THE SCHOOL FOR EXCELLENCE (TSFX)
UNIT 4 MATHEMATICAL METHODS 2009

WRITTEN EXAMINATION 2

Reading Time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

This examination has two sections: Section 1 (multiple-choice questions) and Section 2 (extended-answer questions).

You must complete both parts in the time allocated. When you have completed one part continue immediately to the other part.

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory **DOES NOT** need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.

Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

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SECTION 1 – MULTIPLE CHOICE QUESTIONS

Instructions for Section 1

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers. You should attempt every question.

No marks will be given if more than one answer is completed for any question.

QUESTION 1

The graph of a function has a horizontal asymptote with equation $y = a$ and a vertical asymptote with equation $x = -b$, where a and b are constants. The equation of the function could be

A. $y = \dfrac{4}{x-b} + a$

B. $y = \dfrac{4}{b-x} - a$

C. $y = \dfrac{1}{x+b} - a$

D. $y = \dfrac{1}{x+b} + a$

E. $y = \dfrac{1}{x+a} - b$

QUESTION 2

The quadratic function $f(x) = 3x^2 + 18x + 1$ will have an inverse function for the domain

A. $(-\infty , 3]$

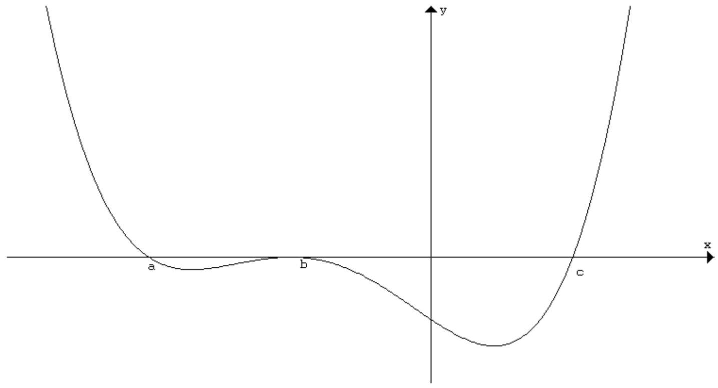
B. $(-\infty , 0]$

C. $(-\infty , -4]$

D. $[-26 , 3]$

E. $[-4 , \infty)$

QUESTION 3



The most likely equation for the graph shown above is

A. $y=(x+a)(x+b)^2(x-c)$

B. $y=-(x+a)(x+b)^2(x-c)$

C. $y=(x-a)(x-b)^2(x+c)$

D. $y=(x-a)(x-b)^2(x-c)$

E. $y=-(x-a)(x-b)^2(x-c)$

QUESTION 4

Consider the two relations $h: R/\{-1\} \rightarrow R$ where $h(x)=\dfrac{1}{|x+1|}+1$ and

$g:(1,\infty) \rightarrow R$ where $g(x)=\log_e(x-1)$

The composite function $g\left(h(x)\right)$ is defined as

A. $-\log_e\left(|x+1|\right)$ for $x \in R$

B. $-\log_e\left(|x+1|\right)$ for $(1,\infty)$

C. $\log_e\left(|x+1|\right)$ for $x \in R^+$

D. $\log_e\left(\dfrac{1}{x+1}\right)$ for $x \in R/\{-1\}$

E. $-\log_e\left(|x+1|\right)$ for $x \in R/\{-1\}$

QUESTION 5

If $a^{2x} - 5a^x + 4 = 0$ where a is a positive real constant, then

A. $x = 0$ only

B. $x = 0$ and $x = 1$

C. $x = 1$ and $x = 4$

D. $x = 0$ and $x = 4$

E. $x = 0$ and $x = \log_a 4$

QUESTION 6

The maximal domain for which the expression $\log_e(x^2) - \log_e(1-x)$ is defined is

A. R^+

B. $R/\{0\}$

C. $(-\infty, 0) \cup (0, 1)$

D. $(-\infty, 0] \cup (0, 1]$

E. $(0, 1)$

QUESTION 7

$3^{4\log_3(x-1)+2}$ may be simplified to

A. $9(x-1)^4$

B. $6(x-1)^4$

C. $2 + (x-1)^4$

D. $9 + (x-1)^4$

E. $4x - 2$

QUESTION 8

The largest set of real values of a for which $|a^2 - 4a| \geq 4$ is

A. $a \geq 2 + 2\sqrt{2}$ and $a \leq 2 - 2\sqrt{2}$ only

B. $a = 2$ only

C. $a \geq 2 + 2\sqrt{2}$, $a = 2$ and $a \leq 2 - 2\sqrt{2}$

D. $a \geq 4$ and $a \leq 0$

E. $a \geq -2 + 2\sqrt{2}$ and $a \leq -2 - 2\sqrt{2}$ only

QUESTION 9

The equation $2\sin^2(\theta) = 3 - 3\cos(\theta)$ for $-\pi \leq \theta \leq \pi$ has solution(s)

A. $\frac{\pi}{3}$ only

B. $-\frac{\pi}{3}$, $\frac{\pi}{3}$ only

C. $-\frac{\pi}{3}$, 0, $\frac{\pi}{3}$

D. $-\pi$, 0, π

E. $-\frac{\pi}{6}$, 0 , $\frac{\pi}{6}$

QUESTION 10

The function $f(x) = a\sin(x) - b\sqrt{3}\cos(x)$ will have a minimum turning point at $x = \frac{\pi}{3}$ if

A. $a = 3b$ and $a < 0$

B. $a = 3b$ and $a > 0$

C. $a = -3b$ and $a > 0$

D. $a = -3b$ and $a < 0$

E. $a = -b$ and $a < 0$

QUESTION 11

If $y = \frac{\log_e(2x)}{x}$ then $\frac{dy}{dx} =$

A. $\frac{1-2\log_e(2x)}{2x^2}$

B. $\frac{1-\log_e(2x)}{x^2}$

C. $\frac{\log_e(2x)-1}{x^2}$

D. $\frac{1-\log_e(2x)}{4x^2}$

E. $\frac{1-x\log_e(2x)}{x^2}$

QUESTION 12

The equation of the **normal** to the curve with equation $y = x^3 - 4x^2 + 7x - 5$ at $x = 2$ is

A. $y - 3x + 5 = 0$

B. $3y - x - 1 = 0$

C. $3y + x + 5 = 0$

D. $y + 3x - 7 = 0$

E. $3y + x - 5 = 0$

QUESTION 13

If $\int_1^3 f(x)\, dx = 10$ then $\int_3^1 (5 - 3f(x))dx$ is equal to

A. -25

B. -20

C. 0

D. 20

E. 25

QUESTION 14

The graph of the function $f(x)$ has a minimum stationary value at the point with co-ordinates $(2\sqrt{3},-1)$. The function $g(x)=5f(-x+1)$ will have the corresponding stationary value of

