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MATHEMATICAL METHODS (CAS) UNITS 3 & 4

TRIAL EXAMINATION 2

2010

Reading Time: 15 minutes
Writing time: 2 hours

Instructions to students

This exam consists of Section 1 and Section 2.
Section 1 consists of 22 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 26 of this exam.
Section 2 consists of 4 extended-answer questions.
Section 1 begins on page 2 of this exam and is worth 22 marks.
Section 2 begins on page 11 of this exam and is worth 58 marks.
There is a total of 80 marks available.
All questions in Section 1 and Section 2 should be answered.
Diagrams in this exam are not to scale except where otherwise stated.
Where more than one mark is allocated to a question, appropriate working must be shown.
Students may bring one bound reference into the exam.
Students may bring a CAS calculator into the exam.
A formula sheet can be found on page 25 of this exam.

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SECTION 1

Question 1

The maximal domain of the function with the rule $y = 1 + \sqrt{x-2}$ is

A. R^+
B. $(1, \infty)$
C. $[1, \infty)$
D. $(2, \infty)$
E. $[2, \infty)$

Question 2

For the function $f(x) = ax^3 + bx^2 + cx + d$,

$$f(1) = 4$$
$$f(-2) = 10$$
$$f'(1) = 4$$
$$\text{and } f'(3) = 36$$

A matrix equation $AX = B$ is to be used to find the values of *a*, *b*, *c* and *d* where

$$X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 10 \\ 4 \\ 36 \end{bmatrix}.$$

The matrix A is

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -8 & 4 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -2 & 1 & 3 \\ -2 & -2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -8 & 4 & -2 & 1 \\ 3 & 2 & 1 & 1 \\ 81 & 6 & 1 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -8 & 4 & -2 & 1 \\ 3 & 2 & 1 & 0 \\ 27 & 6 & 1 & 0 \end{bmatrix}$

Question 3

Let $f: R \rightarrow R,\ f(x) = x(x-2)^2$.

It is true to say that the graph of $y = f(x)$ has

A. a stationary point of inflection.
B. a local minimum located at a point where $x < 0$.
C. a local maximum located at a point where $x < 0$.
D. a point of inflection.
E. three stationary points.

Question 4

If $\int_2^4 f(x)dx = 3$, then $\int_2^4 (1 - 5f(x))dx$ is equal to

A. -2
B. -5
C. -13
D. -14
E. -15

Question 5

The continuous random variable X has a mean of 22 and a standard deviation of 4. The random variable Z has the standard normal distribution. The probability that X is less than 16 is equal to

A. $\Pr(Z < 2)$
B. $\Pr(Z < 1.5)$
C. $\Pr(Z > 1)$
D. $\Pr(Z > 1.5)$
E. $\Pr(Z > 2)$

The information below relates to Questions 6 and 7.

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \dfrac{-3x}{4}(x-2) & \text{if } 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Question 6

The mode of X is

A. 0
B. 0.75
C. 1
D. 1.5
E. 2

Question 7

$\Pr(X < 0.5 | X < 1)$ correct to four decimal places is equal to

A. 0.0781
B. 0.1563
C. 0.1582
D. 0.3125
E. 0.3164

Question 8

Let $f : R \to R, f(x) = |x^2 - 2x|$.
The gradient function, f', is positive for

A. $x \in R^+$
B. $x \in (1, \infty)$
C. $x \in [2, \infty)$
D. $x \in (0,1) \cup (2, \infty)$
E. $x \in [0,1) \cup [2, \infty)$

Question 9

A transformation $T : R^2 \to R^2$ is defined by $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \pi \\ 2 \end{bmatrix}$.
The equation of the image of the curve $y = \cos(3x)$ under this transformation is

A. $y = 2\cos(\pi - x) - 2$
B. $y = 2\cos(-(x + \pi)) - 2$
C. $y = 2\cos(-(x + \pi)) + 2$
D. $y = 2\cos(-(x + 3\pi)) + 2$
E. $y = \dfrac{1}{2}\cos(6(x - \pi)) + 2$

