Neap:

Trial Examination 2010

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 2

Suggested Solutions

SECTION 1

1	A	**B**	C	D	E
2	A	**B**	C	D	E
3	A	B	C	**D**	E
4	A	**B**	C	D	E
5	A	B	C	**D**	E
6	**A**	B	C	D	E
7	A	B	C	D	**E**
8	A	B	**C**	D	E
9	A	**B**	C	D	E
10	A	**B**	C	D	E
11	A	B	**C**	D	E
12	A	B	C	**D**	E
13	**A**	B	C	D	E
14	A	B	C	**D**	E
15	**A**	B	C	D	E
16	A	B	C	**D**	E
17	A	B	C	D	**E**
18	A	B	C	D	**E**
19	A	B	**C**	D	E
20	A	B	**C**	D	E
21	A	B	**C**	D	E
22	A	B	**C**	D	E

SECTION 1

Question 1 **B**

A matrix representation is $\begin{bmatrix} 2 & p \\ 3 & q \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3q \\ 2p \end{bmatrix}$.

A unique solution exists if $\det\left(\begin{bmatrix} 2 & p \\ 3 & q \end{bmatrix}\right) \neq 0$

$$\therefore 2q - 3p \neq 0$$

$$p \neq \frac{2q}{3}$$

Question 2 **B**

A function having the property $f(-x) = -f(x)$ is called an odd function. An odd function is symmetrical through the origin.

Clearly $f(x) = x^2$ is symmetrical about the y-axis, as is $f(x) = \cos(x)$ and $f(x) = \log_e(x^2)$.
In each case, $f(-x) = f(x)$.

The function $f(x) = e^x$ is such that $f(-x) = e^{-x} \neq -e^x$.

Clearly, $f(x) = \frac{2}{x}$ is such that $f(-x) = \frac{2}{-x} = -\frac{2}{x} = -f(x)$.

In addition, it should be noted that the graph of $f(x) = \frac{2}{x}$ is symmetrical through the origin.

Question 3 **D**

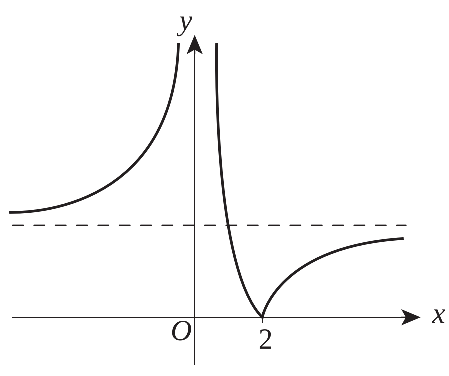
$f(x) = \log_e\left|1 - \frac{2}{x}\right|$ is defined if $\left|1 - \frac{2}{x}\right| > 0$.

Graphing $y = \left|1 - \frac{2}{x}\right|, x \neq 0$

clearly, $y > 0$ when $x \in R\backslash\{2\}$

$$\therefore \left|1 - \frac{2}{x}\right| > 0$$

if $x \in R\backslash\{0, 2\}$



Alternatively, we know that $\left|1 - \frac{2}{x}\right| \geq 0$ for any allowable value of x, since $\frac{2}{x}$ is not defined when $x = 0$, we must exclude $x = 0$. Also, $\left|1 - \frac{2}{x}\right| = 0$ when $1 - \frac{2}{x} = 0$, giving $x = 2$, so we must exclude $x = 2$.

Hence domain $D = R\backslash\{0, 2\}$.

Question 4 **B**

Using a CAS calculator to solve $\left(\sqrt{3}\tan\left(\frac{x}{2}\right)+1=0, x\right)$, gives

$$x=\frac{(6@n1-1)\pi}{3}$$

$$\therefore x=\frac{(6n-1)\pi}{3}$$

$$=\frac{6n\pi}{3}-\frac{\pi}{3}$$

$$=2n\pi-\frac{\pi}{3}$$

Another possible result is $x=2\pi\,\text{constn}(1)-\frac{\pi}{3}$.

