INSIGHT

YEAR 12 *Trial Exam Paper*

2011

MATHEMATICAL METHODS (CAS)

UNIT 4

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring the following items into the examination: blank sheets of paper and/or white out liquid/tape.

Materials provided

- The question and answer book of 22 pages, with a separate sheet of miscellaneous formulas.
- An answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the answer sheet for multiple-choice questions.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the answer sheet for multiple-choice questions inside the front cover of this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet.
Select the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
If more than one answer is selected no marks will be awarded.

Question 1
Where k is a real constant, the simultaneous linear equations

$$kx - 5y = 0$$
$$7x - (k+2)y = k$$

have no solution, provided that

A. $k = 5$

B. $k = -7$

C. $k = 5,\ -7$

D. $5 < k < 7$

E. $k \neq 5,\ -7$

Question 2
At the point (2, 3) on the graph of the function with the rule $y = 2(x-2)^5 + 3$,

A. there is a local maximum.
B. there is a local minimum.
C. there is a stationary point of inflection.
D. the gradient is not defined.
E. there is a point of discontinuity.

Question 3
The maximal domain, D, of the function $f : D \to R$, with the rule $f(x) = \log_e(a - 2x)$ for $a > 0$, is

A. R^+

B. $(-\infty, \frac{a}{2}]$

C. $(-\infty, \frac{a}{2})$

D. $(-\infty, a)$

E. $(-\infty, a]$

Question 4

The range of the function $f:\left[\frac{\pi}{6},\frac{\pi}{2}\right)\to R,\ f(x)=3\cos(2x)$ is

A. $\left[\frac{\pi}{6},\frac{\pi}{2}\right)$

B. $[-3, 3]$

C. $(-3, 0]$

D. $(-3, 1.5]$

E. $(-3, 1.5)$

Question 5

The inverse of the function $f:R^{+}\to R,\ f(x)=\frac{1}{\sqrt{x}}+a$ for $a\in R$ is

A. $f^{-1}:[a,\infty)\to R$, where $f^{-1}(x)=\frac{1}{(x-a)^2}$

B. $f^{-1}:(a,\infty)\to R$, where $f^{-1}(x)=\frac{1}{(x-a)^2}$

C. $f^{-1}:R^{+}\to R$, where $f^{-1}(x)=\frac{1}{(x-a)^2}$

D. $f^{-1}:[a,\infty)\to R$, where $f^{-1}(x)=\frac{1}{x^2}-a$

E. $f^{-1}:(a,\infty)\to R$, where $f^{-1}(x)=\frac{1}{x^2}-a$

Question 6

The continuous random variable, X, has a normal distribution with mean 60 and standard deviation 7. If the random variable Z has the standard normal distribution, then the probability that X is less than 46 is equal to

A. $\Pr(Z>2)$

B. $\Pr(Z>-2)$

C. $\Pr(Z>74)$

D. $\Pr(X>2)$

E. $\Pr(X>-2)$

TURN OVER

Question 7

For $y = e^{3x}\sin(3x)$ the rate of change of y with respect to x when $x = \frac{\pi}{3}$ is

A. 0

B. -3

C. $-3e^{3\pi}$

D. $-3e^{\pi}$

E. $-3e^{\frac{\pi}{3}}$

Question 8

According to a survey, 20% of primary school students at a particular primary school like the colour red. If 10 students from this primary school are selected at random, the probability that at least 6 of them like the colour red is

A. 0.0064

B. 0.0009

C. 0.9936

D. 0.0055

E. 0.9945

Question 9

A transformation $T: R^2 \to R^2$ that maps the curve with the equation $y = \cos(x)$ onto the curve with the equation $y = 1 - 2\cos(3x + \pi)$ is given by

A. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{-\pi}{3} \\ 1 \end{bmatrix}$

B. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{\pi}{3} \\ -1 \end{bmatrix}$

C. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{-\pi}{3} \\ 1 \end{bmatrix}$

D. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{\pi}{3} \\ -1 \end{bmatrix}$

E. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{\pi}{3} \\ -1 \end{bmatrix}$

Question 10

Let X be a discrete random variable with a binomial distribution. The mean of X is 2 and the variance is 1.6. The values of n (number of independent trials) and p (the probability of each trial) are

A. $p = 2$ and $n = 20$

B. $p = 0.8$ and $n = 10$

C. $p = 0.2$ and $n = 10$

D. $p = 0.02$ and $n = 100$

E. $p = 2$ and $n = 100$

Question 11

For the function $f : R \to R,\quad f(x) = (x+a)^2(b-x)$, the subset of R for which the gradient of f is negative is

