

**Year 2010**  
**VCE**  
**Mathematical Methods**  
**CAS**  
**Solutions**  
**Trial Examination 1**



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**Question 1**

$$\Delta = \begin{vmatrix} -3 & p \\ 4 & -5 \end{vmatrix} = 15 - 4p$$

(1)  $\times 4 \Rightarrow -12x + 4py = 4q$

M1

(2)  $\times 3 \Rightarrow 12x - 15y = 60$

i. for a unique solution  $\Delta \neq 0 \Rightarrow p \neq \frac{15}{4}$  and  $q \in R$

A1

ii. for no solution  $\Delta = 0 \Rightarrow p = \frac{15}{4}$  and  $q \neq -15$

A1

iii. for infinitely many solutions  $\Delta = 0 \Rightarrow p = \frac{15}{4}$  and  $q = -15$

A1

**Question 2**

$f(x) = e^{\cos(2x)}$  chain rule

$y = e^{\cos(2x)} = e^u$   $u = \cos(2x)$

$$\frac{dy}{du} = e^u \quad \frac{du}{dx} = -2 \sin(2x)$$

M1

$f'(x) = -2 \sin(2x) e^{\cos(2x)}$

$$f'\left(\frac{\pi}{6}\right) = -2 \sin\left(\frac{\pi}{3}\right) e^{\cos\left(\frac{\pi}{3}\right)} = -2 \left(\frac{\sqrt{3}}{2}\right) e^{\frac{1}{2}}$$

$f'\left(\frac{\pi}{6}\right) = -\sqrt{3}e$

A1

**Question 3**

i. Let  $y = e^{-2x} (2 \cos(3x) - 3 \sin(3x))$  using the product rule

$$\frac{dy}{dx} = -2e^{-2x} (2 \cos(3x) - 3 \sin(3x)) + e^{-2x} (-6 \sin(3x) - 9 \cos(3x))$$

M1

$$\frac{dy}{dx} = e^{-2x} (-4 \cos(3x) + 6 \sin(3x) - 6 \sin(3x) - 9 \cos(3x)) = -13e^{-2x} \cos(3x)$$

so that  $\frac{d}{dx} (e^{-2x} (2 \cos(3x) - 3 \sin(3x))) = -13e^{-2x} \cos(3x)$

A1

ii.  $\int e^{-2x} \cos(3x) dx = \frac{e^{-2x}}{13} (3 \sin(3x) - 2 \cos(3x)) + c$

A1

**Question 4**

$$\sqrt{3} \sin(2x) + \cos(2x) = 0$$

$$\sqrt{3} \sin(2x) = -\cos(2x)$$

$$\tan(2x) = -\frac{1}{\sqrt{3}} \quad \text{M1}$$

$$2x = n\pi + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) \quad \text{A1}$$

$$2x = n\pi - \frac{\pi}{6}$$

$$x = \frac{n\pi}{2} - \frac{\pi}{12} = \frac{\pi}{12}(6n-1) \quad \text{where } n \in \mathbb{Z} \quad \text{A1}$$

**Question 5**

i.  $f: y = e^{2x} - e^{-2x}$   
 $f^{-1}: x = e^{2y} - e^{-2y} \quad \text{let } u = e^{2y} \quad \text{M1}$

$$x = u - \frac{1}{u}$$

$$u^2 - xu - 1 = 0 \quad \text{solve using the quadratic formulae}$$

$$a = 1 \quad b = -x \quad c = -1$$

$$u = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$u = e^{2y} = \frac{x \pm \sqrt{x^2 + 4}}{2} \quad \text{must take the positive, since } e^{2y} > 0 \quad \text{for } y \in \mathbb{R} \quad \text{M1}$$

and the domain and range of  $f$  and  $f^{-1}$  are both  $\mathbb{R}$

$$y = f^{-1}(x) = \frac{1}{2} \log_e \left( \frac{x + \sqrt{x^2 + 4}}{2} \right)$$

ii.  $e^{2x} - e^{-2x} = 4$

$$\text{hence } x = f^{-1}(4) = \frac{1}{2} \log_e \left( \frac{4 + \sqrt{16 + 4}}{2} \right) \quad \text{M1}$$

$$x = \frac{1}{2} \log_e \left( \frac{4 + 2\sqrt{5}}{2} \right)$$

$$x = \frac{1}{2} \log_e (2 + \sqrt{5}) \quad \text{A1}$$

alternatively

$$e^{2x} - e^{-2x} = 4 \quad \text{let } u = e^{-2x}$$

$$u - \frac{1}{u} = 4$$

$$u^2 - 4u - 1 = 0$$

M1

$$u^2 - 4u + 4 = 1 + 4$$

$$(u - 2)^2 = 5$$

$$u - 2 = \pm\sqrt{5}$$

$$u = e^{2x} = 2 \pm \sqrt{5} \quad \text{must take the positive}$$

$$x = \frac{1}{2} \log_e (2 + \sqrt{5})$$

A1

iii.  $\bar{y} = \frac{1}{2-0} \int_0^2 (e^{2x} - e^{-2x}) dx$

A1

$$\bar{y} = \frac{1}{2} \left[ \frac{1}{2} e^{2x} + \frac{1}{2} e^{-2x} \right]_0^2$$