Question 5 **D**

Consider I:

$$f(x)=4x \text{ and } f(y)=4y$$

$$f(2x+2y)=4(2x+2y)$$

$$=2(4x)+2(4y)$$

$$=2f(x)+2f(y)$$

Consider II:

$$f(x)=\frac{x}{2} \text{ and } f(y)=\frac{y}{2}$$

$$f(2x+2y)=\frac{2x+2y}{2}$$

$$=2\left(\frac{x}{2}\right)+2\left(\frac{y}{2}\right)$$

$$=2f(x)+2f(y)$$

Consider III:

$$f(x)=\frac{x^2}{2} \text{ and } f(y)=\frac{y^2}{2}$$

$$f(2x+2y)=\frac{(2x+2y)^2}{2}$$

$$=\frac{4x^2+8xy+4y^2}{2}$$

$$=4\left(\frac{x^2}{2}\right)+4\left(\frac{y^2}{2}\right)+8xy$$

$$\neq 2f(x)+2f(y)$$

$\therefore$ Answer is **D**.

Question 6 **A**

$\Pr(X > 14)$

$= \Pr\left(Z > \frac{14-17}{2}\right)$ where Z = $\frac{X-\mu}{\sigma}$

$= \Pr(Z > -1.5)$ NB: $\sigma^2 = 4$

$= \Pr(Z < 1.5)$ $\Rightarrow \sigma = 2$

Question 7 **E**

Given $h(x) = f(x) \times g(x)$, then by the product rule $h'(x) = f(x) \times g'(x) + g(x)f'(x)$.

But we are given $h'(x) = f(x) \times g'(x)$, thus $f(x) \times g'(x) + g(x)f'(x) = f(x) \times g'(x)$.

For this to occur, $g(x)f'(x) = 0$. But we know $g(x) > 0$, so we must have $f'(x) = 0$.

This means $f(x)$ must be a constant. We know $f(0) = 1$ so clearly $f(x) = 1$.

Question 8 **C**

Given $\frac{dy}{dx} = -\frac{1}{x}$, then $y = -\int \frac{1}{x} dx = -\log_e|x| + c$.

The average rate of change on the interval [1, 4] is determined by $\frac{y(4) - y(1)}{4 - 1}$

$= \frac{(-\log_e(4) + c) - (-\log_e(1) + c)}{3}$

$= -\frac{\log_e 4}{3}$

$= -\frac{2\log_e 2}{3}$

Question 9 **B**

Using CAS to differentiate $f(x) = 2e^{4x^2}$, gives $f'(x) = 16xe^{4x^2}$.

Solving $f'(x) = 3$ on CAS gives $x = 0.168$.

Question 10 **B**

Now $\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2$ giving $\text{Var}(X) = 250 - 15^2 = 25$.

Thus $\sigma_X = \sqrt{25} = 5$

The 95% interval is from $15 - 2 \times 5$ to $15 + 2 \times 5$, i.e. from 5 to 25.

Question 11 C

In order to find the median we need the value of *k*.

As *f* is a probability density function we know that $\int_{50}^{\infty} \frac{k}{x^3} dx = 1$.

Using CAS to solve for *k* gives $k = 5000$.

The median, *m*, is found from solving $\int_{50}^{m} \frac{5000}{x^3} dx = 0.5$.

Using CAS to solve for *m* gives $k = 70.7$. Thus *m* is closest to 71 hours.

Question 12 D

$f(x) = x^{-\frac{3}{2}}$

Dilation of factor 4, parallel to the *x*-axis is the same as a dilation of factor 4 from the *y*-axis.

