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INSIGHT
Trial Exam Paper

2010

MATHEMATICAL METHODS
(CAS)

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 1 hour

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 11 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Let $y = \frac{\sin 2x}{e^x}$. Find $\frac{dy}{dx}$.

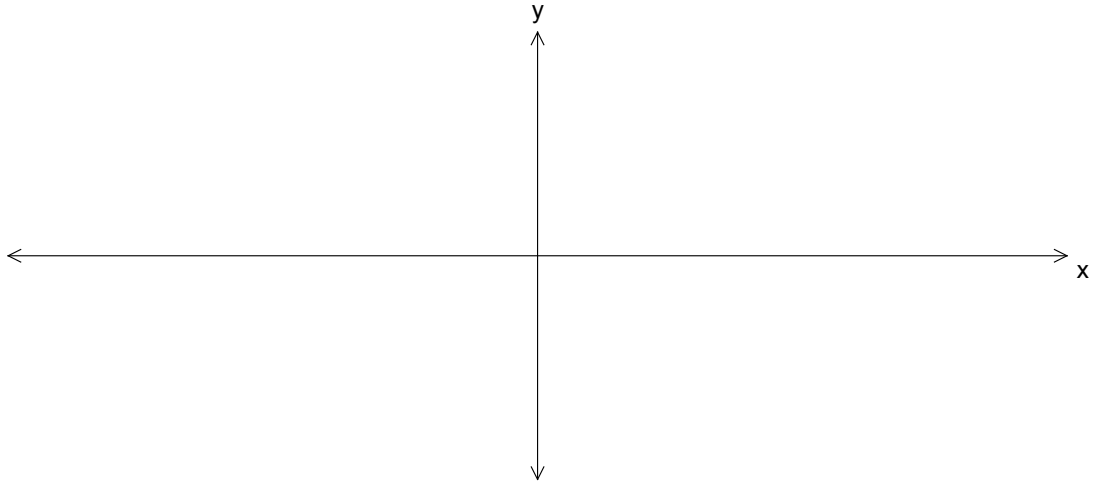
b. If $f(x) = e^{\sqrt{x}}$, find $f'(16)$.

2 + 2 = 4 marks

Question 2

- a. Write $\frac{3x-1}{x+1}$ in the form $\frac{a}{x+1} + b$. State the values of a and b .

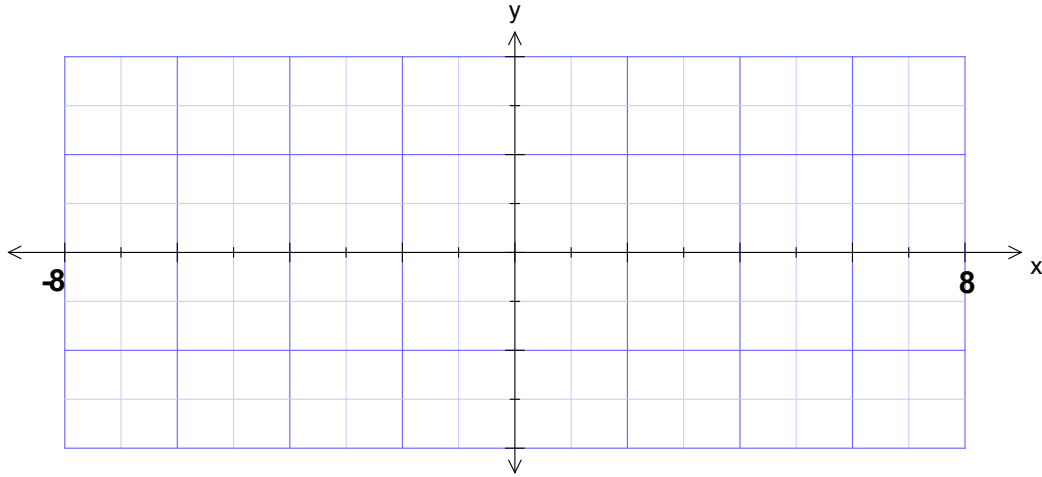
- b. Sketch the graph of $f : R \setminus \{-1\} \rightarrow R$, $f(x) = \frac{3x-1}{x+1}$. Label all axis intercepts as coordinates. Label each asymptote with its equation.