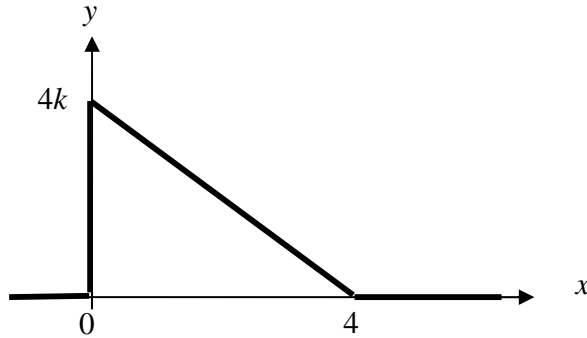
$$\bar{y} = \frac{1}{4} [(e^4 + e^{-4}) - (1+1)]$$

$$\bar{y} = \frac{1}{4} (e^4 + e^{-4}) - \frac{1}{2}$$

A1

**Question 6**

**i.**



area of a triangle  $\frac{1}{2}(4)(4k) = 8k = 1$  M1

$$k = \frac{1}{8}$$

alternatively

$$k \int_0^4 (4-x) dx = 1$$

$$k \left[ 4x - \frac{x^2}{2} \right]_0^4 = k(16 - 8 - 0) = 1$$

M1

$$8k = 1$$

$$k = \frac{1}{8}$$

**ii.** median is  $m$

$$\frac{1}{8} \int_0^m (4-x) dx = \frac{1}{2}$$
A1

$$\left[ 4x - \frac{x^2}{2} \right]_0^m = 4$$

$$4m - \frac{1}{2}m^2 = 4$$

$$m^2 - 8m + 8 = 0$$

M1

$$m^2 - 8m + 16 = 16 - 8 = 8$$

$$(m-4)^2 = \pm\sqrt{8}$$

$$m = 4 \pm 2\sqrt{2} \quad \text{must take negative since } m \in [0, 4]$$

$$m = 4 - 2\sqrt{2}$$

A1

**Question 7**

i.  $y = \frac{x-2}{x+2} = \frac{x+2-4}{x+2} = 1 - \frac{4}{x+2}$

crosses the x-axis at  $y = 0 \Rightarrow x = 2$  (2,0)