Question 10

At the point $(2,7)$ the graph of the curve $y = g(x)$ has a tangent with equation $y = 3x + 1$. At the point $(0,8)$ the graph of the curve $y = 1 + g(x + 2)$ has a tangent with equation

A. $y = \frac{-x}{2} + 8$
B. $y = -3x + 2$
C. $y = 3x - 4$
D. $y = 3x - 8$
E. $y = 3x + 8$

Question 11

If $g(x) = \frac{f(x)}{f(x) + 1}$ then $g'(x)$ is equal to

A. $f'(x)(2f(x) + 1)$
B. $\frac{1}{f(x)+1}$
C. $\frac{f'(x)}{(f(x) + 1)^2}$
D. $\frac{f'(x) + 1}{(f(x) + 1)^2}$
E. $\frac{f'(x)(2f(x) + 1)}{(f(x + 1))^2}$

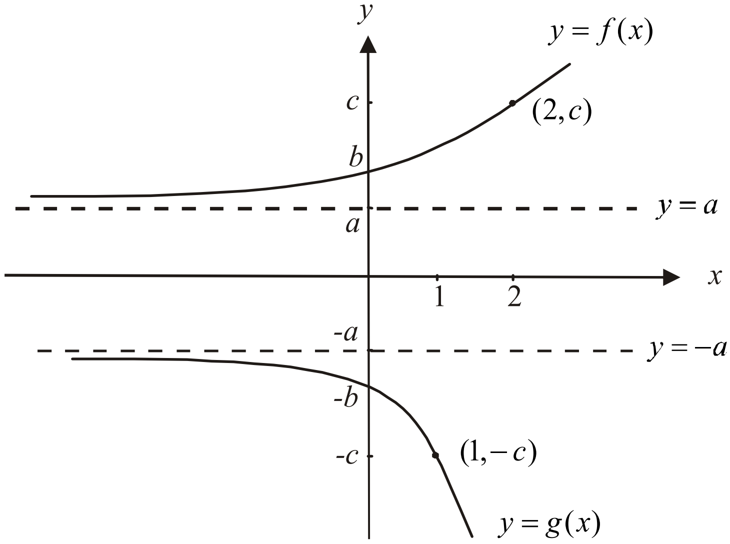
Question 12

A dice has been tampered with so that the probability of throwing a six is 0.2.
Geordie throws the dice 10 times. The probability that he obtains a six more than twice is closest to

A. 0.3222
B. 0.3758
C. 0.4927
D. 0.6241
E. 0.6778

Question 13

The diagram below shows the graphs of $y = f(x)$ and $y = g(x)$. The function *f* has undergone a transformation to become the function *g*.



A transformation $T: R^2 \to R^2$ that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$ is given by

A. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

B. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

C. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$

D. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

E. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2b \end{bmatrix}$

Question 14

Let $f(x) = \log_e(x)$ and $g(x) = x^2 + 1$.
Both functions have a maximal domain.
The range of the function $f(g(x))$ is

A. $(0, \infty)$
B. $[0, \infty)$
C. $(1, \infty)$
D. $[1, \infty)$
E. $R \setminus \{\pm 1\}$

Question 15

For the function $f(x) = |x+2| - 2$ the rate of change of y with respect to x is

A. not defined for $x = 0$
B. zero at $x = -4$
C. negative for $x \in (-\infty, -2]$
D. negative for $x \in (-4, -2) \cup (-2, 0)$
E. never equal to zero

Question 16

The inverse of the function $g: R \setminus \{5\} \to R,\ g(x) = 2 + \dfrac{1}{x-5}$ is given by