A. $(-\infty, -a)$

B. $\left(-a, \dfrac{2b-a}{3}\right)$

C. $(-\infty, -a) \cup (b, \infty)$

D. $(-a, b)$

E. $(-\infty, -a) \cup \left(\dfrac{2b-a}{3}, \infty\right)$

Question 12

The discrete random variable X has a probability distribution as shown.

x	0	1	2	3
$\Pr(X = x)$	$4a$	$2a$	$3a$	a

The median of X is

A. $6a$

B. $5a$

C. 1

D. $\dfrac{a}{6}$

E. 2

Question 13

The tangent at the point (2, 3) on the graph of the curve $y = f(x)$ has the equation $y = 2x - 1$. Therefore, the equation of the tangent at the point (2, –3) on the curve $y = -f(x)$ has the equation

A. $y = -2x - 1$

B. $y = -2x + 1$

C. $y = -2x - 3$

D. $y = -\frac{1}{2}x + 1$

E. $y = -\frac{1}{2}x - 1$

Question 14

If $\int_{-1}^{2} f(x)\,dx = 4$, then $\int_{-1}^{2} (3 - f(x))\,dx$ is equal to

A. –1

B. 7

C. 1

D. –4

E. 5

Question 15

For $y = \frac{1}{(1 - f(x))}$, $\frac{dy}{dx}$ is equal to

A. $\log_e(1 - f(x))$

B. $-\log_e(1 - f(x))$

C. $\frac{-1}{(1 - f(x))^2}$

D. $\frac{f'(x)}{(1 - f(x))^2}$

E. $\frac{-f(x)}{(1 - f(x))^2}$

Question 16

The average value of the function $f : R \setminus \{5\} \to R,\ f(x) = \frac{1}{5-x}$ over the interval $[1,\ k]$ is $\frac{1}{2}\log_e 2$. Hence, the value of k is

A. $\frac{1}{9}$

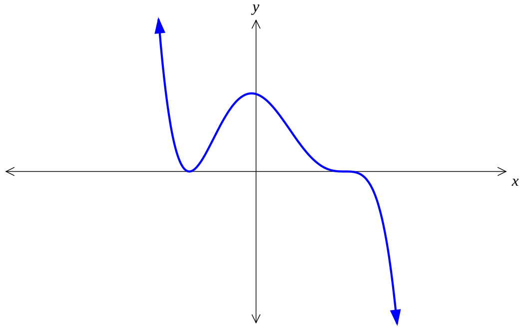
B. e^3

C. -3

D. 3

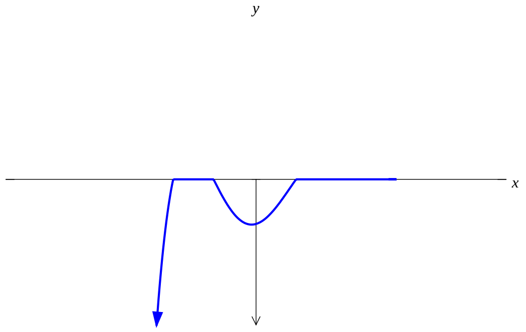
E. 7

Question 17
The graph of $y = f(x)$ is shown below.

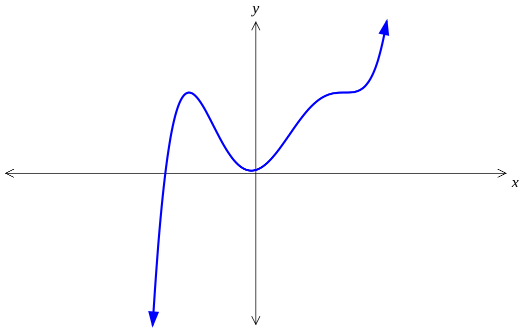


The graph that best represents $f(-x)+2$ is

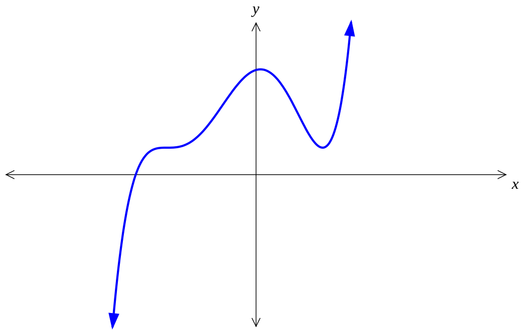
A.