A. a minimum at $(-2\sqrt{3}+1,-5)$

B. a maximum at $(-2\sqrt{3}+1,-5)$

C. a minimum at $(2\sqrt{3}-1,-5)$

D. a maximum at $(2\sqrt{3}-1,-5)$

E. a minimum at $(-2\sqrt{3}-1,-5)$

QUESTION 15

$\int\left(\frac{2}{1-2x}+e^{3x+1}\right)dx$ is equal to

A. $\frac{4}{(1-2x)^2}+3e^{3x+1}+c$

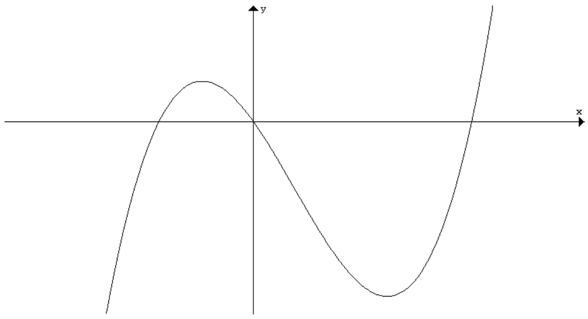
B. $-\frac{4}{(1-2x)^2}+3e^{3x+1}+c$

C. $-\log_e|2x-1|+\frac{1}{3}e^{3x+1}+c$

D. $\log_e|1-2x|+\frac{1}{3}e^{3x+1}+c$

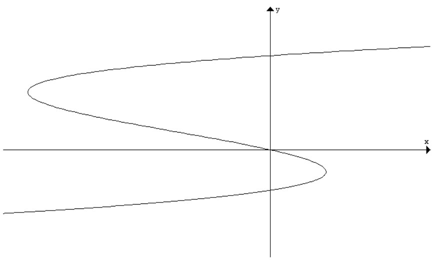
E. $2\log_e|1-2x|+\frac{1}{3}e^{3x+1}+c$

QUESTION 16
The graph of a function $y = f(x)$ is shown here.



The graph of $f'(x)$ could be

A.



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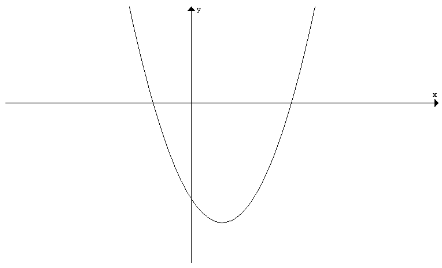

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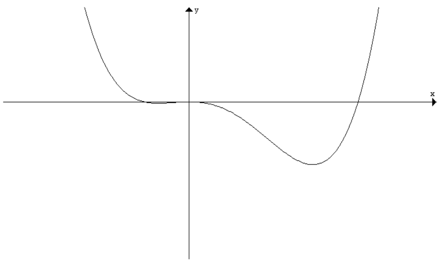

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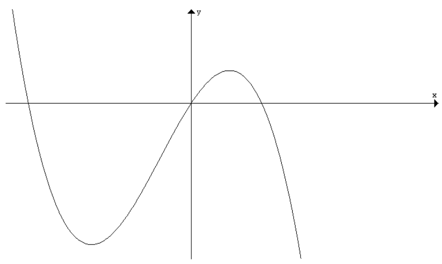
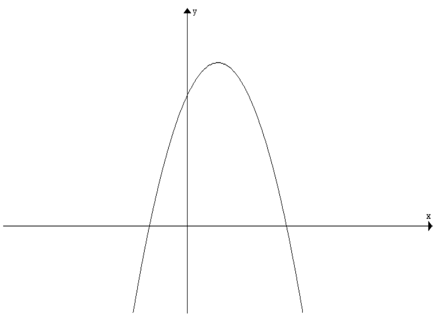


E.



QUESTION 17
The average rate of change of the function $f(x) = (2x-1)e^{3x}$ over $[0, 2]$ is

A. $(8x-3)e^{3x}$

B. $(6x-1)e^{3x}$

C. $\dfrac{3e^6-1}{2}$

D. $\dfrac{3e^6+1}{2}$

E. $3e^6+1$

QUESTION 18
An experiment consists of drawing a number at random from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
Let A = {1, 2, 3, 4, 5} , B = {2, 4, 6, 8, 10} , C = {3, 6, 9} and D = {1, 2, 5, 10}.
For which of the following pairs of events are the events independent?

A. A and B

B. A and C

C. B and C

D. A and D

E. B and D

QUESTION 19
Past records indicate that if it rains on a particular day there is an 80% chance that it will rain the next day. If it does **not** rain on a particular day, the probability that it will rain the next day is ***p***%. It rained on Monday and the probability that it rained on the following Wednesday was 65%. The value of ***p*** is

A. 5

B. 10

C. 15

D. 20

E. 35

QUESTION 20

A Binomial Probability Distribution has a mean of 5 and a variance of 4.
Correct to 4 decimal places, the value of $\Pr(\mu - \sigma \leq X \leq \mu + \sigma)$ is closest to

A. 0.7927

B. 0.8909

C. 0.6800

D. 0.6827

E. 0.9789

QUESTION 21

The weight of 3-year old cats is normally distributed with a mean of 5 kg and a standard deviation of 0.5 kg. The probability that a 3-year old cat weighs greater than 4 kg **given that** it weighs less than 5 kg is closest to

A. 0.1080

B. 0.4124

C. 0.4772

D. 0.9544

E. 0.9545

QUESTION 22

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} ax & 0 \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

where a and b are positive real numbers with $b > a$. If the ***median*** is $\frac{4}{3}$ then

A. $a = \frac{\sqrt{3}}{4}$, $b = \frac{64}{3}$

B. $a = \frac{9}{16}$, $b = \frac{4\sqrt{2}}{3}$

C. $a = \frac{3}{4}$, $b = \sqrt{\left(\frac{8}{3}\right)}$

D. $a = 1$, $b = \sqrt{2}$

E. $a = \frac{16}{9}$, $b = \frac{4\sqrt{2}}{3}$

SECTION 2 – EXTENDED ANSWER QUESTIONS

Instructions For Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1

The graph of the function $f(x) = (x + a)(x - b)^2 + 2$ where a and b are real constants is shown below. It is known that $a > b$.

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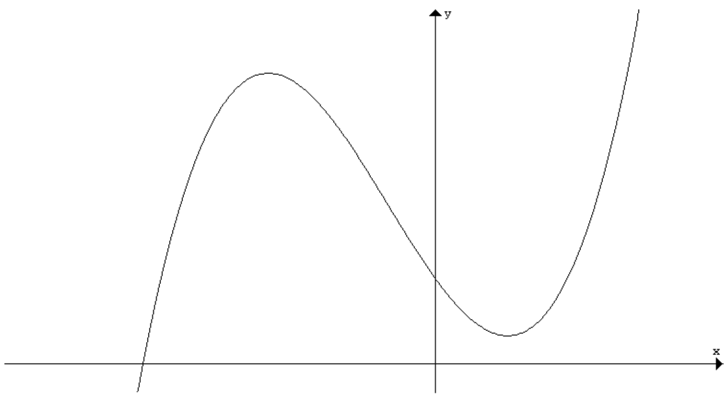


a. Given that $f'(1) = f'\left(-\frac{7}{3}\right) = 0$, use calculus to show that $a = 4$ and $b = 1$.

