2010 Mathematical Methods (CAS) Trial Exam 1 Solutions
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Q1

$-0.5x + y = -1.5$.........(1)

$2x + y = 1$....................(2)

$x - 2y = 3$....................(3)

Equations (1) and (3) represent two superimposed straight lines, and equation (2) represents a straight line cutting across (1) and (3). The coordinates of the intersecting point is:

2×eq(2) + eq(3), $5x = 5$, $x = 1$ and $y = -1$.

Q2a $f(x) = x^2 - 2x$ and

$g(x) = -\frac{1}{2} f(1-2x) + \frac{3}{2} = -\frac{1}{2}\left[(1-2x)^2 - 2(1-2x)\right] + \frac{3}{2}$

$= -\frac{1}{2}\left[4x^2 - 1\right] + \frac{3}{2} = -2x^2 + 2 = -2(x-1)(x+1)$

Q2b $g(x)$ is the result of the following sequential transformations of $f(x)$.

Vertical dilation by a factor of *1/2*
Horizontal dilation by a factor of *1/2*
Reflection in the *x*-axis
Reflection in the *y*-axis
Translation to the right by *1/2* of a unit
Translation upwards by *3/2* units

Q3a Given $f(x) = 1 + \log_e x$

$f(xy) + f\left(\frac{x}{y}\right) = 1 + \log_e(xy) + 1 + \log_e\left(\frac{x}{y}\right)$

$= 1 + \log_e x + \log_e y + 1 + \log_e x - \log_e y$

$= 2(1 + \log_e x) = 2f(x)$

Q3b $f(xy) + f\left(\frac{x}{y}\right) = 0$, $\therefore\ 2(1 + \log_e x) = 0$

$\therefore \log_e x = -1$, $x = e^{-1}$.

Solution set is $\{(x, y) : x = e^{-1} \text{ and } y \in R\}$.

Q4 $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} (\cos^2 x - \sin^2 x)dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cos(2x)dx = \left[\frac{\sin(2x)}{2}\right]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$

$= \frac{\sin\frac{\pi}{3}}{2} - \frac{\sin\frac{\pi}{4}}{2} = \frac{\sqrt{3} - \sqrt{2}}{4}$

Q5a Domain of $f(g(x))$ is *R*.

Q5b Since $g(x) = e^x > 0$, $\therefore\ f(g(x)) > -5$

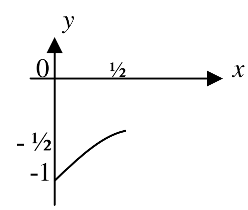
The range of $f(g(x))$ is $(-5, \infty)$.

Q5c $(e^x)^2 + 4e^x - 5 = 8e^x - 8$

$(e^x)^2 - 4e^x + 3 = 0$, $(e^x - 3)(e^x - 1) = 0$

$\therefore e^x = 3$ or $e^x = 1$ $\therefore x = \log_e 3$ or 0

Q6a $h : \left[0, \frac{1}{2}\right] \to R$, $h(x) = \frac{1}{2}\cos\left(\pi x - \frac{\pi}{2}\right) - 1$ can be simplified to $h(x) = \frac{1}{2}\sin(\pi x) - 1$.



The range of *h* is $\left[-1, -\frac{1}{2}\right]$.

Q6b The inverse of $y = \frac{1}{2}\sin(\pi x) - 1$ is $x = \frac{1}{2}\sin(\pi y) - 1$,

$\therefore 2(x+1) = \sin(\pi y)$, $\therefore\ y = \frac{1}{\pi}\sin^{-1}[2(x+1)]$.

Hence $h^{-1} : \left[-1, -\frac{1}{2}\right] \to R$, $h^{-1}(x) = \frac{1}{\pi}\sin^{-1}[2(x+1)]$.

Q6c $h(x) = -\frac{3}{4}$, $\frac{1}{2}\sin(\pi x) - 1 = -\frac{3}{4}$, $\sin(\pi x) = \frac{1}{2}$,

$\pi x = \frac{\pi}{6}$, $x = \frac{1}{6} \in \left[0, \frac{1}{2}\right]$.

Q7a $f(x) = 8x^3 - 12x^2 + 6x - 1 = (8x^3 - 1) - (12x^2 + 6x)$

$= ((2x)^3 - 1) - (12x^2 + 6x) = (2x-1)(4x^2 + 2x + 1) - 6x(2x-1)$

$= (2x-1)(4x^2 - 4x + 1) = (2x-1)(2x-1)^2$

$= (2x-1)^3$

Q7b $\int f(x)dx = \int (2x-1)^3 dx = \frac{(2x-1)^4}{8}$

Q8a $\frac{dV}{dt} = 25$ litres per minute $= 25000$ cm^3 per minute

Q8b $\frac{\Delta h}{\Delta t} = \frac{1.20}{2} = 0.60$ mitres per minute or 60 cm per minute

Q8c Since $\frac{dV}{dt}$ is constant, $\frac{\Delta V}{\Delta t} = \frac{dV}{dt} = 25000\,\text{cm}^3$ per minute

$\frac{\Delta h}{\Delta V} = \frac{\frac{\Delta h}{\Delta t}}{\frac{\Delta V}{\Delta t}} = \frac{60}{25000} = 0.0024\,\text{cm}^{-2}$

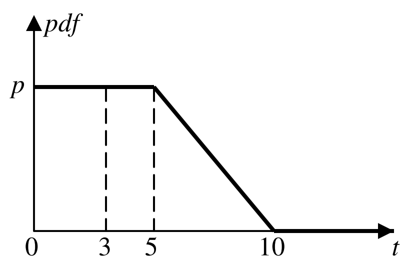
Q9a

5	6	7	8	9	10	11	12
$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Q9b $\Pr(Y \geq 1) = 1 - \Pr(Y = 0) = 1 - \left(\frac{5}{6}\right)^3 = \frac{91}{216} \approx 0.4213$

Q10 Let 7:30 am be $t = 0$, 7:35 am be $t = 5$ and 7:40 am be $t = 10$.

A probability density function which describes the distribution of probability in missing the train if you arrive at the station after time *t* is shown below.



Total area under graph $= \frac{1}{2}(5+10)p = 1$, $\therefore p = \frac{2}{15}$.

If you arrive after 7:33 am ($t = 3$),

$\Pr(miss.the.train) =$ area under graph (0 to 3) $= \frac{2}{15} \times 3 = 0.4$

$\Pr(catch.the.train) = 1 - 0.4 = 0.6$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