crosses the y-axis at  $x = 0 \Rightarrow y = -1$  (0,-1), graph shape

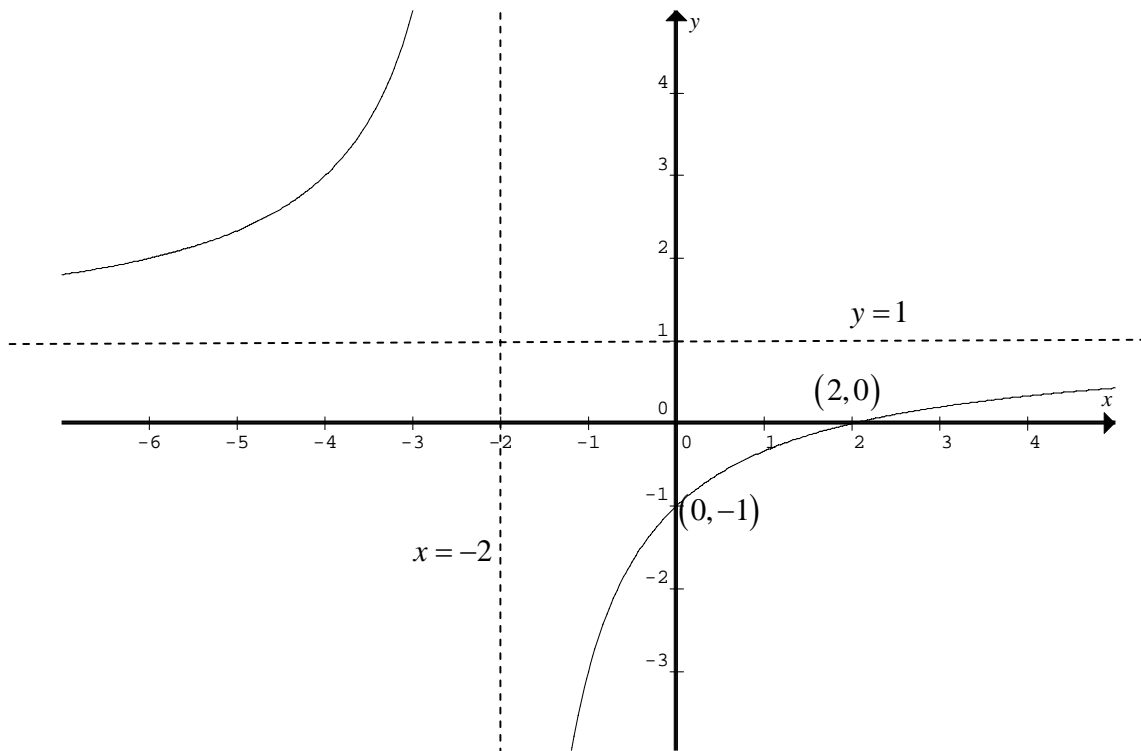
G1

$x = -2$  is a vertical asymptote

$y = 1$  is a horizontal asymptote,

both intercepts and both asymptotes

A1



ii. The required area is  $A = -\int_0^2 \frac{x-2}{x+2} dx$  since the area is below the x-axis