A. $g^{-1}: R \setminus \{2\} \to R,\ g^{-1}(x) = 5 + \dfrac{1}{x-2}$

B. $g^{-1}: R \setminus \{5\} \to R,\ g^{-1}(x) = 5 + \dfrac{1}{x-2}$

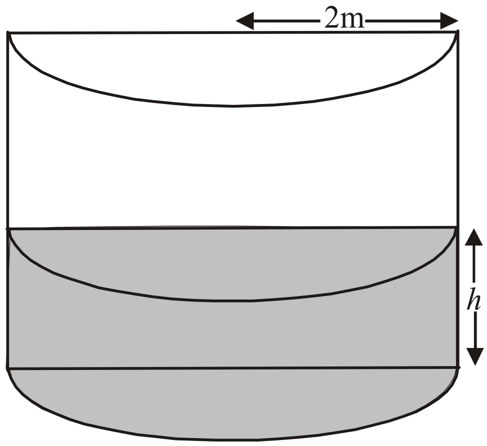
C. $g^{-1}: R \setminus \{5\} \to R,\ g^{-1}(x) = 2 + \dfrac{1}{x-5}$

D. $g^{-1}: R \setminus \{2\} \to R,\ g^{-1}(x) = 2 + \dfrac{1}{x-5}$

E. $g^{-1}: R \to R,\quad g^{-1}(x) = -2 + \dfrac{1}{x-5}$

Question 17

A water tank in the shape of a prism with a semi-circular cross section and a radius of 2 metres is being filled with water. The height, h, of the water in the tank is increasing at the rate of 0.02m per hour.



The rate at which the volume of water in the tank V, is increasing in cubic metres per hour is given by

A. $\dfrac{1}{100\pi}$
B. $\dfrac{1}{2\pi}$
C. 0.04π
D. 0.2π
E. 10π

Question 18

Let $f:(-\pi,\pi)\rightarrow R, f(x)=\left|\tan\left(\dfrac{x}{2}\right)\right|$.

The number of solutions to the equation $f(x)=1$ is/are

A. 0
B. 1
C. 2
D. 3
E. 4

Question 19

The area enclosed by the graph of $y = e^{2(x+1)} - 1$, the x axis and the y axis is given by

A. $\int_{-1}^{0} (e^{2(x+1)} - 1)dx$

B. $\int_{-1}^{0} (1 - e^{2(x+1)})dx$

C. $\int_{-1}^{0} (e^{2(x+1)} + 1)dx$

D. $\int_{-1}^{e^2-1} (e^{2(x+1)} - 1)dx$

E. $\int_{-1}^{e^2-1} (1 - e^{2(x+1)})dx$

Question 20

Eight identical balls numbered 1 – 8 are placed in a box. One ball is randomly selected.
The sample space for this activity is $\{1,2,3,4,5,6,7,8\}$.
Which pair of events listed below are independent of one another?

A. $\{1,2,3,4\}$ and $\{5,6,7,8\}$
B. $\{1,2,3,4\}$ and $\{1,4,5,8\}$
C. $\{1,2,3\}$ and $\{2,4,6,8\}$
D. $\{1,2,3\}$ and $\{1,8\}$
E. $\{1,2\}$ and $\{2,8\}$

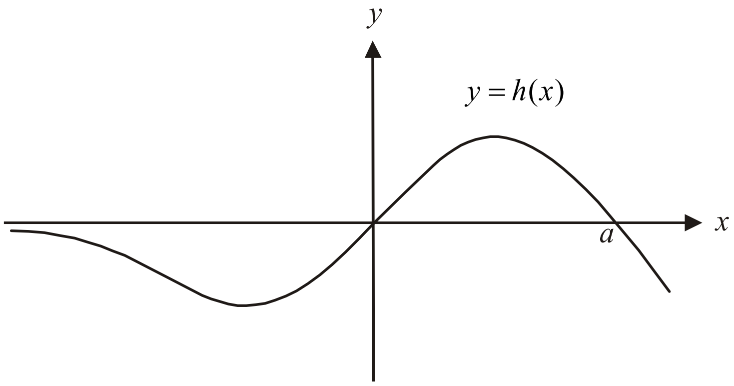
Question 21

When the approximation formula $f(x+h) \approx f(x) + hf'(x)$ is used with $f(x) = \sqrt{x}$, and $x = 5$, an approximate value for $\sqrt{4.7}$ is given by