4 marks

b. Find the coordinates of the turning point of the graph of $y = f(x)$ at $x = 1$.

1 mark

c. If the coordinates of the other turning point of the graph of $y = f(x)$ is $\left(-\frac{7}{3}, c\right)$, find the value of c correct to 2 decimal places.

1 mark

d. Find the real values of m for which the equation $f(x) = m$ has three distinct solutions. (Non-integer values are to be given correct to 2 decimal places).

2 marks

The graph of $y = f(x)$ is dilated by a factor of k from the *y*-axis to form another function $g(x)$ so that the **horizontal** distance between the two turning points is 10 units.

e. Find this value of k.

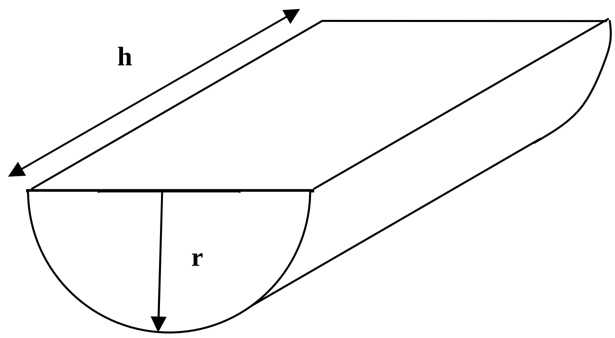
2 marks

f. Hence, or otherwise, write down the coordinates of the turning points of the graph with equation $y = g(x)$. (Non-integer values are to be given correct to 2 decimal places).

2 marks

Total = 12 marks

QUESTION 2



A quantity of precious metal whose volume is $500\,cm^3$ is melted and moulded into a shape of length ***h*** centimetres and uniform semi-circular cross section of radius ***r*** centimetres, as shown in the diagram.

a. Find an equation for A**,** the total surface area, in terms of h and r.

2 marks

b. Hence show that $A = \pi r^2 + \dfrac{1000(2+\pi)}{\pi r}$.

2 marks

c. Using calculus, find the value of r, correct to 2 decimal places, for which this surface area is a minimum.

2 marks

d. Write down the minimum surface area, correct to 2 decimal places.

1 mark

To make this precious metal into a beautiful piece of jewellery, it is decided to cover the surface area with gold leaf. The flat surface area is easier to cover and costs \$***p*** per cm^2 whereas the curved surface area is more difficult to coat with gold leaf and costs \$***q*** per cm^2, where ***p*** and ***q*** are constants with ***q*** > ***p***.

e. Using the information that you used in parts **a.** and **b.**, show that an expression for the ***total cost*** (in dollars) of covering this piece of jewellery in terms of ***r***, ***p*** and ***q*** is given by $C = \boldsymbol{p}\left(\pi r^2 + \dfrac{2000}{\pi r}\right) + \dfrac{1000}{r}\boldsymbol{q}$.

2 marks

f. Find the value of r in terms of ***p*** and ***q*** for which this cost is a minimum.

3 marks

g. Hence, or otherwise, if $\boldsymbol{p} = 15$ and $\boldsymbol{q} = 35$ write down

i. the minimum cost (correct to the nearest dollar).

1 mark

ii. the radius (in centimetres, correct to 2 decimal places).

1 mark

h. For safety reasons, the height of the jewellery piece must **not** be less than 10 centimetres in length. Find the minimum cost to cover the surface area with gold leaf, if $\boldsymbol{p} = 15$ and $\boldsymbol{q} = 35$. Give your answer correct to the nearest dollar.

2 marks

Total = 16 marks

QUESTION 3
Speeds of vehicles recorded by a speed-camera along a Victorian country road are normally distributed. Over a one month period, 20% of cars exceeded 94 km/h and 1% of cars travelled at more than 122 km/h.

a. Set up two equations which would enable you to find the mean $\boldsymbol{p}$ and standard deviation $\boldsymbol{q}$ of these recorded speeds. Non-integer values are to be rounded to 3 decimal places.

2 marks

b. Solve the equations to show that the values of $\boldsymbol{p}$ and $\boldsymbol{q}$ are 78.1 and 18.9 km/h, respectively. Find the percentage of cars recorded travelling at less than the speed limit of 80 km/h. Give your answer correct to the nearest percentage.

2 marks

c. Of the drivers exceeding 80 km/h on this country road, what is the probability that they were travelling at more than 100km/h? Give your answer correct to 3 decimal places.

3 marks

d. Drivers on this country road are fined if they exceed the speed limit. The penalties are shown in the table below.

Recorded speed of car	**Amount of penalty**	**Probability**
Below 80 km/h	zero	
From 80 km/h to under 100 km/h	\$220	
From 100 km/h to under 110 km/h	\$440	
Over 110 km/h	\$500	

Fill in the probability column, giving each value correct to 2 decimal places.

2 marks

e. Calculate the expected value of the penalty for speeding drivers, correct to the nearest ten dollars.

2 marks

f. In a particular hour, the speed camera on this country road recorded 48 vehicles exceeding 100 km/h. How many cars passed the speed camera during that hour?

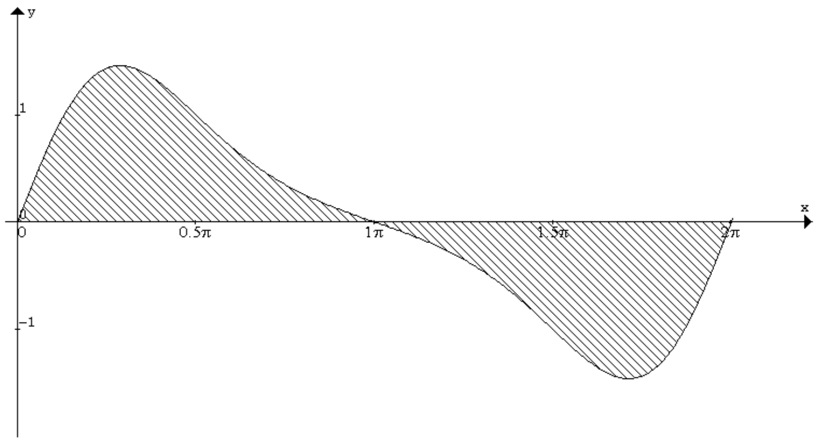
2 marks

g. Six drivers who passed this speed camera stopped further along this country road for a cup of coffee. What is the probability (correct to 4 decimal places) that at least two of them were fined for speeding?

3 marks

Total = 16 marks

QUESTION 4
A flower-bed is to be planted in a garden. The boundary of this flower-bed is the *x*- axis and the curve with equation $f(x) = \sin(x)e^{\cos(x)}$ for $0 \le x \le 2\pi$ where x is measured in metres.



a. **i.** Using calculus, find the derivative of $e^{\cos(x)}$.