$\therefore$ transformation is $f\left(\frac{x}{4}\right) = \left(\frac{x}{4}\right)^{-\frac{3}{2}}$

$= \left(\frac{1}{4}x\right)^{-\frac{3}{2}}$

$= 8(x)^{-\frac{3}{2}}$ as $\left(\frac{1}{4}\right)^{-\frac{3}{2}} = 8$

$= 8(f(x))$ i.e. a factor 8 from the *x*-axis.

Question 13 A

Let *X* represent the number of medium-size jumpers in the box of 20 jumpers. Then *X* follows a binomial distribution with $n = 20$ and $p = 0.6$.

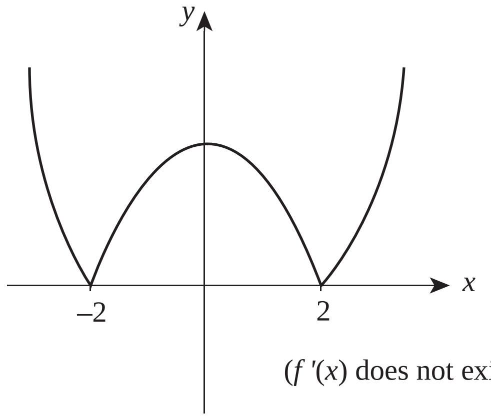
We require $\Pr(X > 17) = \Pr(X = 18) + \Pr(X = 19) + \Pr(X = 20)$

Applying the binomial probability formula gives:

$\Pr(X > 17) = {}^{20}C_{18}(0.6)^{18}(0.4)^2 + {}^{20}C_{19}(0.6)^{19}(0.4) + (0.6)^{20}$

Question 14 D

The graph of $f(x)$:



($f'(x)$ does not exist at $x = \pm 2$)

$$f(x) = \begin{cases} x^2 - 4 & x \in R\backslash(-2, 2) \\ 4 - x^2 & x \in (-2, 2) \end{cases}$$

$$f'(x) = \begin{cases} 2x & x \in R\backslash[-2, 2] \\ -2x & x \in (-2, 2) \end{cases}$$ as $\{-2, 2\}$ must be removed.

Note: CAS gives $f'(x) = 2\text{signum}(x^2 - 4)$, which can be interpreted as above.

Question 15 A

Given $y = \log_e\sqrt{\dfrac{1}{f(x)}}$, it is useful to simplify before differentiating.

Applying appropriate log laws:

$$\log_e\sqrt{\frac{1}{f(x)}} = \log_e\left(\frac{1}{f(x)}\right)^{\frac{1}{2}} = \frac{1}{2}\log_e\left(\frac{1}{f(x)}\right) = -\frac{1}{2}\log_e(f(x))$$

Now with $y = -\frac{1}{2}\log_e(f(x))$ we have $\dfrac{dy}{dx} = -\dfrac{1}{2} \cdot \dfrac{f'(x)}{f(x)}$.

Alternatively, with direct use of CAS:

$$\frac{d}{dx}\left(\ln\left(\sqrt{\frac{1}{f(x)}}\right)\right) = \frac{-\frac{d}{dx}(f(x))}{2f(x)}$$

Question 16 **D**

$f(x)$ has the range $[-4, \infty)$, $\therefore$ dom f^{-1} is $[-4, \infty)$

$\therefore$ only **C** or **D** is possible.

range of f^{-1} = dom $f = (-\infty, -2]$

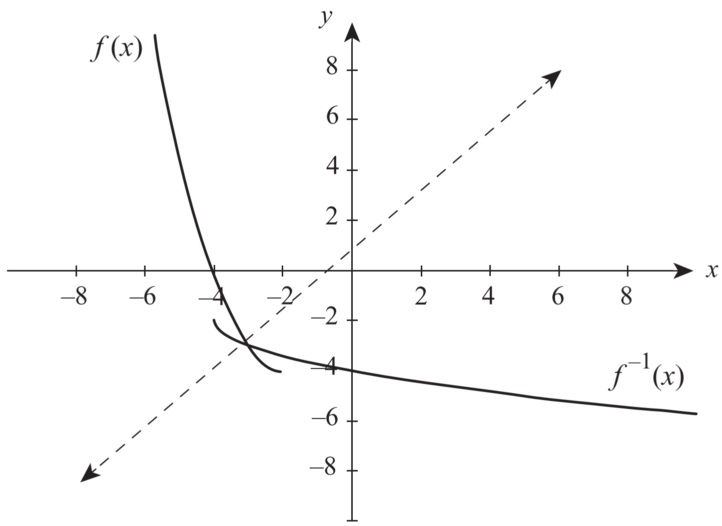
$\therefore$ The inverse function f^{-1} must have values less than –2.