A. $f(0.3) + 5f'(0.3)$
B. $f(0.7) - 5f'(0.7)$
C. $f(4.7) - 5f'(4.7)$
D. $f(5) - 0.3f'(5)$
E. $f(5) + 0.3f'(5)$

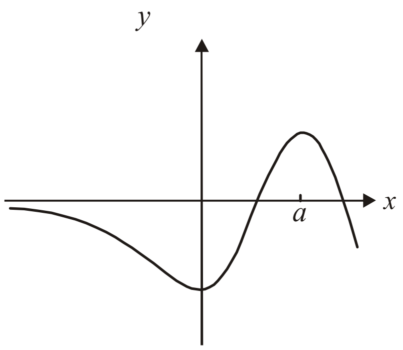
Question 22

The graph of the function h is shown below.

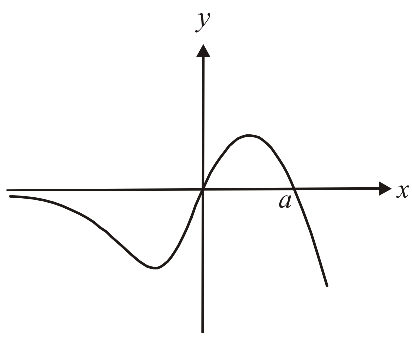


The graph of an antiderivative function could be

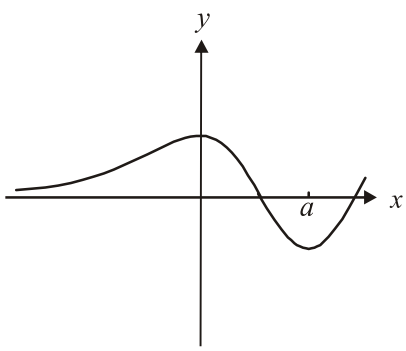
A.



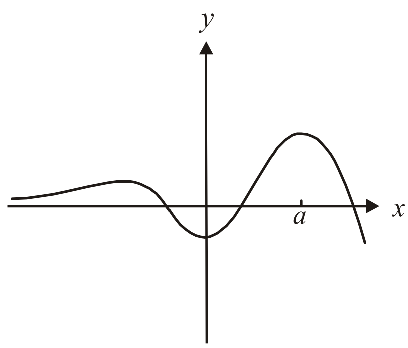
B.



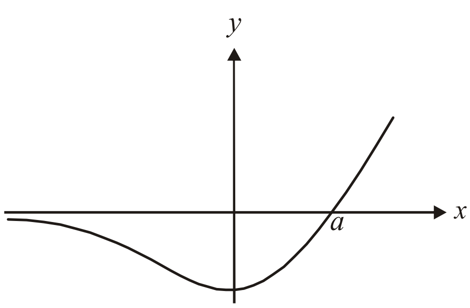
C.



D.



E.

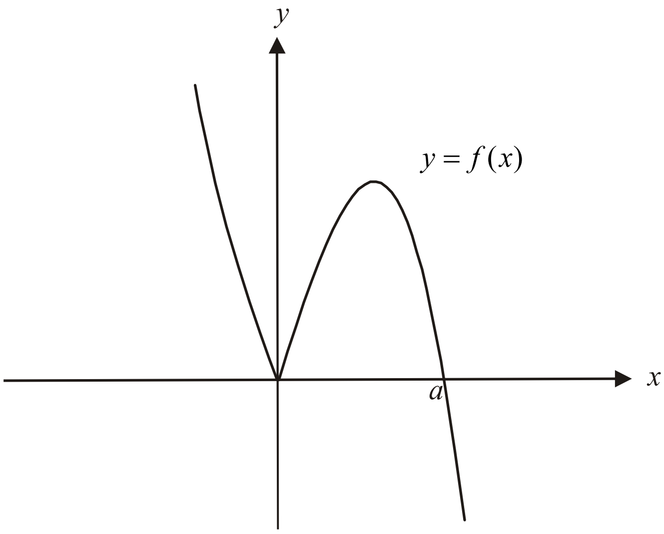


SECTION 2

Answer all questions in this section.

Question 1

Let $f: R \rightarrow R,\ \ f(x) = 3|x| - x^3$.
The graph of $y = f(x)$ is shown below.



