent: $y-1=\dfrac{1}{2}(x+1)$ and so $2y=x+3$. **A1**

(b) Area required $= \displaystyle\int_{-3}^{-1}\left(\frac{1}{2}x+\frac{3}{2}\right)\mathrm{dx} - \int_{-2}^{-1}\sqrt{(x+2)}\,\mathrm{dx}$ **A1**

$$=\left[\frac{x^2}{4}+\frac{3x}{2}\right]_{-3}^{-1}-\int_{-2}^{-1}(x+2)^{0.5}\,\mathrm{dx}$$

$$=\left(\frac{1}{4}-\frac{3}{2}\right)-\left(\frac{9}{4}-\frac{9}{2}\right)-\left[\frac{1}{1.5}(x+2)^{1.5}\right]_{-2}^{-1}$$

$$=1-\frac{2}{3}(1-0)$$

$=\dfrac{1}{3}$ **A1**

Total = 5 marks

END OF SOLUTIONS TO EXAMINATION 1