$\therefore$ The answer is **D**.

OR

Rule for inverse is given by

$$\begin{aligned} x &= y^2 + 4y \\ x + 4 &= y^2 + 4y + 4 \\ (y+2)^2 &= x + 4 \\ y + 2 &= \pm\sqrt{x-4} \\ y &= -2 - \sqrt{x-4} \quad \text{as} \quad y \le -2 \end{aligned}$$



Question 17 **E**

$\Pr(X = x) = kx! \Rightarrow k(0! + 1! + 2! + 3! + 4!) = 1$

Thus $34k = 1 \Rightarrow k = \frac{1}{34}$

The probability function is given by:

X	0	1	2	3	4
Pr(*X* = *x*)	$\frac{1}{34}$	$\frac{1}{34}$	$\frac{2}{34}$	$\frac{6}{34}$	$\frac{24}{34}$

We now require Pr(0, 0) + Pr(1, 1) + Pr(2, 2) + Pr(3, 3) + Pr(4, 4). As each value of X is independent, the required probability is:

$$\left(\frac{1}{34}\right)^2 + \left(\frac{1}{34}\right)^2 + \left(\frac{2}{34}\right)^2 + \left(\frac{6}{34}\right)^2 + \left(\frac{24}{34}\right)^2 = \frac{309}{578}$$

Question 18 E

The average value of a function *f* with rule $f(x) = \cos(x)$ for the interval [–2, 4] is defined as

$$\frac{1}{4-(-2)}\int_{-2}^{4}\cos(x)dx = \frac{1}{6}[\sin(x)]_{-2}^{4}$$

$$= \frac{1}{6}(\sin(4) - \sin(-2))$$

$$= \frac{\sin 4 + \sin 2}{6} \text{ as } \sin(-2) = -\sin(2)$$

Question 19 C

$f\left(\frac{x}{2}\right)$ is a dilation of factor 2 from the *y*-axis.

So the asymptote at $x = -1$ becomes $x = -2$, similarly (2, 0) becomes (4, 0).

Then $\left|f\left(\frac{x}{2}\right)\right|$ reflects section of $f\left(\frac{x}{2}\right)$ from $x = 0$ to $x = 2$ about the *x*-axis.

Question 20 C

$e^{2x}(x^2 + 1)\sqrt{x^2 - 1}\log_e(x^2) = 0$ is solved by applying the Null Factor Law.

$\therefore e^{2x} = 0$ or $x^2 + 1 = 0$ or $\sqrt{x^2 - 1} = 0$ or $\log_e(x^2) = 0$.

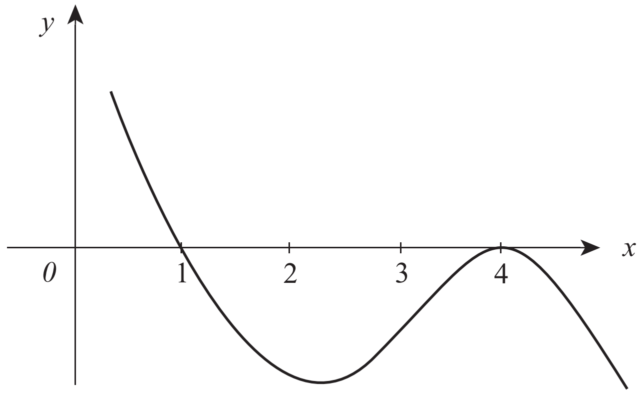
Now $e^{2x} > 0$ for all *x*, $x^2 + 1 \geq 1$ for all *x*.