The x-intercepts occur at the points $(0,0)$ and $(a,0)$.

a. Find the value of a.

1 mark

b. Find the coordinates of the local maximum.

1 mark

c. For $x > 0$, find the values of x for which the graph of f is strictly decreasing.

1 mark

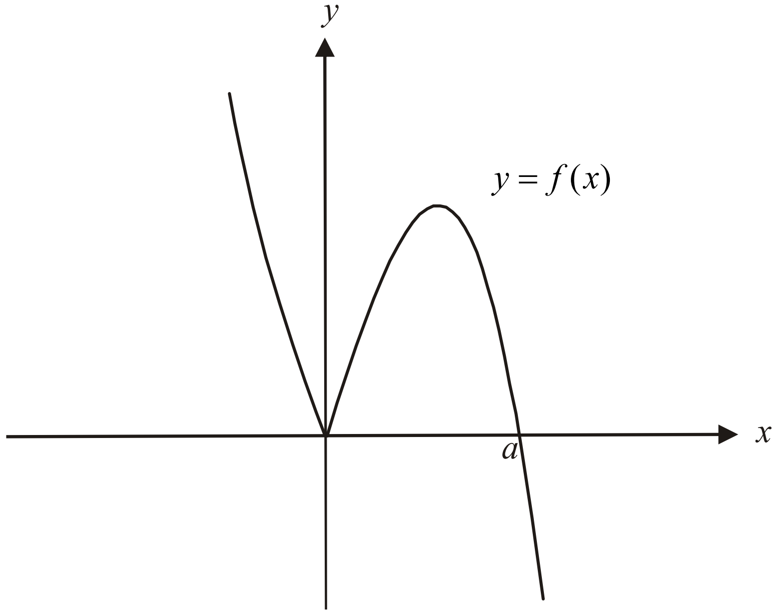
d. The function g has the same rule as the function f but $d_g = [c, \infty)$. The inverse function $g^{-1}(x)$ exists.

i. Find the minimum value of c.

____

ii. On the set of axes below, which shows the graph of $y = f(x)$, sketch the graph of $y = g^{-1}(x)$. Indicate clearly on your graph any endpoints or intercepts.

$1+2=3$ marks



e. Find the area enclosed by the graph of $y = f(x)$ and the x-axis.

____

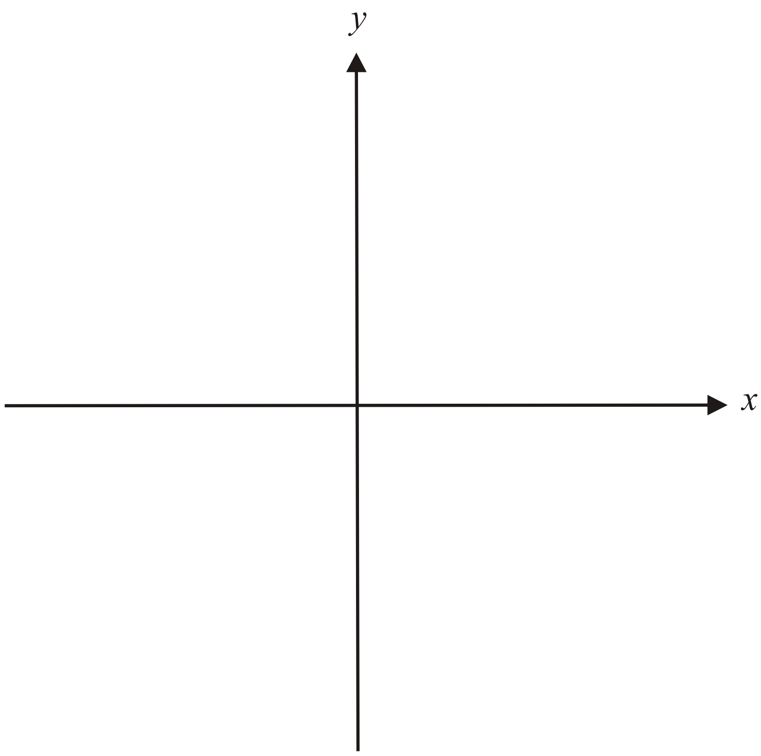
____

____

2 marks

f. **i.** Find the derivative function $f'(x)$.

ii. Sketch the graph of $y = f'(x)$ on the set of axes below.