2 + 2 = 4 marks

Question 3

Sketch the graph of $y = 2 \cos\left(\frac{\pi x}{8}\right) + 2$ for $x \in [-8, 8]$. Label axis intercepts and endpoints with their coordinates.



3 marks

Question 4

Ciara is a very good netballer. She plays goal shooter in her team and knows that from past experience her probability of scoring a goal depends on the success of her previous attempt. She knows that if she has scored a goal previously then her probability of scoring a goal on the next attempt is 0.7. If she is unsuccessful on the previous attempt, her probability of being unsuccessful on the next attempt is 0.8.

The probabilities associated with each state are represented in the transition matrix:

$$\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$$

- a.** If she has scored a goal, what is the chance of her not scoring on her next attempt?

- b.** She has five shots at goal. Her first attempt is a goal. What is the probability that it takes her until her final shot to score another goal?

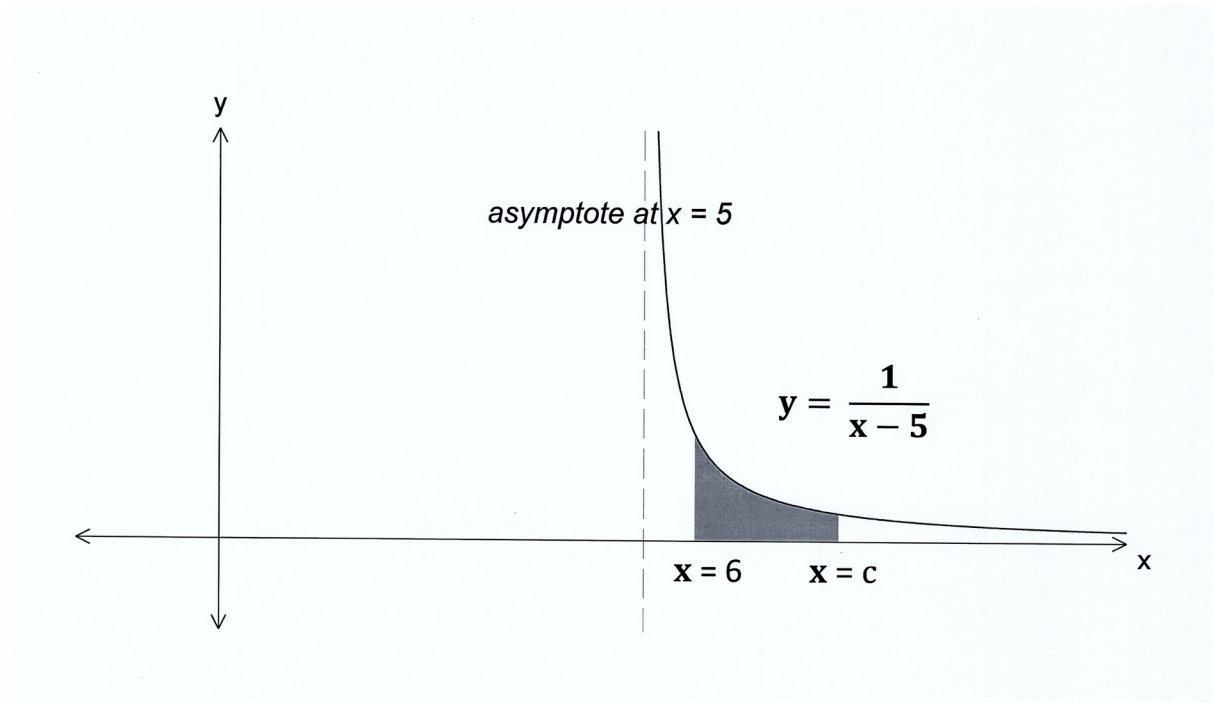
- c.** Find the probability of Ciara scoring a goal in the long term.

1 + 2 + 2 = 5 marks

TURN OVER

Question 5

The shaded area is equal to $\frac{1}{2}$. Find the value of c .