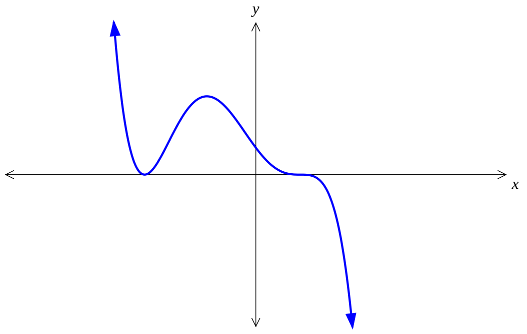
B.



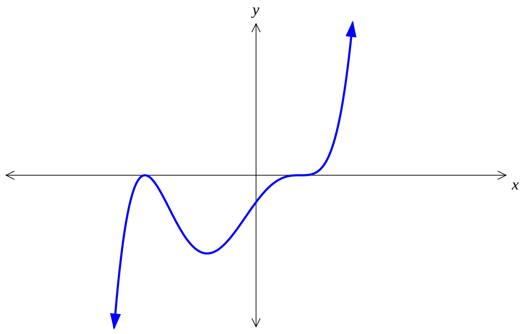
C.



D.



E.



Question 18
The exact area of the region bounded by the curve $f(x) = e^{2x} - 2e^{x} - 3$, the x-axis and the y-axis is

A. $3\log_e 3$

B. $4 - 3\log_e 3$

C. $4\log_e 3$

D. $4 - 3\log_e 3$

E. 3

Question 19
The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sin(x), \text{ if } 0 < x < \pi \\ 0, \text{ otherwise} \end{cases}$$

The $\Pr(X \geq k)$ is closest to

A. $\dfrac{\cos\left(\frac{1}{2}\right)}{2}$

B. $\dfrac{1 - \sin\left(\frac{1}{2}\right)}{2}$

C. $\dfrac{\sin\left(\frac{1}{2}\right)}{2}$

D. $\dfrac{1 - \cos\left(\frac{1}{2}\right)}{2}$

E. $\dfrac{\cos\left(\frac{1}{2}\right) + 1}{2}$

Question 20

The interval [0, 4] is divided into n equal subintervals by the points

$x_0, x_1, ..., x_{n-1}, x_n,$ where $0 = x_0 < x_1, < ...... < x_{n-1} < x_n = 4.$

Let $\delta x = x_i - x_{i-1}$ for $i = 1, 2, 3....., n.$

Then $\lim_{\delta x \to 0} \sum_{i=1}^{n} (x_i^2 \delta x)$ is equal to

A. $\frac{64}{3}$

B. 64

C. 16

D. 8

E. 21

Question 21

The velocity of a car moving in a straight line in metres per second is given by

$v(t) = 8 - \frac{1}{2}t^2, \ t \geq 0.$

Hence, the distance, in metres, travelled in the first 5 seconds is

A. –12.5

B. 23.5

C. 21.3

D. 19.2

E. 11.3

Question 22

Let $f(x) = a - b\sin(x)$, where a and b are real numbers and $b > 0$.
Then $f(x) > 0$ for all real values of x if

A. $a > b$

B. $b > a$

C. $-a > b$

D. $b > -a$

E. $-a < b$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Where a numerical answer is required an exact value must be given unless otherwise specified
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated diagrams in this book are **not** drawn to scale.

Question 1

Hugo wishes to compete in the State-wide Junior Schools Athletics carnival. His chosen event is discus.

The longest throwing distance recorded by his school is 51.60 metres.
The current State-wide junior schools record is 55.80 metres.

To be selected to compete he must throw at least his own school's record.

Hugo knows that the distance X, in metres, he can throw the discus follows a normal distribution with a mean of 50.55 metres and a standard deviation of 3.90.

a. Complete the following table. Give probabilities, correct to 3 decimal places

Distance thrown	Probability
Better than own school record	
Better than own school record but less than State-wide record	
Better than State-wide record	

3 marks

b. 85% of Hugo's discus throws travel at least A metres. Find the value of A, correct to 2 decimal places.

2 marks

c. On a particular attempt, Hugo throws the discus a distance that is less than the State-wide record. What is the probability that the distance thrown is farther than the school's record? Give your answer correct to 2 decimal places.

2 marks

d. During practice, Hugo throws five discuses. What is the probability that more than three of them are farther than the school's record? Give your answer correct to 2 decimal places.

2 marks

Riley is another competitor trying out for the State-wide athletics carnival.

The distance he is able to throw the discus is normally distributed with a mean of 51.20 metres and a standard deviation of σ.

e. He knows from past experience that 20% of his throws reach the State-wide record distance. Find the value of the standard deviation, σ, correct to 2 decimal places.