$2+3=5$ marks

g. Hence find the values of x for which the function $f'(x)$ has a unique value.

2 marks

Total 15 marks

Question 2

A city fun run is held each year. The time, in minutes, taken by each of the entrants to complete the fun run is normally distributed with a mean of 45 minutes and standard deviation of 3 minutes.

a. What percentage of entrants, to the nearest percent, record a time of less than 40 minutes?

1 mark

The fastest 30% of entrants receive a medallion.

b. What was the slowest time needed in order to receive a medallion? Express your answer in minutes correct to 3 decimal places.

1 mark

A randomly selected entrant who received a medallion is interviewed after the race.

c. What is the probability, correct to 4 decimal places, that this entrant recorded a time of less than 40 minutes?

1 mark

Five entrants in the fun run are randomly selected to receive a prize from one of the sponsors.

d. What is the probability that exactly two of these entrants have already received a medallion?

2 marks

The time taken, in minutes, for each entrant to register for the fun run is a continuous random variable X, with probability density function given by

$$f(x)=\begin{cases}\dfrac{5}{4x^2} & \text{if } 1\le x\le 5\\ 0 & \text{elsewhere}\end{cases}$$

e. Find the probability that an entrant takes less than 2 minutes to register.

2 marks

f. Find the variance of X.
Express your answer correct to 4 decimal places.

3 marks

Race entrants must have their entrant number clearly visible. They can opt to wear a bib showing the number or have the number temporarily tattooed to the upper part of their arm.

Race organizers noted over the years that 70% of entrants who wore a bib one year opted for a tattoo the next and 60% of entrants who had a tattoo one year opted for a tattoo again the next year.

Jane wore a bib the first year she entered the race.

g. **i.** What is the probability that Jane opted for a tattoo in each of the next three years?

ii. What is the probability that Jane opts for a bib in at least two of the next three years? Express your answer correct to 4 decimal places.

iii. What is the probability that Jane will wear a bib in the tenth year? Express your answer correct to 4 decimal places.

iv. What is the probability that Jane opts for a tattoo in the long term? Express your answer correct to 4 decimal places.

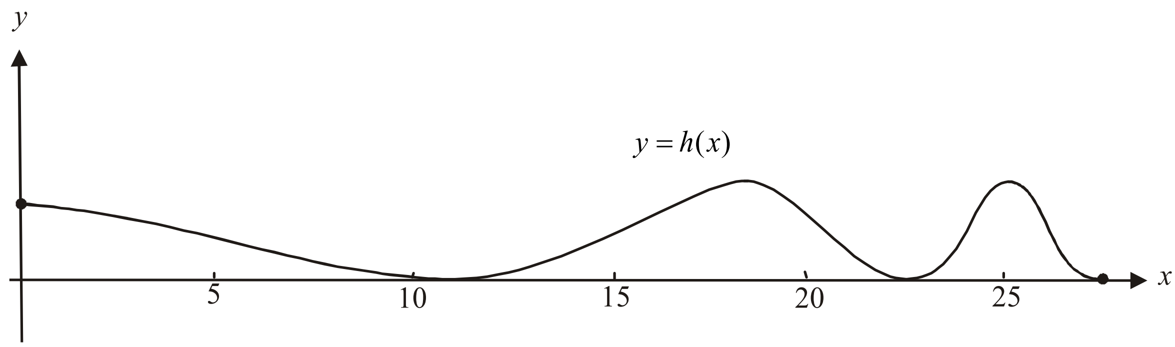
$1+2+3+1=7$ marks
Total 17 marks

Question 3

At a recycling station there are piles of landfill.

