

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2



2009 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: C

Explanation: $x=0$, y intercept is 6 and C is the only one that gives this result (or do long division and other features also become clear)

Question 2

Answer: B

Explanation: product rule: $\ln(x^2 - x) + \frac{x(2x-1)}{x(x-1)}$
 $\ln(x^2 - x) + \frac{(2x-1)}{(x-1)}$

Question 3

Answer: A

B
Explanation: sub $x=e$, $y= -2$, sub $x=5$, $y=-2\ln(5)$
giving $[-2\ln(5), -2)$

Question 4*Answer:* D*Explanation:* swap x and y for inverse

$$x = (y - 2)^2 + 3$$

$$\pm \sqrt{x - 3} = y - 2, \text{ due to domain only negative}$$

$$y = -\sqrt{x - 3} + 2, \text{ domain of } f(x) \text{ is the range of the inverse function}$$

$$f^{-1} : [3, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = -\sqrt{x - 3} + 2$$

Question 5*Answer:* B

$$\begin{aligned} \text{Explanation: } y &= g\left(-\frac{1}{2}(x-2)\right) \\ &= g\left(-\frac{x}{2}+1\right) \Rightarrow g\left(1-\frac{x}{2}\right) \end{aligned}$$

Question 6*Answer:* C

$$\text{Explanation: let } e^x = a, \Rightarrow a^2 - a - 6 = 0$$

$$(a - 3)(a + 2) = 0$$

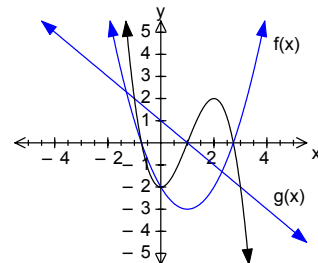
$$e^x = 3, e^x = -2, \Rightarrow x = \ln(3), \text{ only}$$

Question 7*Answer:* D

Explanation: graph has been translated 2 units left, absolute valued, reflected in the x -axis then translated 1 unit down

Question 8*Answer:* C

Explanation: product of functions key points are $x=0, 1, -1$



Question 9*Answer:* C

$$\text{Explanation: } V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2, r = 1$$

$$\frac{dV}{dr} = 4\pi$$

Question 10*Answer:* A

$$\text{Explanation: } [2x^2 - 5x]_0^k = 3$$

$$2k^2 - 5k - 3 = 0$$

$$(2k + 1)(k - 3) = 0$$

$$k = -\frac{1}{2}, 3$$

Question 11*Answer:* B

$$\text{Explanation: } 0.3 = \frac{\Pr(A \cap B)}{0.4}$$

$$\Pr(A \cap B) = 0.12$$

$$0.6 = \Pr(A) + 0.4 - 0.12$$

$$\Pr(A) = 0.32$$

Question 12*Answer:* E

$$\text{Explanation: } 0.7 + 6k = 1$$

$$k = 0.05$$

$$E(X) = 0 + 0.1 + 0.9 + 0.6 = 1.6$$

Question 13

Answer: E

Explanation: $1 - \text{binomcdf}(30, 0.85, 26) = 0.3217$

Question 14

Answer: A

Explanation: turning point at (2,2), point of inflection at (5,7) makes it a quartic curve

Question 15

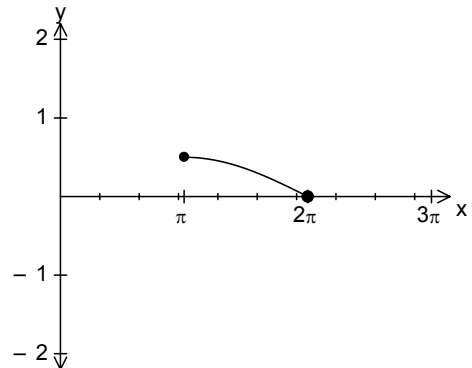
Answer: B

Explanation: $\frac{\Pr(1.6 < X < 1.8)}{\Pr(X < 1.8)} = \frac{\text{normalcdf}(1.6, 1.8, 1.5, 0.25)}{\text{normalcdf}(-10^{99}, 1.8, 1.5, 0.25)} = 0.2594$