2 marks

ii. Hence, find the ***exact*** area of the garden bed.

2 marks

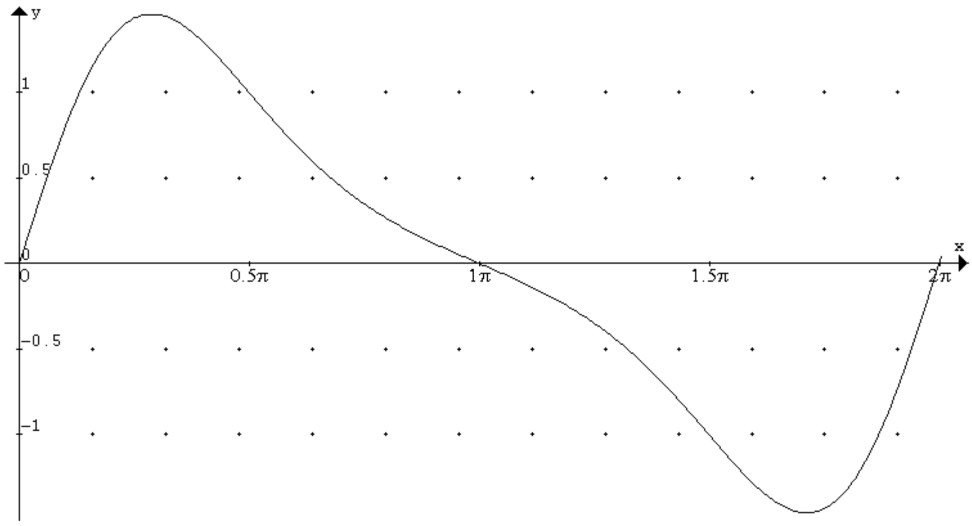
b. **i.** Find $f'(x)$.

2 marks

ii. Hence show algebraically that $f'(x) = 0$ if $\cos(x) = \frac{\sqrt{5}-1}{2}$.

2 marks

The boundary of the flower-bed whose equation is $f(x) = \sin(x)e^{\cos(x)}$ is shown on the diagram below. It is decided to install a watering system for this flower-bed. The area covered by this watering system is contained by the graph of the composite function $y = f(g(x))$ where $g(x) = \sqrt{(x^2 + 1)}$ for $x \geq 0$ and the x- axis (similar to that of the garden bed).



c. Write down the equation of $f(g(x))$.

1 mark

d. **i.** Using your calculator, sketch the graph of $y = f(g(x))$ for $0 \leq x \leq 2\pi$ on the grid above, giving any intercepts to 2 decimal places.

ii. Write down the coordinates of any points of intersection between the two curves, giving answers correct to 2 decimal places.

2 + 1 = 3 marks

e. **i.** This watering system does not cover the entire area of the garden. Write down the definite integrals that would represent the area **not** covered.
(It is not necessary to evaluate this area.)

2 marks

Total = 14 marks

End of Paper

THE SCHOOL FOR EXCELLENCE
UNIT 3 & 4 MATHEMATICAL METHODS 2009
COMPLIMENTARY WRITTEN EXAMINATION 2 - SOLUTIONS

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MARKING SCHEME (EXTENDED ANSWER QUESTIONS)

- ($A\,4\times\frac{1}{2}\downarrow$) means four answer half-marks rounded **down** to the next integer. Rounding occurs at the end of a part of a question.
- M1 = 1 **M**ethod mark.
- **A**1 = 1 **A**nswer mark (it **must** be this or its equivalent).
- **H**1 = 1 consequential mark (**H**is/**H**er mark…correct answer from incorrect statement or slip).

SECTION 1 – MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11
D	C	D	E	E	C	A	C	C	D	B

12	13	14	15	16	17	18	19	20	21	22
E	D	A	C	B	D	E	A	A	E	B

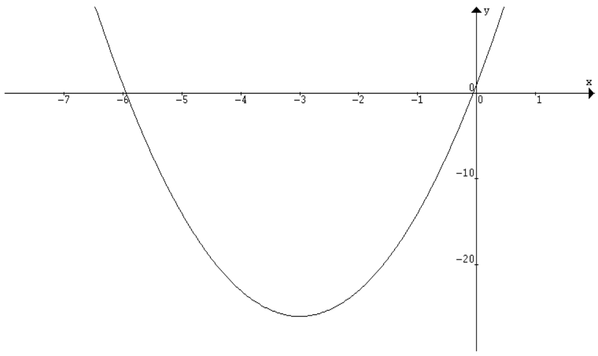
QUESTION 1

The vertical asymptote of a rational function occurs when the denominator (bottom line) of the fraction is zero. So $(x+b)$ must be the bottom line. The rest of the equation (ignoring the fraction) must read $y = a$ for the horizontal asymptote.

The answer is D.

QUESTION 2

The graph of $f(x) = 3x^2 + 18x + 1$ is shown below.



An inverse function exists if the function is one-to-one. The turning point of $f(x)$ occurs at $x = -3$ so any interval to the left of this value, or right of it, will ensure that the function is one-to-one.

The answer is C.

QUESTION 3

Because the graph touches the *x*-axis at $x = b$ there must be a factor of $(x-b)^2$ in the equation.

$\pm(x-c)$ is also a factor as is $\pm(x-a)$. The choices are between **D** or **E**.

Y Intercept:

Substituting $x = 0$ into $y = (x-a)(x-b)^2(x-c)$ gives the value ab^2c which is negative as $a < 0$. As the graph cuts the *y*-axis at a negative value, this is consistent with option D.

The answer is D.

QUESTION 4

$$g\left(h(x)\right) = \log_e\left(\frac{1}{|x+1|} + 1 - 1\right)$$

$$= \log_e\left(\frac{1}{|x+1|}\right)$$

$$= \log_e 1 - \log_e(|x+1|)$$

$$= -\log_e(|x+1|)$$

The domain of *g* (*h*(*x*)) is the same as the domain of *h*(*x*) which is ***R*** / {−1}.

The answer is E.

QUESTION 5

Let $m = a^x$ in the equation $a^{2x} - 5a^x + 4 = 0$.

Then $m^2 - 5m + 4 = 0$ and so $(m-4)(m-1) = 0$.

Therefore $m = 4$ and so $a^x = 4$ which means that $x = \log_a 4$

Also $m = 1$ and so $a^x = 1$ which means that $x = \log_a 1 = 0$.

The answer is E.

QUESTION 6

$\log_e(x^2)$ is defined for $R\,/\,\{0\}$ and $\log_e(1-x)$ is defined for $(-\infty\,,\,1\,)$.
The expression is defined for the intersection of these two sets which is $(-\infty, 0) \cup (0,1)$.

The answer is C.