3 marks

Question 6

The duration of telephone calls to the home loan department of a mortgage broker is a random variable X minutes with probability density function

$$f(x) = \begin{cases} \frac{1}{4} e^{-\frac{x}{4}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

- a.** Find $\Pr(4 < X < 8)$

- b.** Find the probability that a telephone call will last more than 12 minutes.

- c.** Find the value of a such that 90% of telephone calls last less than a minutes.

2 + 2 + 3 = 7 marks

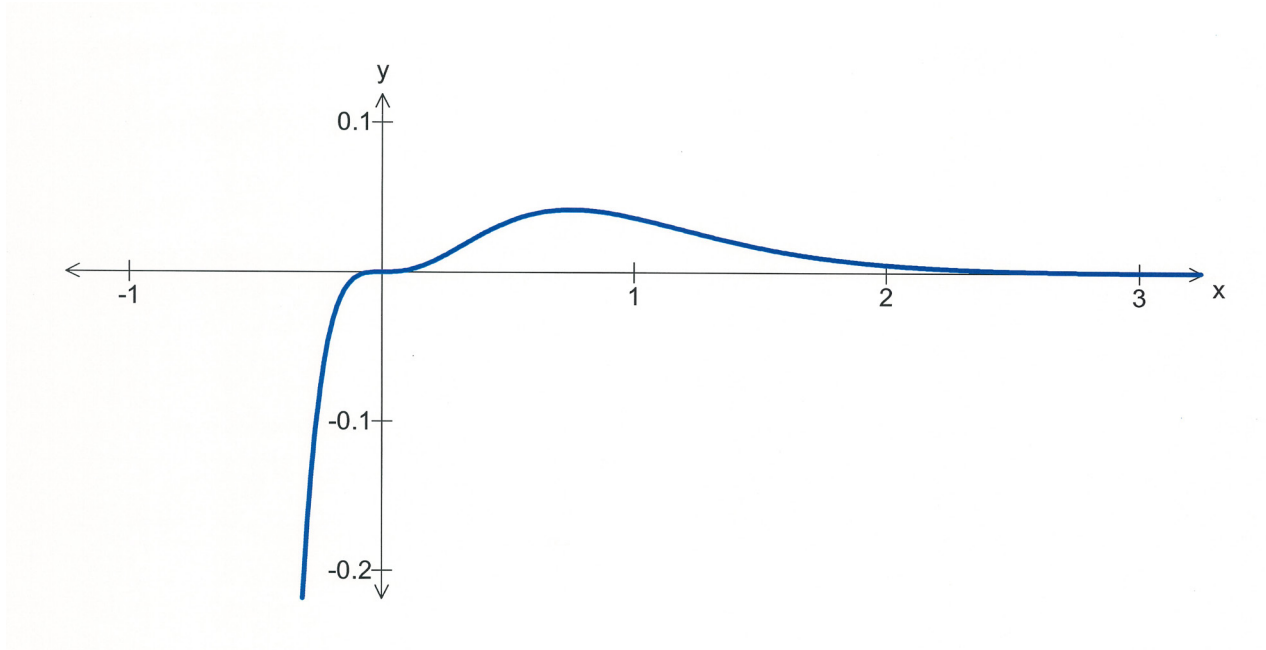
TURN OVER

Question 7

Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x^3e^{-4x}$

- a. $f'(x)$ may be written in the form $f'(x) = e^{-4x}(ax^2 + bx^3)$ where a and b are real constants. Find the values of a and b .

The graph of $y = f(x)$ is as shown.



- b. Find the exact coordinates of the two stationary points and state their nature.

- c. Find the equation of the tangent to the curve $y = f(x)$ where $x = 1$.

2 + 3 + 2 = 7 marks

Question 8

A transformation is defined by the matrix $\begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}$. Find the equation of the image of the graph of the line with the equation $y = 3x + 5$ under this transformation.