Question 16

Answer: C

Explanation: mode is the highest point, $x = \pi$



Question 17

Answer: E

Explanation: let $y = 0$

$$0 = 4 \sin\left(\frac{x}{2}\right) + 2, x \in [-\pi, \pi]$$

$$-\frac{1}{2} = \sin\left(\frac{x}{2}\right)$$

$$\frac{x}{2} = -\frac{\pi}{6}, -\frac{5\pi}{6} \Rightarrow x = -\frac{\pi}{3}, -\frac{5\pi}{3}$$

Question 18

Answer: A

Explanation: $\int_{-1}^3 (5) dx - \frac{1}{2} \int_{-1}^3 (g(x)) dx$
 $[5x]_{-1}^3 - \frac{1}{2}(-6)$
 $15 + 5 + 3 = 23$

Question 19

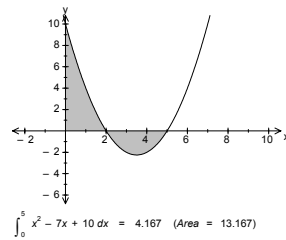
Answer: A

Explanation: $\int \left(\frac{4}{3 - 2(x-1)} \right) dx$
 $\int \left(\frac{4}{1 - 2x} \right) dx$
 $-\frac{4}{2} \ln|1 - 2x| + c \Rightarrow -2 \ln|1 - 2x| + c$

Question 20

Answer: A

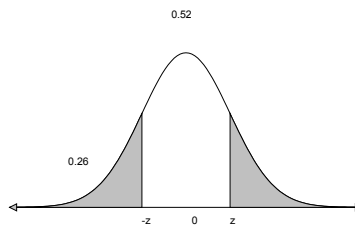
Explanation:
 $\int_0^2 (x^2 - 7x + 10) dx - \int_2^5 (x^2 - 7x + 10) dx = 13 \frac{1}{6} \text{ sq. units}$



Question 21

Answer: A

Explanation: $\text{invNorm}(0.26,0,1) = 0.6433$



Question 22

Answer: D

Explanation: $x = 5, y = 26 \ln(16), \text{ and } x = 3, y = 26 \ln(10)$

$$\begin{aligned} \text{avrate} &= \frac{26 \ln(16) - 26 \ln(10)}{5 - 3} \\ &= \frac{26 \left(\ln \left(\frac{16}{10} \right) \right)}{2} \\ &= 13 \ln \left(\frac{8}{5} \right) \end{aligned}$$

SECTION 2: Analysis Questions

Question 1

a. Turning point at $x = 1 \therefore b = 1$, x intercept at $x = 6 \therefore c = 3$,

At back door $(0,3)$ sub into equation $3 = a(-1)^2(-6)$

$$3 = -6a$$

$$a = -\frac{1}{2}, b = 1, c = 6$$

M1+A1
2 marks

b. expand (or use product rule) or use CAS calculator.

$$y = -\frac{1}{2}x^3 + 4x^2 - \frac{13}{2}x + 3$$

$$\frac{dy}{dx} = -\frac{3}{2}x^2 + 8x - \frac{13}{2}, \frac{dy}{dx} = 0$$

$$0 = -3x^2 + 16x - 13$$

$$\therefore TP \text{ is } \left(4\frac{1}{3}, 9\frac{7}{27}\right)$$

M2+A1
3 marks

c. $(0,3)(4,9) \Rightarrow m = \frac{9-3}{4-0} = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x - 0)$$

$$y = \frac{3}{2}x + 3$$

M1+A1
2 marks

d. Find point on curve $Area = \int_0^4 \frac{3x + 6 + x^3 - 8x^2 + 13x - 6}{2} dx$

$$\int_0^4 8x + \frac{x^3}{2} - 4x^2 dx$$

$$\left[4x^2 + \frac{1}{8}x^4 - \frac{4}{3}x^3 \right]_0^4$$

$$10\frac{2}{3} \text{ sq units}$$

M1+A1
2 marks

e. Let $x=2, y=2$ ($f(2)=2$)