A cross-section of some of these piles is shown on the graph below where the height, *h*, in metres above ground level is given by the function.

$$h:[0, 10\log_e(5\pi)] \to R,\ h(x) = \cos(e^{\frac{x}{10}}) + 1$$



The x-axis represents ground level and x, in metres, represents the distance from a fence at the perimeter of the recycling station.

a. The function h is a composite function where $h(x) = f(g(x))$.

i. Write down the rule for $g(x)$ and for $f(x)$.

ii. Explain whether or not the composite function $g(f(x))$ exists.

$1+1=2$ marks

b. Find $h'(x)$.

1 mark

c. Find the coordinates of the point closest to the fence that is not covered in landfill.

1 mark

d. Find the

i. maximum height of the landfill.

ii. distance(s) from the fence where this maximum height occurs. Express your answer(s) as an exact value.

$1+2=3$ marks

e. A bulldozer is to be used to flatten out the piles of landfill represented by the function *h*. The surface will then be horizontal.
How high above the ground will the landfill be after this has happened? Express your answer in metres correct to 2 decimal places.

2 marks

f. A straight metal pole has to be inserted into the landfill at right angles to the slope of the pile. The pole is pushed through until its end touches the ground so that the temperature through the pile can be recorded.

The pole is inserted at a point for which $x \in [5,10]$ and the gradient of the pole when it has been inserted is 6.5.
Find the horizontal distance of the end of the pole from the fence. Express your answer correct to 3 decimal places.

3 marks
Total 12 marks

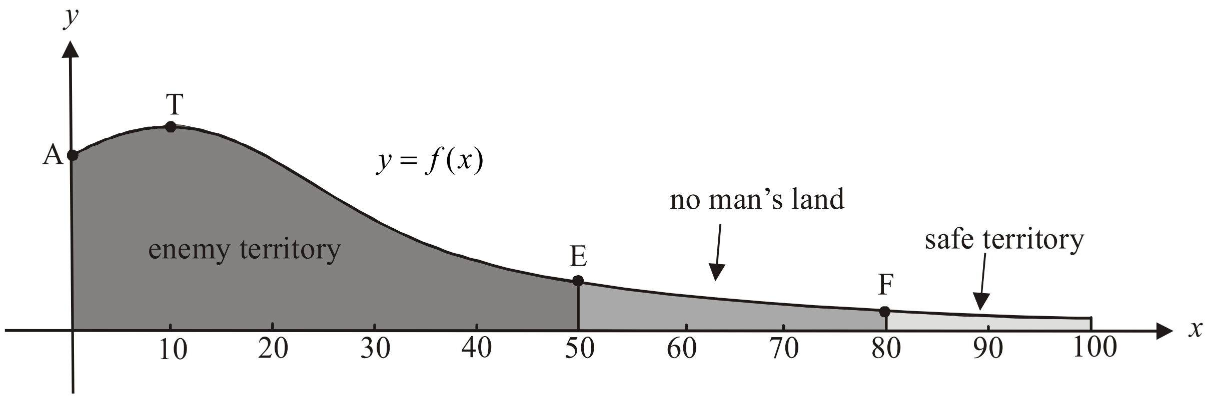
Question 4

Victoria James is a spy.
She is attempting to flee enemy territory and her escape is being planned.

The cross-section of the terrain surrounding her is given by the function

$$f:[0,100]\to R, f(x)=\frac{2500}{x^2-20x+600}$$

where *f* represents the height in metres of the ground above sea level and *x* represents the horizontal distance in metres from Victoria's starting point at *A*. The graph of $y=f(x)$ is shown below.



At point *A*, Victoria is hidden behind the top, *T*, of a small hill which shields her from the enemy guard post located at point *E*.
At point *F*, there are friendly guards and safe territory extends to the right of *F* and beyond the point where $x=100$.
Between points *E* and *F* is "no-man's land". Victoria is trying to make it to point *F* or beyond.

a. Find the coordinates of point *A*. Express values in exact form.

1 mark

b. How far vertically below point *T* is Victoria initially.