QUESTION 7

$$4\log_3(x-1) + 2 = \log_3(x-1)^4 + 2\log_3 3$$

$$= \log_3(x-1)^4 + \log_3 3^2$$

$$= \log_3(x-1)^4 + \log_3 9$$

$$= \log_3 9(x-1)^4$$

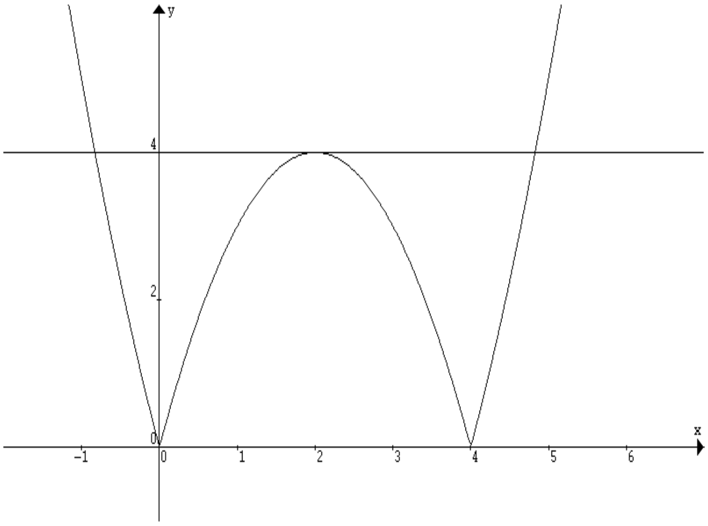
Therefore $3^{4\log_3(x-1)+2} = 3^{\log_3 9(x-1)^4} = 9(x-1)^4$

The answer is A.

QUESTION 8

$$\left|a^2-4a\right|=a^2-4a$$
$$=-(a^2-4a)=4a-a^2$$

To solve $\left|a^2-4a\right|=4$, find the points of intersection of the graphs $y=\left|a^2-4a\right|$ and $y=4$.



Either $a^2-4a=4$ or $4a-a^2=4$

If $a^2-4a=4$ then $a^2-4a-4=0$ so $a=\dfrac{4\pm\sqrt{32}}{2}$ = $2\pm2\sqrt{2}$

If $4a-a^2=4$ then $a^2-4a+4=0$. Hence $a=2$.

From the graph, $\left|a^2-4a\right|\geq 4$ if $a\geq 2+2\sqrt{2}$ or $a=2$ or $a\leq 2-2\sqrt{2}$

The answer is C.

QUESTION 9

$$2\sin^2(\theta)=3-3\cos(\theta)$$
$$2(1-\cos^2(\theta))=3-3\cos(\theta)$$
$$2-2\cos^2(\theta)=3-3\cos(\theta)$$
$$2\cos^2(\theta)-3\cos(\theta)+1=0$$
$$(2\cos(\theta)-1)(\cos(\theta)-1)=0$$

Hence $\cos(\theta)=0.5$ or $\cos(\theta)=1$

For $-\pi\leq\theta\leq\pi$, $\cos(\theta)=0.5$ has solutions $-\dfrac{\pi}{3},\dfrac{\pi}{3}$ and $\cos(\theta)=1$ has solution 0.

The answer is C.

QUESTION 10

If $f(x) = a\sin(x) - b\sqrt{3}\cos(x)$ then $f'(x) = a\cos(x) + b\sqrt{3}\sin(x)$

$0 = a\cos(x) + b\sqrt{3}\sin(x)$ at any turning points.

$b\sqrt{3}\sin(x) = -a\cos(x)$

$\dfrac{\sin(x)}{\cos(x)} = \tan(x) = -\dfrac{a}{b\sqrt{3}}$

Now $\tan\left(\dfrac{\pi}{3}\right) = \sqrt{3}$ and so $-\dfrac{a}{b\sqrt{3}} = \sqrt{3}$

Therefore $a = -3b$ and so the answer is either alternative C or D.

Only alternative D gives a minimum value at the required value of x.

The answer is D.

QUESTION 11

Using the Quotient Rule: $\dfrac{dy}{dx} = \dfrac{x \times \dfrac{d}{dx}(\log_e(2x)) - \log_e(2x) \times \dfrac{d}{dx}(x)}{x^2}$

$$= \frac{x \times \dfrac{2}{2x} - \log_e(2x) \times 1}{x^2}$$

$$= \frac{1 - \log_e(2x)}{x^2}$$

The answer is B.

QUESTION 12

If $y = x^3 - 4x^2 + 7x - 5$ then $\dfrac{dy}{dx} = 3x^2 - 8x + 7$

At $x = 2$, $\dfrac{dy}{dx} = 3 \times 4 - 8 \times 2 + 7 = 3$

Gradient of tangent is 3. Therefore, gradient of the normal is $-\dfrac{1}{3}$.

As $y = 1$ at $x = 2$:

Equation of the normal is: $y - 1 = -\dfrac{1}{3}(x - 2)$

$$3y - 3 = -x + 2$$

$$3y + x - 5 = 0$$

The answer is E.

QUESTION 13

$$\int_3^1 (5-3f(x))dx = -\int_1^3 (5-3f(x))dx$$
$$= \int_1^3 (3f(x)-5)dx$$
$$= 3\int_1^3 f(x)dx - \int_1^3 5dx$$
$$= 3\times 10 - [5x]_1^3$$
$$= 30-(15-5)$$
$$= 20$$

The answer is D.

QUESTION 14

The function *g* (*x*) is obtained from *f* (*x*) through the following three transformations:

- A dilation from the *x* – axis (or parallel to the *y* – axis) by a factor of 5 which results in the minimum value being at $(2\sqrt{3}, -5)$.
- A reflection in the *y* – axis which now means that the minimum is at $(-2\sqrt{3}, -5)$.
- Finally there is a translation of 1 unit to the right which results in the minimum now being at $(-2\sqrt{3}+1, -5)$

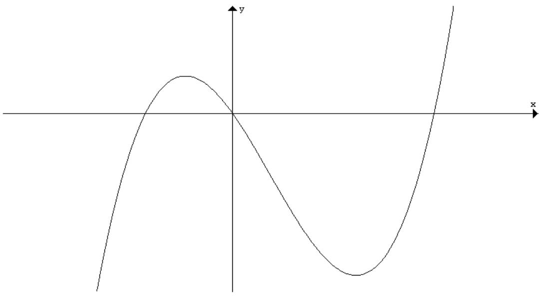
The answer is A.

QUESTION 15

$\int \left(\frac{f'(x)}{f(x)}\right) dx = \log_e |f(x)|$ and so $\int \left(\frac{2}{1-2x} + e^{3x+1}\right) dx = -\log_e |\ 2x-1\ | + \frac{1}{3}e^{3x+1} + c$

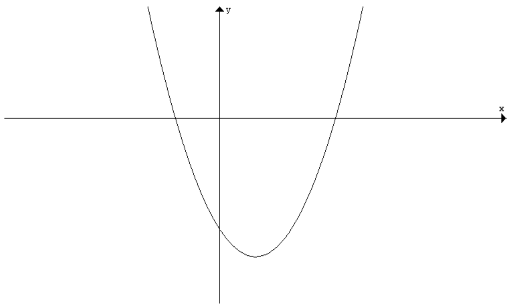
The answer is C.