$$\frac{dy}{dx} = -\frac{3}{2}x^2 + 8x - \frac{13}{2}, x=2$$

$$m_t = 3.5, \Rightarrow m_n = -\frac{2}{7}$$

$$y-2 = -\frac{2}{7}(x-2)$$

$$\therefore g(x) = -\frac{2x}{7} + 2\frac{4}{7}$$

M2+A1
3 marks

f. Sub $y=0, x=9$ so no, it won't fit before the side gate

A1
1 mark

Question 2

a. $a^x = \frac{b}{b^x}$

$$a^x b^x = b$$

$$(ab)^x = b$$

$$\log_{ab} b = x$$

$$x = \frac{\ln(b)}{\ln(ab)}$$

$$\frac{\ln(b)}{\ln(a) + \ln(b)}$$

M1+A1
2 marks

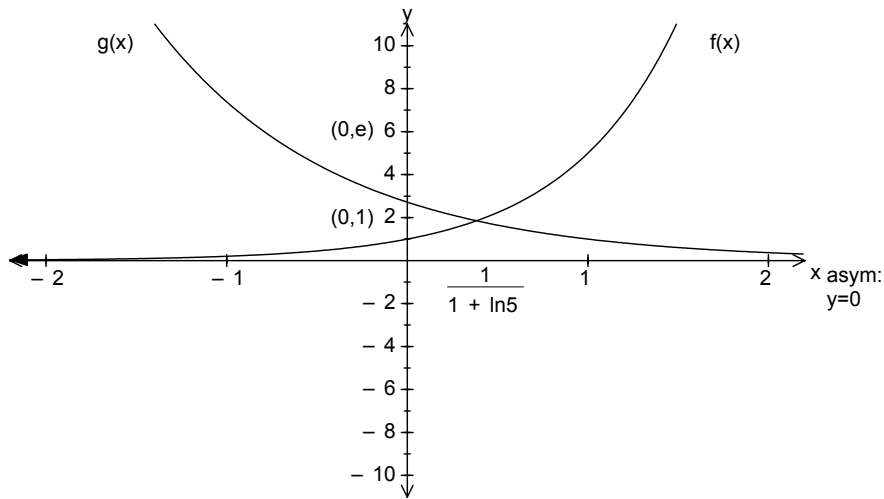
b.
$$\frac{\ln(4)}{\ln(9) + \ln(4)}$$

$$\frac{2 \ln(2)}{2 \ln(3) + 2 \ln(2)}$$

$$\frac{\ln(2)}{\ln(6)}$$

M1+A1
2 marks

c. Correct shape, show important points, $(0,e)$, $(0,1)$



A2
2 marks

d. point of intersection at $x = \frac{1}{1 + \ln(5)}$