$$A = \int_0^2 \left( \frac{4}{x+2} - 1 \right) dx$$

A1

$$A = [4 \log_e |x+2| - x]_0^2$$

M1

$$A = (4 \log_e (4) - 2) - (4 \log_e (2) - 0)$$

$$A = 4 \log_e (2) - 2$$

$$A = \log_e (16) - 2 \text{ and } A > 0$$

$$a = 16 \quad b = -2$$

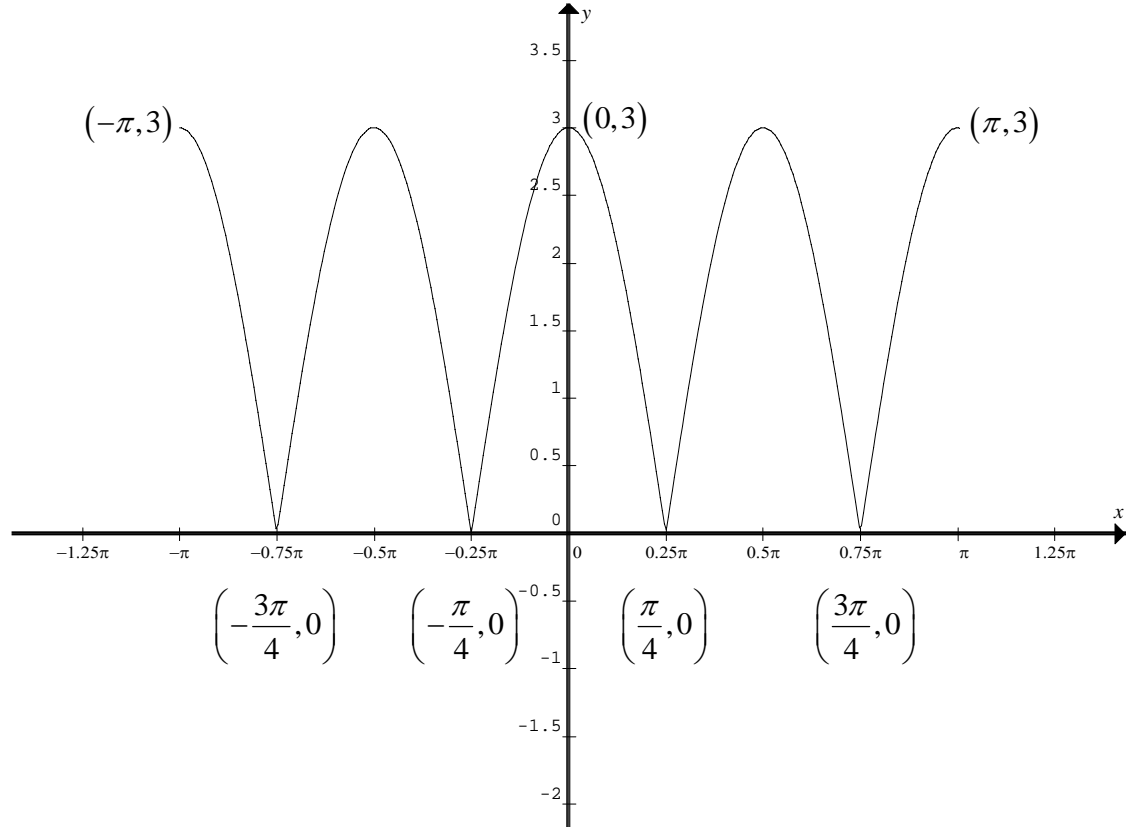
A1

**Question 8**

$f(g(x)) = f(-3\cos(2x)) = |-3\cos(2x)| = 3|\cos(2x)|$ , graph shape G1

and domain of  $f(g(x)) = \text{domain } g = [-\pi, \pi]$  correct period

and end-points  $(\pm\pi, 3)$ , axial intercepts and end-points. A1



**Question 9**

Area of a triangle  $A = \frac{1}{2}bc \sin(\theta) = \frac{1}{2}(4)(9)\sin(\theta) = 18\sin(\theta)$

$$A = 18\sin(\theta) \Rightarrow \frac{dA}{d\theta} = 18\cos(\theta)$$

Now given  $\frac{d\theta}{dt} = 1^\circ/\text{min} = \frac{\pi}{180} \text{ c/min}$  A1

By the chain rule  $\frac{dA}{dt} = \frac{dA}{d\theta} \frac{d\theta}{dt} = \frac{18\pi}{180} \cos(\theta)$  M1

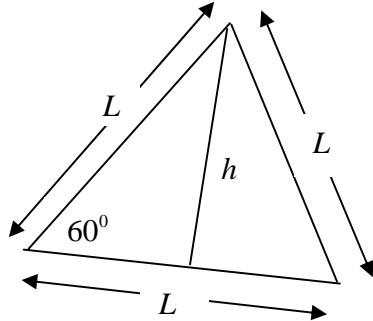
$$\left. \frac{dA}{dt} \right|_{\theta=45^\circ} = \frac{\pi}{10} \cos(45^\circ)$$

$$= \frac{\pi\sqrt{2}}{20} \text{ cm}^2/\text{min} \quad \text{A1}$$



**Question 10**

a.



Each triangle is equilateral, the area of one triangle is

$$A = \frac{1}{2} Lh = \frac{1}{2} L^2 \sin(60^\circ) = \frac{\sqrt{3}L^2}{4} \quad \text{A1}$$

Area of the top is six triangles, so the volume \$V\$

$$V = 6 \left( \frac{\sqrt{3}L^2}{4} \right) H = 10$$

$$H = \frac{20}{3\sqrt{3}L^2} = \frac{20\sqrt{3}}{9L^2} \quad \text{A1}$$

b. The total surface area is 12 triangles ( top and bottom ) and 6 rectangles

$$A = 12 \left( \frac{\sqrt{3}L^2}{4} \right) + 6HL \quad \text{M1}$$

$$A = 3\sqrt{3}L^2 + 6L \left( \frac{20\sqrt{3}}{9L^2} \right) \text{ since } L > 0 \text{ and } H > 0$$

$$A = 3\sqrt{3}L^2 + \frac{40\sqrt{3}}{3L}$$

c. for a minimum surface area

$$\frac{dA}{dL} = 6\sqrt{3}L - \frac{40\sqrt{3}}{3L^2} = 0 \quad \text{A1}$$

$$6\sqrt{3}L = \frac{40\sqrt{3}}{3L^2}$$

$$L^3 = \frac{40}{18}$$

$$L = \sqrt[3]{\frac{20}{9}} \text{ metres} \quad \text{A1}$$

**Question 11**

i. total number of balls  $r + b + w$

$$\Pr(\text{same colour}) = \Pr(RRR) + \Pr(BBB) + \Pr(WWW)$$

$$= \frac{r(r-1)(r-2)}{(r+b+w)(r+b+w-1)(r+b+w-2)}$$

$$+ \frac{b(b-1)(b-2)}{(r+b+w)(r+b+w-1)(r+b+w-2)} \quad \text{M1}$$

$$+ \frac{w(w-1)(w-2)}{(r+b+w)(r+b+w-1)(r+b+w-2)}$$

$$= \frac{r(r-1)(r-2) + b(b-1)(b-2) + w(w-1)(w-2)}{(r+b+w)(r+b+w-1)(r+b+w-2)} \quad \text{A1}$$

ii.  $\Pr(\text{all different colour})$

$$= \Pr(RBW) + \Pr(RWB) + \Pr(WRB) + \Pr(WBR) + \Pr(BWR) + \Pr(BRW)$$

$$= \frac{6rbw}{(r+b+w)(r+b+w-1)(r+b+w-2)} \quad \text{A1}$$

**END OF SUGGESTED SOLUTIONS**