QUESTION 16



The graph shown above resembles that of a cubic function and so its derivative function will resemble a parabola.

The gradient on the left and right of the function is positive and so the best alternative is shown alongside.



The answer is B.

QUESTION 17

$f(x) = (2x-1)e^{3x}$

Substitute $x=0$ into $(2x-1)\,e^{3x}$: $-e^0 = -1$

Substitute $x=2$ into $(2x-1)\,e^{3x}$: $3e^6$

The average rate of change is $\dfrac{3e^6-(-1)}{2-0} = \dfrac{3e^6+1}{2}$

The answer is D.

QUESTION 18

If two events X and Y are independent then $\Pr(X \cap Y) = \Pr(X).\Pr(Y)$.

Now $A \cap B = \{2, 4\}, A \cap C = \{3\}, B \cap C = \{6\}, A \cap D = \{1, 2, 5\}, B \cap D = \{2, 10\}$

Test whether:

$\Pr(A).\Pr(B) = \Pr(A \cap B)$? Left side = $\dfrac{5}{10} \times \dfrac{5}{10} = \dfrac{1}{4}$ Right side = $\dfrac{2}{10}$ No!

$\Pr(A).\Pr(C) = \Pr(A \cap C)$? Left side = $\dfrac{5}{10} \times \dfrac{3}{10} = \dfrac{3}{20}$ Right side = $\dfrac{1}{10}$ No!

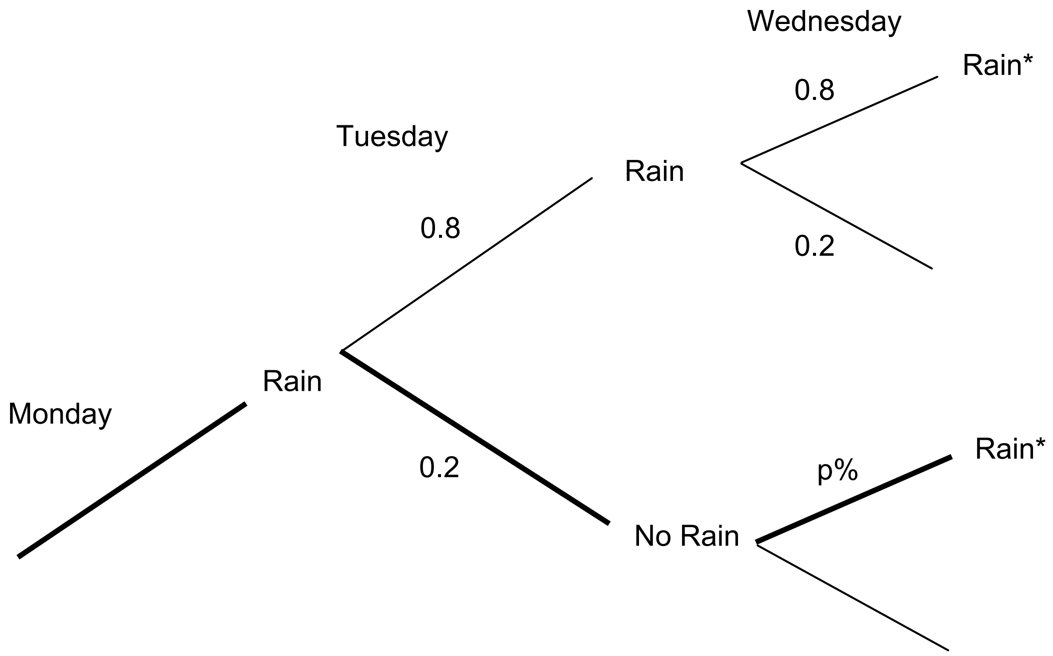
$\Pr(B).\Pr(C) = \Pr(B \cap C)$?   Left side = $\frac{5}{10} \times \frac{3}{10} = \frac{3}{20}$ Right side = $\frac{1}{10}$ No!

$\Pr(A).\Pr(D) = \Pr(A \cap D)$?   Left side = $\frac{5}{10} \times \frac{4}{10} = \frac{1}{5}$ Right side = $\frac{3}{10}$ No!

$\Pr(B).\Pr(D) = \Pr(B \cap D)$?   Left side = $\frac{5}{10} \times \frac{4}{10} = \frac{1}{5}$ Right side = $\frac{2}{10} = \frac{1}{5}$ Yes!

The answer is E.

QUESTION 19



Probability of rain on Wednesday $= 0.8 \times 0.8 + 0.2 \times \frac{p}{100}$

$= \frac{64}{100} + \frac{2p}{1000}$

$\frac{65}{100} = \frac{64}{100} + \frac{2p}{1000}$ and so $p = 5$.

The answer is A.

QUESTION 20

Binomial Distribution with $np = 5$ and $npq = 4$

So $5q = 4$ (substituting np = 5). Therefore $q = \frac{4}{5}$ which gives $p = \frac{1}{5}$

If $p = \frac{1}{5}$ then $\frac{n}{5} = 5$ and so $n = 25$

$\mu - \sigma = 5 - 2 = 3$ and $\mu + \sigma = 5 + 2 = 7$ and so find the Binomial cdf for $3 \le X \le 7$.

This is binomcdf(25, 0.2, 7) – binomcdf(25, 0.2, 2) = 0.8909 – 0.0982 = 0.7927

The answer is A.

QUESTION 21

$$Normalcdf\,(4, 5, 5, 0.5) = 0.4772499$$

$$\Pr(X > 4 \mid X < 5) = \frac{0.4772499}{0.5} = 0.954499$$

The answer is E.

QUESTION 22

The sum of the probabilities is 1.

Therefore $\int_0^b ax\,\mathrm{dx} = \left[\frac{a}{2}x^2\right]_0^b = 1$ and so $\frac{ab^2}{2} = 1$ (Equation 1)

Now $\int_0^{\frac{4}{3}} ax\ dx = 0.5$

$\left[\frac{a}{2}x^2\right]_0^{\frac{4}{3}} = 0.5$ and so $\frac{a}{2} \times \frac{16}{9} = \frac{1}{2}$. (Equation 2)

Hence $a = \frac{9}{16}$

Substituting for a in equation 1 gives $b^2 = \frac{32}{9}$ and so $b = \frac{4\sqrt{2}}{3}$.

The answer is B.

