Year 2010

VCE

Mathematical Methods CAS

Trial Examination 1

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Victorian Certificate of Education
2010

STUDENT NUMBER

Figures										Letter
Words										

MATHEMATICAL METHODS
AND
MATHEMATICAL METHOD (CAS)

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Find the values of p and q for which the equations

$$-3x + py = q$$
$$4x - 5y = 20$$

have

i. a unique solution

ii. no solution

iii. infinitely many solutions.

4 marks

Question 2

Let $f(x) = e^{\cos(2x)}$. Find $f'\left(\frac{\pi}{6}\right)$.

2 marks

Question 3

i. Differentiate $e^{-2x}\left(2\cos(3x) - 3\sin(3x)\right)$

ii. Hence find $\int e^{-2x}\cos(3x)\,dx$

2 + 1 = 3 marks

Question 4

Find the general solution to $\sqrt{3}\sin(2x)+\cos(2x)=0$.

3 marks

Question 5

Consider the function $f:R\to R$, $f(x)=e^{2x}-e^{-2x}$

i. Show that the rule for the inverse function is given by

$$f^{-1}(x)=\frac{1}{2}\log_e\left(\frac{x+\sqrt{x^2+4}}{2}\right)$$

ii. Hence or otherwise solve $e^{2x} - e^{-2x} = 4$.

iii. Find the average value of the function over $0 \leq x \leq 2$

2 + 2 + 2 = 6 marks

Question 6

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} k(4-x) & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

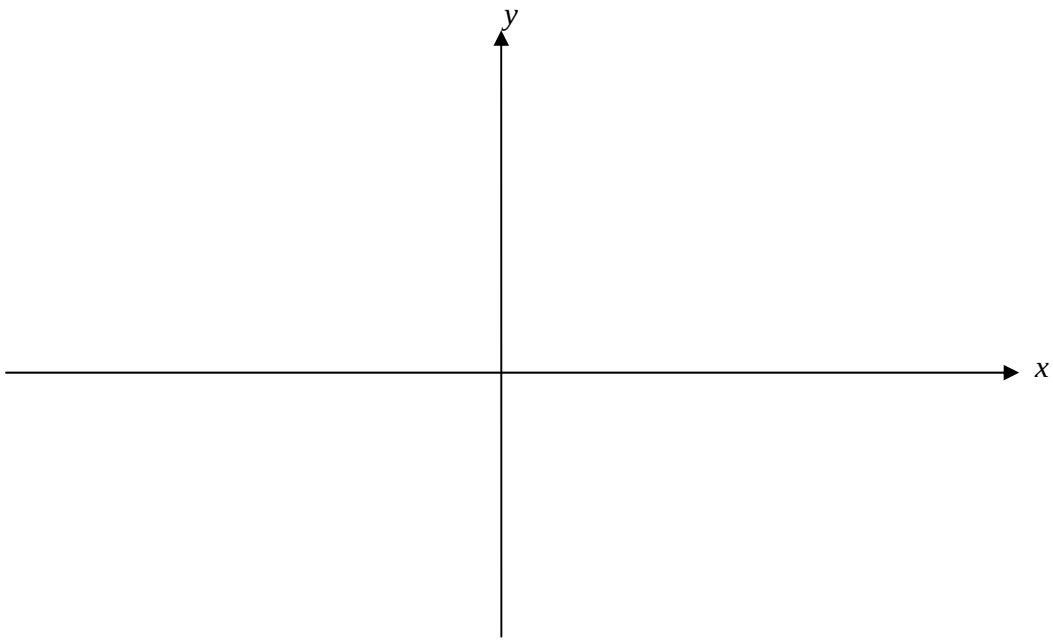
i. Show that $k = \frac{1}{8}$.

ii. Find the median value.

1 + 3 = 4 marks

Question 7

i. Sketch the graph of $y = \dfrac{x-2}{x+2}$ on the axes below, clearly stating the coordinates of all axial intercepts and the equations of any asymptotes.



ii. Find the area bounded by the graph $y = \dfrac{x-2}{x+2}$ and the coordinate axes. Give your answer in the form $\log_e(a)+b$.