$\sqrt{x^2 - 1} = 0$ if $x = \pm 1$, and $\log_e(x^2) = 0$ if $x^2 = 1 \Rightarrow x = \pm 1$.

Thus two different real solutions.

Question 21 C

Using knowledge of repeated factors, the graph of $g(x)$ is:



$\therefore$ From the graph, alternative **C** is the only possibility for $g'(x) > 0$.

or $g'(x) = 2(x-4)(1-x) + (x-4)^2(-1) = 0$

$= (x-4)[2-2x-(x-4)] = 0$ for stationary points

$= (x-4)(6-3x) = 0$

$x = 4$ or $x = 2$ which confirms $g'(x) > 0$ for $x \in (2, 4)$.

Question 22 C

The graphs do not necessarily intersect. Consider an example being $f(x) = -e^{-x}$ and $g(x) = e^{-x}$. However, the graphs could intersect. Consider a simple example like $f(x) = 2x$ and $g(x) = x$. If the graphs happen to intersect once, they cannot intersect again because $f(x)$ grows faster than $g(x)$.

SECTION 2

Question 1

a. $f'(x) > 0$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{x^{\frac{3}{2}}} = 0$ for stationary point — M1

$x = 4$ using CAS.

$\therefore$ Strictly increasing $x > 4$, so increasing on $(4, \infty)$. — A1

Alternative solution:

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{x^{\frac{3}{2}}}$ — M1

$= \frac{1}{2x^{\frac{3}{2}}}(x - 4)$

$f'(x) > 0$ if $x > 4$, since $x^{\frac{3}{2}} > 0$ for all $x > 0$.

On a CAS, define $f(x)$ and solve the equation $\left(\frac{d}{dx}(\ f(x) > 0, x)\right)$, which gives $x > 4$ directly. — A1

b. Absolute minimum at $x = 4$.

$f(4) = -1$

$\therefore$ Absolute minimum of $f(x)$ is –1. — A1

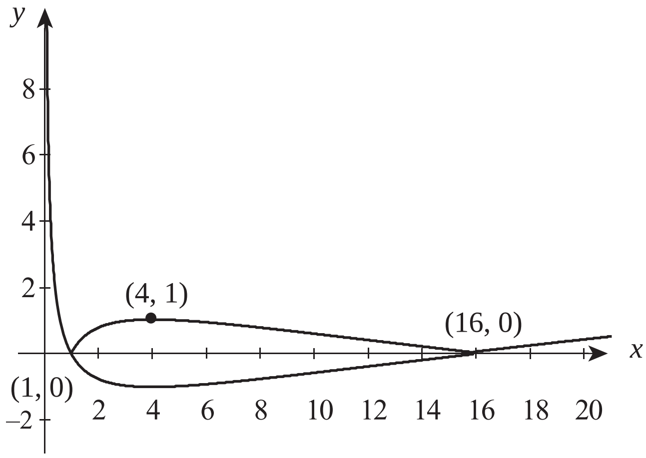
c. Tangent at $x = 16$, $f'(16) = \frac{3}{32}$ — A1

Now $f(16) = 0$.

Equation of the tangent: $y - 0 = \frac{3}{32}(x - 16)$

$y = \frac{3}{32}x - \frac{3}{2}$ — A1

d.



