MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1

2010 Trial Examination

SOLUTIONS

Question 1

a. $\frac{dy}{dx} = -2e^{-2x}\sin(3x-1) + 3e^{-2x}\cos(3x-1)$

M1+A1
2 marks

b. $f'(x) = \frac{3xe^{3x} - e^{3x}}{x^2} = \frac{e^{3x}(3x-1)}{x^2}$

$f'(1) = 2e^3$

M2+A1
3 marks

Question 2

a. $\int e^{3\pi x-1}dx = \frac{e^{3\pi x-1}}{3\pi} + c$

M1+A1
2 marks

b. $\left[-\frac{3}{2}\log_e(9-2x)\right]_m^4 = 1$

$-\frac{3}{2}\log_e(9-8) + \frac{3}{2}\log_e(9-2m) = 1$

$\log_e(9-2m) = \frac{2}{3}$

$e^{\frac{2}{3}} = 9-2m$

$2m = 9 - e^{\frac{2}{3}}$

$m = \frac{9-e^{\frac{2}{3}}}{2}$

M2+A1
3 marks

Question 3

a. The transformation matrix is $\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$, which represent a dilation by a factor of 2 away from the y −axis, a dilation by a factor of 3 away from the x−axis and a reflection in the x−axis.

M1+A1

2 marks

b. $y = \cos(x)$

$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ -3y \end{bmatrix}$

$u = 2x \text{ and } v = -3y$

$x = \frac{u}{2} \text{ and } y = \frac{v}{-3}$

$\frac{v}{-3} = \cos\left(\frac{u}{2}\right)$

$y' = -3\cos\left(\frac{x'}{2}\right)$

M1+A1

2 marks

c. $\cos\left(\frac{x'}{2}\right) = 0,\ -\frac{\pi}{2} \le \frac{x'}{2} \le \frac{\pi}{2}.$

$\frac{x'}{2} = \frac{\pi}{2}$ or $\frac{x'}{2} = \frac{3\pi}{2} \Longrightarrow$ is not in the domain ∴ not part of the solution

$x' = \pi$

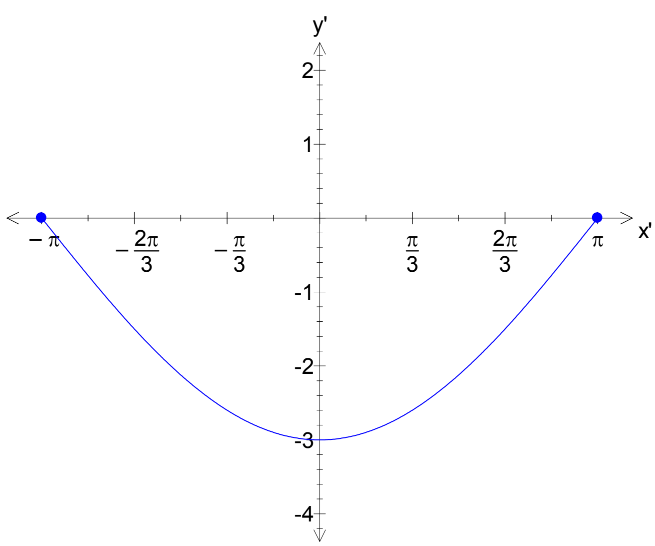
$\frac{x'}{2} = \frac{3\pi}{2} - 2\pi = \frac{-\pi}{2}$

$x' = -\pi$

M2+A1

3 marks

d.



A2

2 marks

Question 4

$\frac{dV}{dt} = 8$ m^3/minute, $r = \frac{h}{3}$ and $\frac{dV}{dh} = \frac{\pi h^2}{9}$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$8 = \frac{\pi h^2}{9} \times \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8\times 9}{4\pi} = \frac{18}{\pi}$ m/min

M2+A1
3 marks

Question 5

a. $x \in (1, \infty)$

A1
1 mark

b. $x = \frac{1}{(y-1)^2} + 2$

$x - 2 = \frac{1}{(y-1)^2}$

$(y-1)^2 = \frac{1}{x-2}$

$y = \frac{1}{\sqrt{x-2}} + 1$

$f^{-1}(x) = \frac{1}{\sqrt{x-2}} + 1, \; x > 2$

M2+A1
3 marks

Question 6

$f(x) = \sqrt[4]{x} = x^{\frac{1}{4}}, \quad f'(x) = \frac{1}{4}x^{\frac{-3}{4}}, \quad x = 16, \quad h = -0.08$

$f(x+h) \approx f(x) + hf'(x)$

$\sqrt[4]{15.92} \approx \sqrt[4]{16} - 0.08 \times \frac{1}{4}(16)^{\frac{-3}{4}}$

$\sqrt[4]{15.92} \approx 2 - 0.08 \times \frac{1}{32}$

$\sqrt[4]{15.92} \approx 2 - \frac{1}{400} \approx 1\frac{399}{400} \, or \; 2 - 0.0025 \approx 1.9975$

M3+A1
4 marks

Question 7

a. $\int_0^2 -x+4-x(x-1)^2 dx = \int_0^2 -x^3+2x^2-2x+4dx$

$\left[-\frac{x^4}{4}+\frac{2x^3}{3}-x^2+4x\right]_0^2 = -4+\frac{16}{3}-4+8 = \frac{16}{3}$ units^2

M2+A1
3 marks

b. $h'(x) = 3x^2-4x+1$

$h'(0) = 1$

$m_{normal} = -1 \quad and\ at\ (0,0)$

$y-0 = -1(x-0)$

$y = -x$

M1+A1
2 marks

Question 8

a. $0 \le p(x) \le 1$

$p(-1) = \frac{1}{24}(7-2\times -1) = \frac{9}{24}$

$p(0) = \frac{1}{24}(7-2\times 0) = \frac{7}{24}$

$p(1) = \frac{1}{24}(7-2\times 1) = \frac{5}{24}$

$p(2) = \frac{1}{24}(7-2\times 2) = \frac{3}{24}$

$\sum p(x) = \frac{9+7+5+3}{24} = 1$

M1+A1
2 marks

b.

i. mean= $E(X) = \sum x \times p(x) = -1\times\frac{9}{24}+1\times\frac{5}{24}+2\times\frac{3}{24} = \frac{2}{24} = \frac{1}{12}$

A1
1 mark

ii. $P(X > -1|X \le 1) = \frac{(X>-1 \cap X\le 1)}{X\le 1} = \frac{7+5}{24}\times\frac{24}{21} = \frac{12}{21} = \frac{4}{7}$

M1+A1
2 marks