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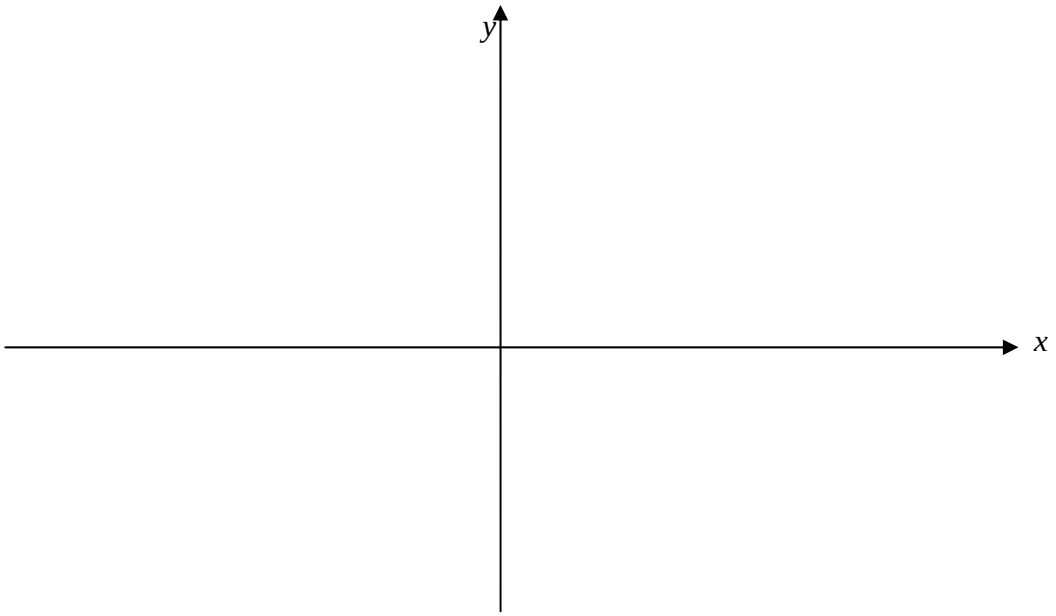
2 + 3 = 5 marks

Question 8

Let $f : R \to R$, $f(x) = |x|$ and $g : [-\pi, \pi] \to R$ $g(x) = -3\cos(2x)$

Sketch the graph of $f(g(x))$ on the axes below. Label all axial intercepts and label end-points with their coordinates.



2 marks

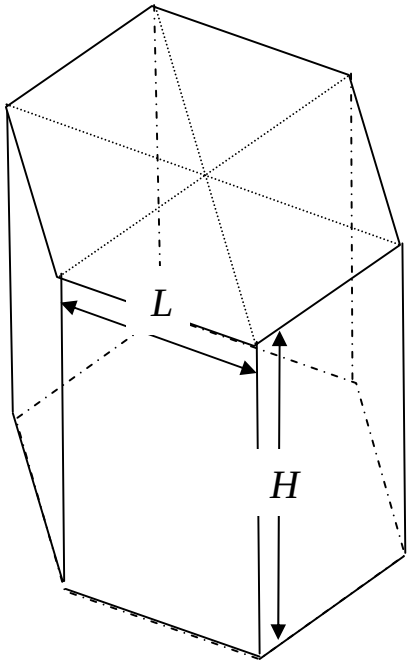
Question 9

A triangle has two sides of lengths 4 cm and 9 cm. The angle between them is increasing at a rate of 1^0 per minute. Find the rate in cm^2 per minute at which the area of the triangle is increasing, when the angle is 45^0.

3 marks

Question 10

A satellite in the form of a hexagonal prism is to be launched into space. It is to be entirely covered with solar panels, that is at the top and bottom and all sides. These consist of equilateral triangles each with a length of L and rectangular sides of height H, as shown in the diagram.



Due to size restrictions in the rocket ship, launching the satellite, the satellite is to have a volume of 10 m^3.

a. Find an expression for H in terms of L.

b. Show that the total surface area A m^2, of the satellite is given by

$$A = 3\sqrt{3}L^2 + \frac{40\sqrt{3}}{3L}$$

c. Find the value of L, for which the total surface area of the satellite is a minimum. You do not have to find the minimum surface area or verify that it is a minimum.

2 + 1 + 2 = 5 marks

Question 11

An urn contains r red balls, b black balls and w white balls. Three balls are drawn at random from the urn, one at a time, without replacement.

i. Find the probability that the three balls are all of the same colour?

ii. Find the probability that the three balls are all of different colours?

2 + 1 = 3 marks

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c \ , \ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\,f(x)\,dx$