SECTION 2 – EXTENDED ANSWER QUESTIONS

QUESTION 1

a. $f'(x) = (x+a)\times 2(x-b) + 1\times(x-b)^2$ (using the Product Rule) **M1**

$= (x-b)[2(x+a)+(x-b)]$

$= (x-b)(3x+2a-b)$ **A1**

$= 0$ if $x = b$ or $x = \dfrac{b-2a}{3}$ **A1**

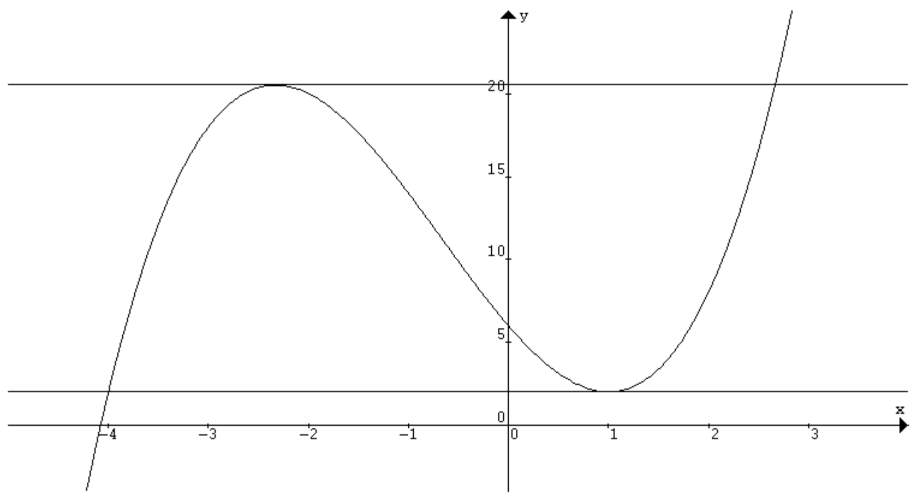
Since $f'(1) = 0$ then b could be 1. If this is the case, see if $a = 4$ satisfies the other stationary value. $-\dfrac{7}{3} = \dfrac{1-2a}{3}$ so $-7 = 1-2a$ which means that $a = 4$, as req. **M1**

b. $f(x) = (x+4)(x-1)^2 + 2$

When $x = 1$, $y = (1+4)(1-1)^2 + 2 = 2$ and so the turning point is at $(1, 2)$. **A1**

c. When $x = -\dfrac{7}{3}$, $y = (-\dfrac{7}{3}+4)(-\dfrac{7}{3}-1)^2 + 2 = 20.52$ and so $c = 20.52$ **A1**

d. The lines $y = 2$ and $y = 20.52$ have been drawn showing that each of them meets the graph at two points.



If $2 < m < 20.52$ then the equation $f(x) = m$ will have three distinct solutions.

A $4\times\dfrac{1}{2}\downarrow$ **(2 , < , <, 20.52)**

e. If the turning points of $f(x)$ are at $(1, 2)$ and $\left(-\frac{7}{3}, 20.52\right)$ then the horizontal distance between them is $1 - -\frac{7}{3} = \frac{10}{3}$ units. This would need to be multiplied by 3 to give the required result of being 10 units apart. Hence $k = 3$.

M1 (horizontal distance idea)
A1 (k = 3)

f. (-7, 20.52) and (3, 2) **A2** (1 for each pair)

Total = 12 marks

QUESTION 2

a. Total area = Two end semi-circles + flat surface + curved surface **M1**

$$A = 2\times(\frac{1}{2}\pi\ r^2) + 2r\times h + \frac{1}{2}\times 2\pi\ r\ h$$

$$A = \pi\ r^2 + 2rh + \pi\ r\ h$$ **A1**

b. Volume = 500 = $\frac{1}{2}\pi\, r^2 h$ and so $h = \frac{1000}{\pi\, r^2}$ **A1**

$$A = \pi\ r^2 + 2rh + \pi\ r\ h$$

$$= \pi\ r^2 + (2r + \pi\ r\ )h$$

$$= \pi\ r^2 + (2r + \pi\ r\ )\times\frac{1000}{\pi\ r^2}$$ **M1**

$$= \pi\ r^2 + (2 + \pi\ )r\times\frac{1000}{\pi\ r^2}$$ which when cancelling the r gives

$$A = \pi\, r^2 + \frac{1000(2+\pi)}{\pi\, r}$$, as required.

c. $A = \pi\, r^2 + \frac{1000(2+\pi)}{\pi}\times r^{-1}$

$$\frac{dA}{dr} = 2\pi\, r - \frac{1000(2+\pi)}{\pi}\times r^{-2}$$ **H1**

$= 0$ for a minimum value

Therefore $2\pi^2 r^3 = 1000(2+\pi)$ and so $r = \left(\frac{1000(2+\pi)}{2\pi^2}\right)^{\frac{1}{3}}$ = 6.39 cm **A1**

d. 384.40 cm^2 (do not accept 384.4 cm^2) **A1**

e. $C = p(\pi r^2 + 2rh) + q \times \pi r \times \dfrac{1000}{\pi r^2} = p(\pi r^2 + 2r \times \dfrac{1000}{\pi r^2}) + q \times \pi r \times \dfrac{1000}{\pi r^2}$ **M2**

(Give a method mark for each part, curved and flat)

$$\therefore C = p\left(\pi r^2 + \frac{2000}{\pi r}\right) + \frac{1000}{r} q$$

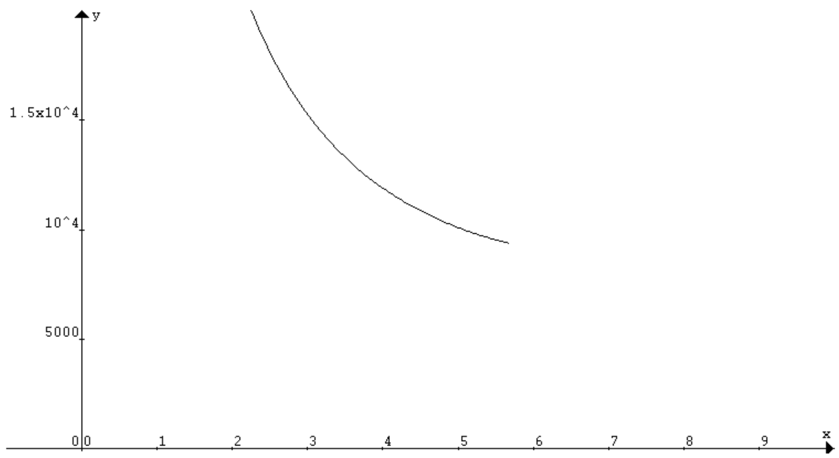
f. $\dfrac{dC}{dr} = p\left(2\pi r - \dfrac{2000}{\pi r^2}\right) - \dfrac{1000}{r^2} q$ **H1**

For minimum cost $\dfrac{dC}{dr} = 0$ and so $2\pi r\ p = \dfrac{2000p + 1000\pi q}{\pi r^2}$ **M1**

Hence $r = \left(\dfrac{1000(2p + q\pi)}{2\pi^2 p}\right)^{\frac{1}{3}}$ **A1**

g. **i.** \$8578
ii. 7.79 cm **A2**

h. Since $h = \dfrac{1000}{\pi r^2}$ then if $h = 10$, $r = \sqrt{\dfrac{100}{\pi}}$ (= 5.64 cm to 2 decimal places). **A1**



The minimum cost occurs when $r = 5.64\,cm$, and is approximately \$9396. **A1**

Total = 16 marks

QUESTION 3

a. Invnorm(0.8) = $\frac{94-p}{q}$ and Invnorm(0.99) = $\frac{122-p}{q}$ **M1**

$0.842q = 94 - p$ and $2.326q = 122 - p$ **A1**

b. (i) $0.842q = 94 - p$

(ii) $2.326q = 122 - p$

Taking (i) from (ii) gives $1.484q = 28$ and so $q = 18.8679...$ which rounds to 18.9 as required.