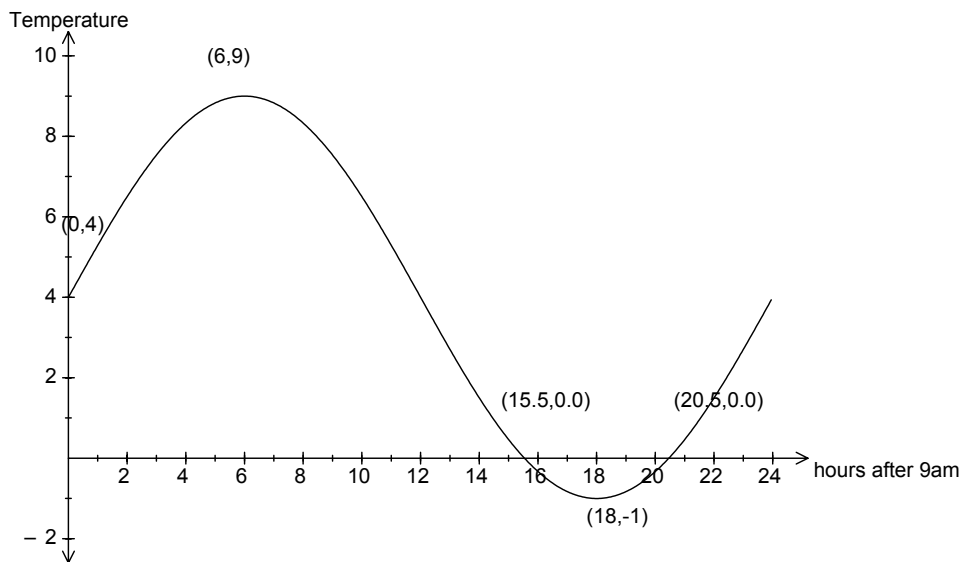
M1+A1
2 marks

Question 3

a. Temperature range is $[-1,9]$

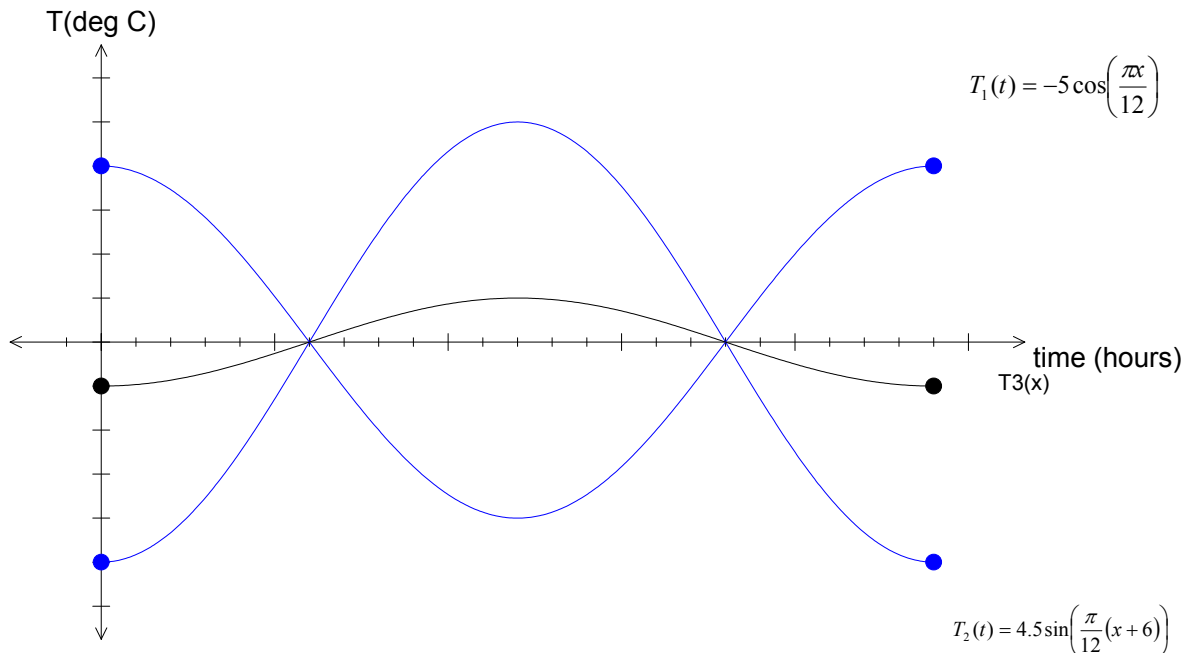
A1
1 mark

b.



A3
3 marks

c.



A1
1 mark

d. [-1,1]

A1
1 mark

Question 4

a. $V = \pi \left(\frac{x}{2}\right)^2 l$

$$100 = \pi \left(\frac{x}{2}\right)^2 l$$

$$l = \frac{400}{\pi x^2}$$

M1+A1
2 marks

b. $V = a^2 l$

$$= \frac{x^2}{2} \times \frac{400}{\pi x^2}$$

$$= \frac{200}{\pi} \text{ cm}^3$$

M1+A1
2 marks

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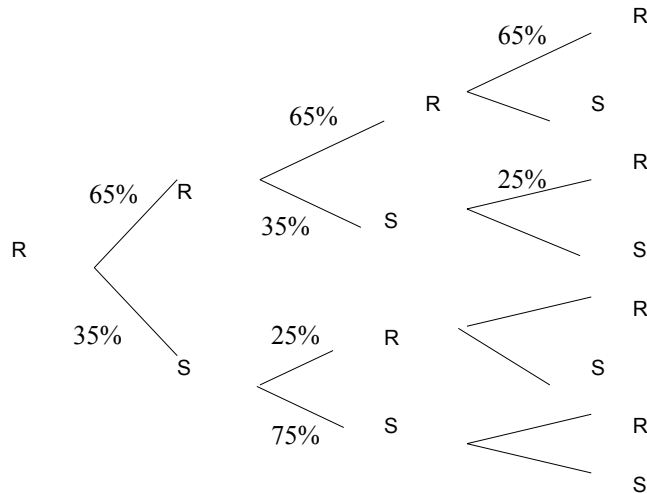
c. $\Pr(X \leq 2) = \text{binomcdf}(100, .03, 2) = 0.4198$

M1+A1
2 marks

d. $\Pr(X \geq 9) = 1 - \text{binomcdf}(10, 0.4198, 1) = 0.0025$

M1+A1
2 marks

e.