1 mark

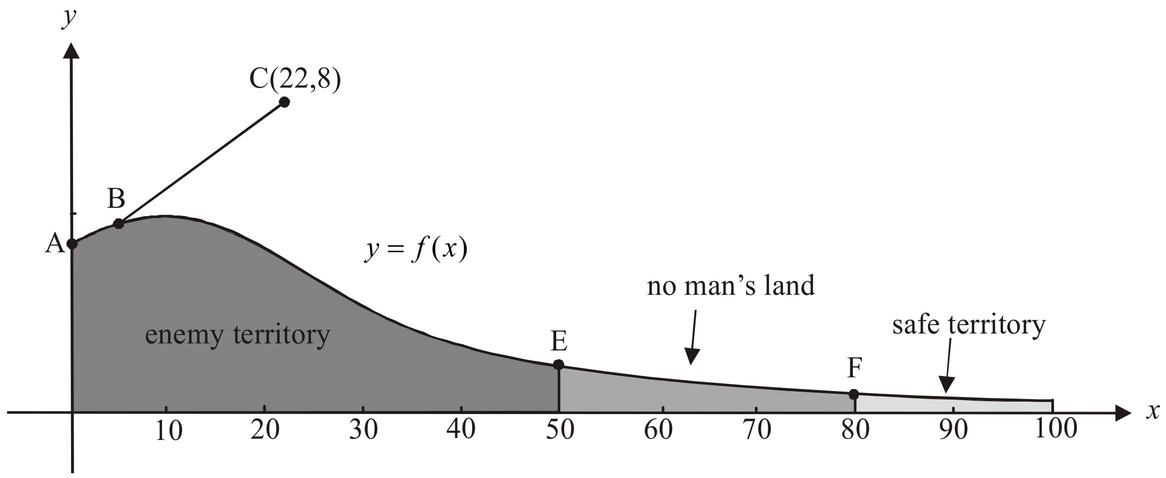
Victoria has propulsion equipment attached to her. This will enable her to become airborne at point B where the gradient of the curve is $\frac{40}{441}$.

c. Find the coordinates of point B.

2 marks

Once airborne at point B, Victoria is to follow a path which is a straight line with a gradient of $\frac{4}{21}$ until she reaches point $C(22,8)$.

This path from B to C is shown below.



d. Find the equation of BC.

1 mark

e. How far above the ground would Victoria be at point C? Express your answer in metres correct to 1 decimal place.

1 mark

After Victoria reaches point $C(22,8)$, her flight path is to change. Her new flight path is to be given by a function with the rule

$$y = -\frac{1}{k}(x-22)^2 + 8$$

f. What is the maximum height above sea level that Victoria will have whilst on this new flight path?

1 mark

g. Find the value(s) of k, expressed correct to 2 decimal places, which would see Victoria land

i. at the enemy guard post at point E.

ii. in safe territory.

$1+2=3$ marks

Victoria's movements are being tracked by a rescue squad. They believe that her proposed flight will be severely affected by strong winds.
Their proposed flight path for the latter part of her escape flight is given by

$$y = -\frac{1}{450}(x-22)^2 + 8 .$$

They now believe that this path will undergo a transformation defined by the matrix $\begin{bmatrix} 0.8 & 0 \\ 0 & 0.4 \end{bmatrix}$ because of the winds.

h. Find the equation of the transformed flight path.

2 marks

It is decided to wait for the winds to subside. When they do, Victoria flies successfully to the point $C(22, 8)$ and then follows the path proposed by the rescue squad given by

$$y = -\frac{1}{450}(x-22)^2 + 8$$

i. Explain whether or not Victoria lands in safe territory.

2 marks
Total 14 marks

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad$ $\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad$ $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$ $\qquad$ $\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad$ $\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad$ $\int \cos\ (ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ $\qquad$ quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ $\qquad$ approximation: $f(x+h) \approx f(x) + hf'(x)$

Transition Matrices

$S_n = T^n \times S_0$

Probability

$\Pr(A) = 1 - \Pr(A')$ $\qquad$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$ $\qquad$ variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x\, p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$

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MATHEMATICAL METHODS (CAS)

TRIAL EXAMINATION 2

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ▬ C D E

The answer selected is B. Only one answer should be selected.

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E