Substituting for *q* in (i) gives $p = 78.113$ which rounds to 78.1, as required. **M1**

Normalcdf(-10^{10}, 80, 78.1, 18.9) = 0.5400 so the answer is 54% **A1**

c. Normalcdf (100, 10^{10}, 78.1, 18.9) = 0.12328 **A1**

Required probability = $\frac{0.12328}{0.46}$ **M1**

$= 0.268$ **A1**

d.

Recorded speed of car	Amount of penalty	Probability
Below 80 km/h	zero	0.54
From 80 km/h to under 100 km/h	$220	0.34
From 100 km/h to under 110 km/h	$440	0.08
Over 110 km/h	$500	0.04 or 0.05

(**A** $4 \times \frac{1}{2} \downarrow$)

e. Mean = $\frac{0 \times 0.54 + \$220 \times 0.34 + \$440 \times 0.08 + \$500 \times 0.04}{0.46}$ **M1**

$= \frac{(0 + 68 + 35.2 + 20)}{0.46}$

$= \frac{123.2}{0.46}$

$= \$270$ to the nearest $10 (or $280 if 0.05 was used) **A1**

f. The proportion of the population exceeding 100 km/h is 0.12 (from the table).

Hence $0.12x = 48$ and so $x = \dfrac{48}{0.12} = 400$. **M1**

400 cars pass in the hour. **A1**

If students use 0.13 then the mark scheme is:

The proportion of the population exceeding 100 km/h is 0.13 (from the table).

Hence $0.13x = 48$ and so $x = \dfrac{48}{0.13} = 369.23$. **M1**

369 (or 370) cars pass in the hour. **A1**

g. Pr(Speeding) = 0.46

Binomial Distribution: $(0.46 + 0.54)^6$ **M1**

Pr(at least two) = Pr(2) + Pr(3) + Pr(4) + Pr(5) + Pr(6) or 1 – [Pr(0) + Pr(1)] **A1**

$$= 1 - \left[0.54^6 + \binom{6}{1} \times 0.54^5 \times 0.46 \right]$$

$= 0.8485$ **A1**

Total = 16 marks

QUESTION 4

a. **i.** $y = e^{\cos x}$

Let $u = \cos(x)$ and so $\dfrac{du}{dx} = -\sin(x)$ **M1**

$y = e^u$ and so $\dfrac{dy}{du} = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= e^{\cos(x)} \times -\sin(x)$$

$$= -\sin(x)e^{\cos(x)}$$ **A1**

ii. $2\int_o^{\pi} \sin(x)e^{\cos(x)}dx = 2\left[-e^{\cos(x)}\right]_0^{\pi}$ **A1**

$= 2(-e^{-1} + e^1)$ **A1**

b. **i.** $f(x)=\sin(x)e^{\cos(x)}=uv$

Let $u=\sin x$ and $v=e^{\cos x}$ then $\dfrac{du}{dx}=\cos(x)$ and $\dfrac{dv}{dx}=-\sin(x)e^{\cos(x)}$ **M1**

$f'(x)=v\dfrac{du}{dx}+u\dfrac{dv}{dx}$ = $\cos(x)e^{\cos(x)}$ − $\sin^2(x)e^{\cos(x)}$ **A1**

ii. If $f'(x)=0$ then $\cos(x)e^{\cos(x)}$ − $\sin^2(x)e^{\cos(x)}=0$

Therefore $e^{\cos(x)}(\cos(x)$ − $\sin^2(x))$ = 0

Now $e^{\cos(x)}$ can never be zero so $\cos(x)-\sin^2(x)=0$

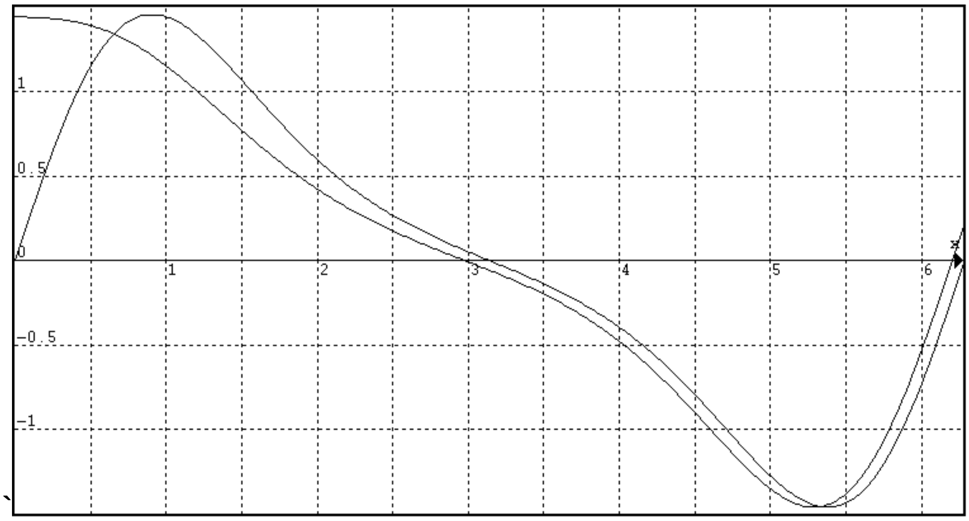
Hence $\cos(x)-(1-\cos^2(x))=0$ and so $\cos^2(x)+\cos(x)-1=0$ **M1**

Using the quadratic formula, $\cos(x)=\dfrac{-1\pm\sqrt{1+4}}{2}$ **A1**

One of these values corresponds with what needed to be found.

c. $f(g(x))$ = $\sin\sqrt{(x^2+1)}\,.\,e^{\cos\sqrt{(x^2+1)}}$ **A1**

d. **i.**



Intercepts (0, 1.44), (2.98, 0), (6.20, 0). Coordinate format not necessary here. **A1**

Shape with two points of intersection at approximately (0.7, 1.3) and (5.3, -1.4) **H1**

ii. (0.65, 1.34) and (5.33, 1.46) **A1**

e. $$\int_{0.65}^{2.98}[f(x)-g(f(x))]dx+\int_{2.98}^{\pi}f(x)dx+\int_{5.33}^{6.20}[f(x)-g(f(x))]dx+\left|\int_{6.20}^{2\pi}f(x)dx\right|$$

The two "difference integrals" with correct lower terminals. **M1**
All four integrals correct. **A1**

Total = 14 marks