Pr (round in April | round in Jan)

$$= (0.65)^3 + 0.65 \times 0.35 \times 0.25 + 0.35 \times 0.25 \times 0.65 + 0.35 \times 0.75 \times 0.25$$

$$= 0.4540$$

M2+A1
3 marks

f. Pr (square on at least 2 of next 3 | round in Jan)

$$= 0.65 \times 0.35 \times 0.75 + 0.35 \times 0.25 \times 0.35 + 0.35 \times 0.75 \times 0.25 + 0.35 \times (0.75)^2$$

$$= 0.4638$$

M1+A1
2 marks

g.

i. $\frac{6}{5} \int_1^m (x^2 - x) dx = 0.5$

$$\frac{6}{5} \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_1^m = 0.5$$

$$\frac{6}{5} \left[\frac{m^3}{3} - \frac{m^2}{2} - \frac{1}{3} + \frac{1}{2} \right] = 0.5$$

Intersect on calculator $m = 1.75$ therefore median cost is \$1.75

M2+A1
3 marks

ii. $\frac{6}{5} \int_1^{1.6} (x^2 - x) dx = 0.3024$

M1+A1
2 marks

h.

i. $\Pr(Z < z) = 0.11 \Rightarrow z = \text{invNorm}(0.11, 0, 1) = -1.2265$

$\sigma = \frac{107 - 108}{-1.2265} = 0.82$

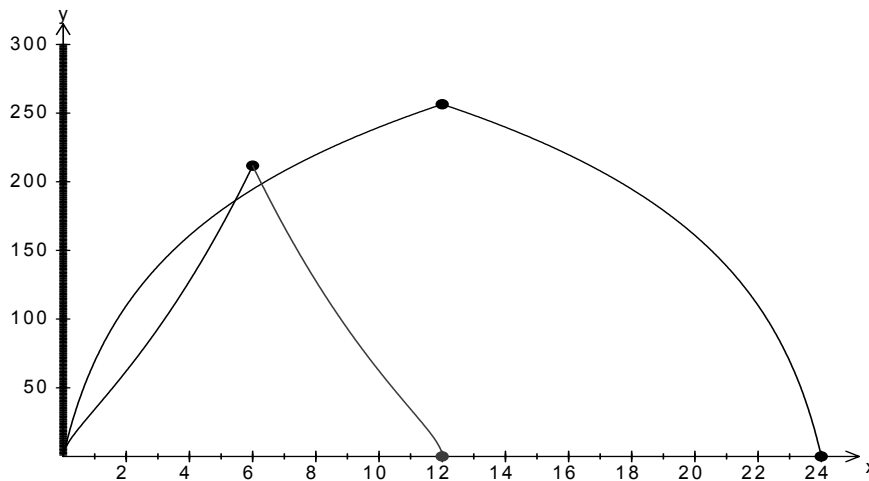
M2+A1
3 marks

ii. $\mu \pm 2\sigma \Rightarrow [106.36, 109.64]g$

A1
1 mark

Question 5

a.



A1
1 mark

- b. i. reflected in y axis, then translated 24 units right
ii. reflected in y axis then translated 12 units right

A2
2 marks

- c. i. On calculator $A = 256.49 g$
ii. $B = 211.56 g$

A2
2 marks

- d. $198.54025 \times 2 = 397.08g$ (2 x y part of intersection of A_2 and B_1)

A1
1 mark

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e. 6.28 hours = 6 hours 17min (*x* part of intersection of A_2 and B_1)

A1
1mark

f. *X* parts of intersection of A_1 and $y=125$ and A_2 and $y=125$ times are 3.9244 and 8.0756 hours
Giving 4.1512 hours or 4 hours 9 min

M1+A1
2 marks