2 marks

Question 9

For the simultaneous linear equations:

$$mx - 6y = 6$$

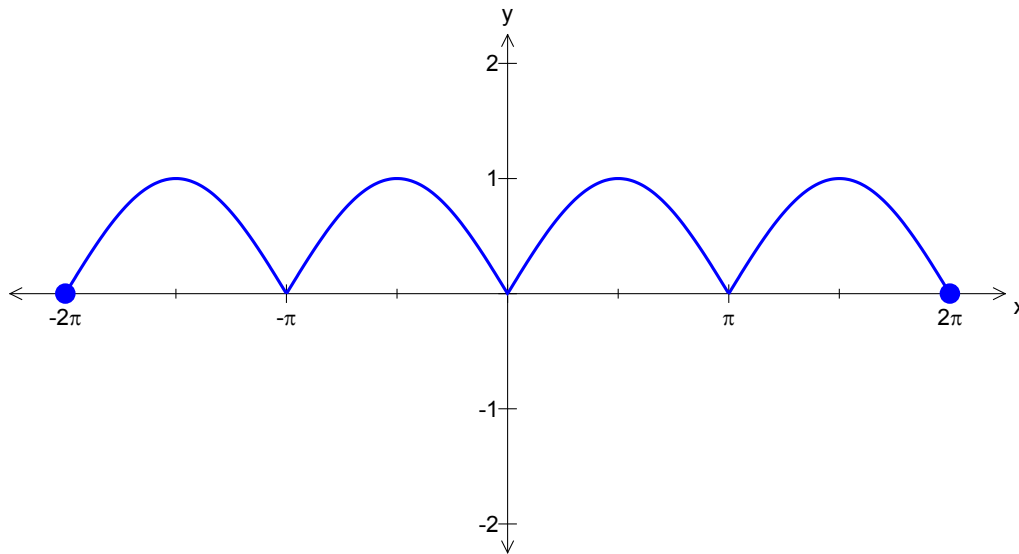
$$4x - my = m$$

find the values of m for which the equations have infinitely many solutions.

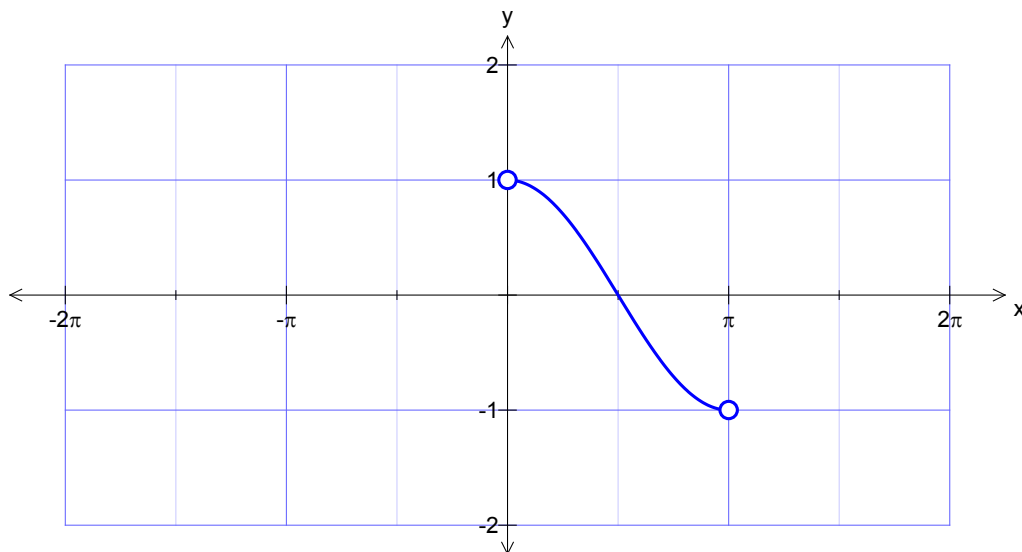
2 marks

Question 10

The graph of the function $f : [-2\pi, 2\pi] \rightarrow \mathbb{R}$, $f(x) = |\sin(x)|$ is shown below



- a. Part of the graph of the derivative function is drawn on the axes given. Complete the graph of the derivative function, f' on the axes given.



- b. State the rule for the derivative function.

1 + 2 = 3 marks

END OF QUESTION AND ANSWER BOOK