Section $x \in [1, 16]$ A1

Section $x \in (0, 1] \cup [16, \infty)$ A1

Correct points (4, 1), (1, 0), (16, 0) A1

e. $g(x) = \begin{cases} \dfrac{4}{\sqrt{x}} + \sqrt{x} - 5, & \text{for } x \in (0, 1] \cup [16, \infty) \\ 5 - \dfrac{4}{\sqrt{x}} - \sqrt{x}, & \text{for } x \in (1, 16) \end{cases}$

$g'(x) = \begin{cases} \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{\frac{3}{2}}}, & \text{for } x \in (0, 1) \cup (16, \infty) \\ \dfrac{2}{x^{\frac{3}{2}}} - \dfrac{1}{2\sqrt{x}}, & \text{for } x \in (1, 16) \end{cases}$ A1 A1

NB: $g'(x)$ does not exist at $x = 1$ or $x = 16$.

f. Area $= \displaystyle\int_1^{16} -f(x)dx$ or $\left|\displaystyle\int_1^{16} f(x)dx\right|$ A1

$= 9$ A1

g. $\displaystyle\int_1^{a} -f(x)dx = \frac{9}{2}$ A1

$a = 6.25$ A1

h. $g(h(x)) = |f(h(x))| = |f(x^2)| = \left|\frac{4}{\sqrt{x^2}} - \sqrt{x^2} - 5\right|$ A1

$= \left|\frac{4}{|x|} - |x| - 5\right|$

$= \left|\frac{|x^2| - 5|x| + 4}{x}\right|$ M1

The smallest value of $|f(h(x))|$ is zero.

This occurs if $|x^2| - 5|x| + 4 = 0$ A1

Using CAS $x = \pm1, \pm4$

Question 2

a. **i.** The amplitude of the function $N(t) = 68 + k\sin(at)$ is k. Thus $62 = 68 \pm k$, giving $k = \pm 6$, but $k > 0$ so $k = 6$. A1

ii. Given $N(22) = 62$, we have $62 = 68 + 6\sin(22a)$.

i.e. $\sin(22a) = -1$ M1

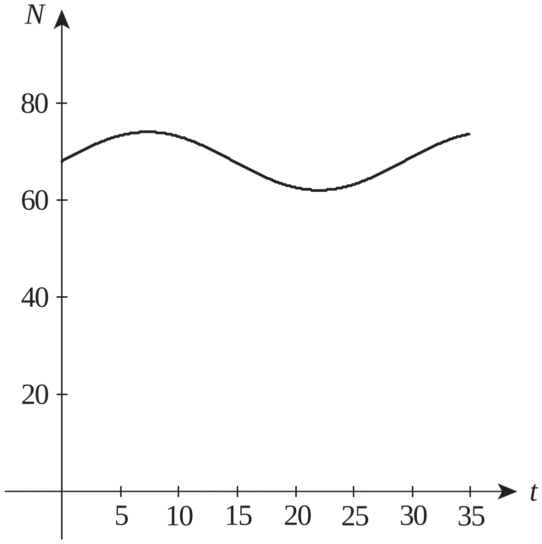
Thus, $22a = \frac{3\pi}{2} + 2n\pi,\ n \in Z$ A1

But $a > 0$ and a minimum first occurs at the 22 minute mark.

$\Rightarrow 22a = \frac{3\pi}{2}$ i.e. $n = 0$

$\therefore a = \frac{3\pi}{4}$

b. From observation of the graph of $N(t) = 68 + 6\sin\left(\frac{3\pi t}{44}\right)$ on CAS,



we can see that the maximum value of N occurs at $t = \frac{22}{3}$ and the minimum value occurs at $t = 22$. M1

Thus the traffic flow is increasing for $0 < t < \frac{22}{3}$ and $22 < t < 35$. A1

c. To find the total number of cars passing through the intersection, we compute $\int_0^{35} N(t)dt$.

Using CAS we get $\int_0^{35} 68 + 6\sin\left(\frac{3\pi t}{44}\right)dt = 2398.22$. M1

Thus 2398 cars. A1

d. The average value over the interval $5 \le t \le 15$ is given by $\frac{1}{15-5}\int_5^{15} N(t)dt$. M1

$= \frac{1}{10}\int_5^{15} 68 + 6\sin\left(\frac{3\pi t}{44}\right)dt = 72.1$ cars per minute. A1

e. **i.** The average rate of change of traffic flow is between $t = 5$ and $t = 15$. A1

ii. Given that $\frac{N(15) - N(5)}{10} = \frac{3}{5}\left(\sin\left(\frac{45\pi}{44}\right) - \sin\left(\frac{15\pi}{44}\right)\right)$

$= \frac{3}{5}\left(\sin\left(\pi + \frac{\pi}{44}\right) - \cos\left(\frac{\pi}{2} - \frac{15\pi}{44}\right)\right)$ M1

$= \frac{3}{5}\left(-\sin\left(\frac{\pi}{44}\right) - \cos\left(\frac{7\pi}{44}\right)\right)$

$= -\frac{3}{5}\left(\sin\left(\frac{\pi}{44}\right) + \cos\left(\frac{7\pi}{44}\right)\right)$ as required A1

iii. $\frac{N(15) - N(5)}{15 - 5} = -0.569 = -0.6$ cars per minute per minute. A1

Question 3

a. Let X_S represent the volume in litres of wine contained in a standard bottle.

Then $X_S \sim N(\mu = 0.760, \sigma = 0.008)$.

$\Pr(X_S < 0.750)$ using CAS gives $0.1056498\ldots = 0.1056$, correct to 4 decimal places. A1

b. **i.** Let X_L represent the volume in litres of wine contained in a large bottle.

We have $\Pr(X_L > 1.023) = 0.08$ and $\Pr(X_L < 0.994) = 0.04$.

Using $Z = \frac{X_L - \mu}{\sigma}$ gives $\Pr\left(Z > \frac{1.023 - \mu}{\sigma}\right) = 0.08$. M1

Thus $\frac{1.023 - \mu}{\sigma} = \text{invnorm}(0.92) = 1.4051$.

$\mu + 1.4051\sigma = 1.023$ (equation 1) A1

Similarly $\Pr\left(Z < \frac{0.994 - \mu}{\sigma}\right) = 0.04$.

Thus $\frac{0.994 - \mu}{\sigma} = \text{invnorm}(0.04) = -1.7507$.

$\mu - 1.7507\sigma = 0.994$ (equation 2) A1

ii. Using CAS to solve equations 1 and 2 (**Question 3 b i.**) gives $\mu = 1.010$ and $\sigma = 0.009$. A1

c. Define the random variable Y as the number of bottles in the box which contain less than 0.750 litres.

Then $Y \sim \text{Bi}(n = 10, p = 0.1056)$. M1

We require $\Pr(Y \geq 3)$. Using CAS gives $\Pr(Y \geq 3) = 0.126$. A1

d. **i.** $\begin{matrix} & S & L \\ S & 0.75 & 0.65 \\ L & 0.25 & 0.35 \end{matrix}$ or $\begin{matrix} & L & S \\ L & 0.35 & 0.25 \\ S & 0.65 & 0.75 \end{matrix}$ A1

ii. The estimated percentage of standard and large bottles sold after a further three weeks is given by S_3 where $S_3 = T^3 \times S_0$. The initial percentage of bottles of standard and large size is described by S_0.

Thus $S_0 = \begin{bmatrix} 40 \\ 60 \end{bmatrix}$ M1

$$S_3 = \begin{bmatrix} 0.75 & 0.65 \\ 0.25 & 0.35 \end{bmatrix}^3 \begin{bmatrix} 40 \\ 60 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7225 & 0.7215 \\ 0.2775 & 0.2785 \end{bmatrix} \begin{bmatrix} 40 \\ 60 \end{bmatrix}$$

$$= \begin{bmatrix} 72.19 \\ 27.81 \end{bmatrix}$$ M1

Equivalent calculation possible if T given in alternative form in **Question 3 d i.**

Therefore at the end of week 3 it is estimated that 72.2% of sales are for the standard bottles and 27.8% are for the large bottles. A1

iii. The long term is represented by the steady state probability. As we have the transition

matrix $T = \begin{bmatrix} 0.75 & 0.65 \\ 0.25 & 0.35 \end{bmatrix}$, M1

the steady state probability required $= \dfrac{0.25}{0.25 + 0.65} = \dfrac{25}{90} = \dfrac{5}{18}$ as required. A1

Alternative solution:

Let the steady state be $\begin{bmatrix} p \\ 1-p \end{bmatrix}$.

Then $\begin{bmatrix} 0.75 & 0.65 \\ 0.25 & 0.35 \end{bmatrix}\begin{bmatrix} p \\ 1-p \end{bmatrix} = \begin{bmatrix} p \\ 1-p \end{bmatrix}$. M1

Multiplying out gives $0.75p + 0.65(1-p) = p$ and another equivalent equation;

solving gives $p = \dfrac{65}{90} = \dfrac{13}{18}$ and hence $1 - p = \dfrac{5}{18}$ as required. A1

e. We have from **Question 3 a.** that the probability that a standard bottle is less than 750 mL is 0.1057.

Let X_L represent the volume in litres of wine contained in a large bottle.

Then $X_L \sim N(\mu = 1.010, \sigma = 0.009)$.

$\Pr(X_L < 1.000)$ using CAS gives 0.1333. A1

Consider 100 bottles of each size:

The expected number of standard bottles for which the final glass is short is 10.56. Similarly for 100 large bottles, the expected number for which the final glass is short is 13.33.

Standard bottle: Out of 600 glasses, $\dfrac{10.56}{600} = 0.0176$ is the expected proportion of wasted glasses.

Large bottle: Out of 800 glasses, $\dfrac{13.33}{800} = 0.0166$ is the expected proportion of wasted glasses.

$\therefore$ There is a greater proportion of wasted wine from standard bottles. A1

Question 4

a. $f(5) = 0.2(5)^3 - 1.8(5)^2 + 3(5) + 12.6$

$= 7.6$ A1

$f'(x) = 0.6x^2 - 3.6x + 3$

$f'(5) = 0.6(5)^2 - 3.6(5) + 3 = 0$ M1

$\therefore$ Stationary point at $x = 5$.

$\left.\begin{aligned} f'(4) &= -1.4 \\ f'(6) &= 3 \end{aligned}\right\}$ $\therefore$ there is a local minimum at $x = 5$. A1

b. $f(6) = 9$ A1

c. The equation of a parabola with a turning point at (7, 10) is $y = a(x-7)^2 + 10$. M1

It is also required to pass through (6, 9):

$9 = a(6-7)^2 + 10$

$\therefore a = -1$ A1

$\therefore y = -(x-7)^2 + 10$

$= -x^2 + 14x - 39$ A1

d. $\lim_{x \to 6^-} f'(x) = 0.6(6)^2 - 3.6(6) + 3 = 3$ M1

$\lim_{x \to 6^+} f'(x) = -2(6) + 14 = 2$ M1

$\therefore$ As $\lim_{x \to 6^-} f'(x) \neq \lim_{x \to 6^+} f'(x)$, the derivative does not exist at $x = 6$. $\therefore$ Function is not smooth. A1

e. A cubic is required of the form $y = a(x-h)^3 + k$, which possesses a point of inflection at (7, 10) and passes through (6, 9).

$\therefore y = a(x-7)^3 + 10$ M1

$9 = a(6-7)^3 + 10 \Rightarrow a = 1$

$\therefore y = (x-7)^3 + 10$ A1

$\frac{dy}{dx} = 3(x-7)^2$

$\lim_{x \to 6^+} f'(x) = 3(6-7)^2 = 3$

$\therefore$ As $\lim_{x \to 6^-} f'(x) = \lim_{x \to 6^+} f'(x)$, the function is smooth at $x = 6$. A1