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2 marks

On the day of competition, Marcus, a discus competitor from another school, knows that his chances of throwing better than the State-wide junior schools' record depend only on what he has thrown at the previous attempt. If he throws farther than the State-wide record on one attempt, then there is a 60% chance that he'll throw farther than the State-wide record on the next attempt. If he throws less than the State-wide record on one attempt, then there is a 30% chance he'll throw farther than the State-wide record on the next attempt.

f. State the transition matrix for this information.

---

1 mark

g. If Marcus throws farther than the State-wide record on his first attempt, what is the probability that he throws farther than the State-wide record on exactly two of his next four attempts? Give your answer correct to 4 decimal places.

---

3 marks

h. What is the probability that Marcus throws farther than the State-wide record on his tenth attempt, if he throws farther than the State-wide record on his first attempt? Give your answer correct to 3 decimal places.

---

1 mark

Marcus has a probability of 0.429 of throwing farther than the State-wide record on his tenth attempt.

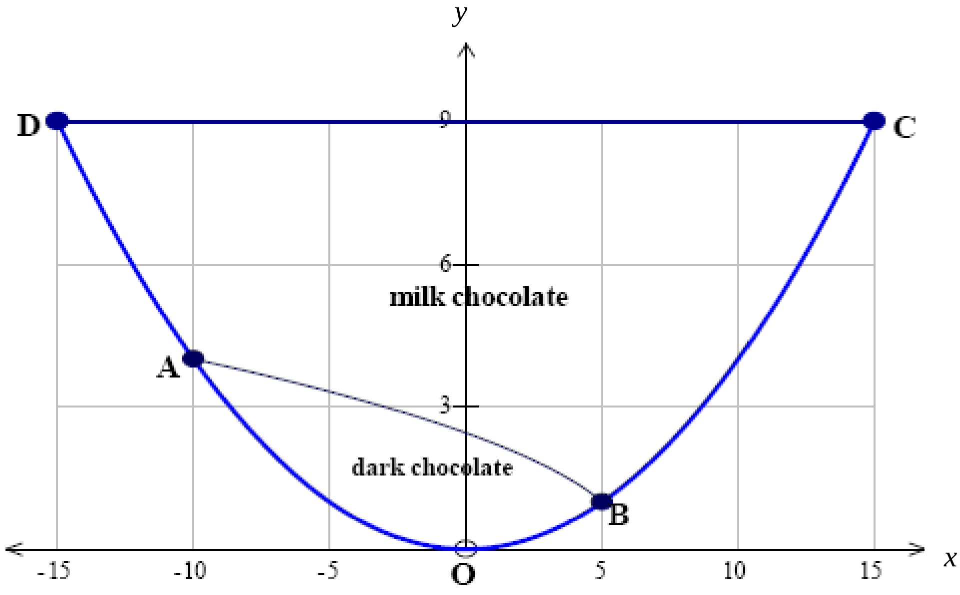
i. To be eligible for National selection, Marcus has to have thrown at least 650 throws that are farther than the State-wide record, from 1200 attempts. Based on his current performance, is this possible for Marcus? Explain.

---

2 marks
Total = 18 marks

SECTION 2 – Question 1 – continued

Question 2
A vertical section of a chocolate mousse in a bowl is illustrated in the diagram below. The diagram shows the arrangement of dark chocolate and milk chocolate in the mousse.
The axes *OX* and *OY* are as indicated. All measurements are in centimetres.



a. The chocolate mousse is placed in a paraboli-shaped bowl and the outer curved edge is described by the equation $y = ax^2$. Show that the value of $a = \frac{1}{25}$.

1 mark

b. The parabolic-shaped bowl is filled to the top with mousse. Find the cross-sectional area of the mousse (both milk and dark), correct to the nearest cm^2.

2 marks

c. The curve *AB* separates the dark chocolate from the milk chocolate. The equation of the curve *AB* is given by $y = \sqrt{6 - x}$.

i. Find the coordinates of points *A* and *B*.

ii. Hence, write an expression that will determine the cross-sectional area of the dark chocolate.

iii. What is the cross-sectional area of the dark chocolate?

iv. Find the ratio of milk chocolate to dark chocolate mousse in the bowl. State the ratio in simplest form.

v. It is decided that the ratio of milk chocolate to dark chocolate mousse needs to be 1 : 1. This is to be achieved by altering the amount of milk chocolate mousse that is added. The milk chocolate mousse will still form an upper horizontal level but will not reach to the top of the bowl. Find the new height of the mousse in the bowl.

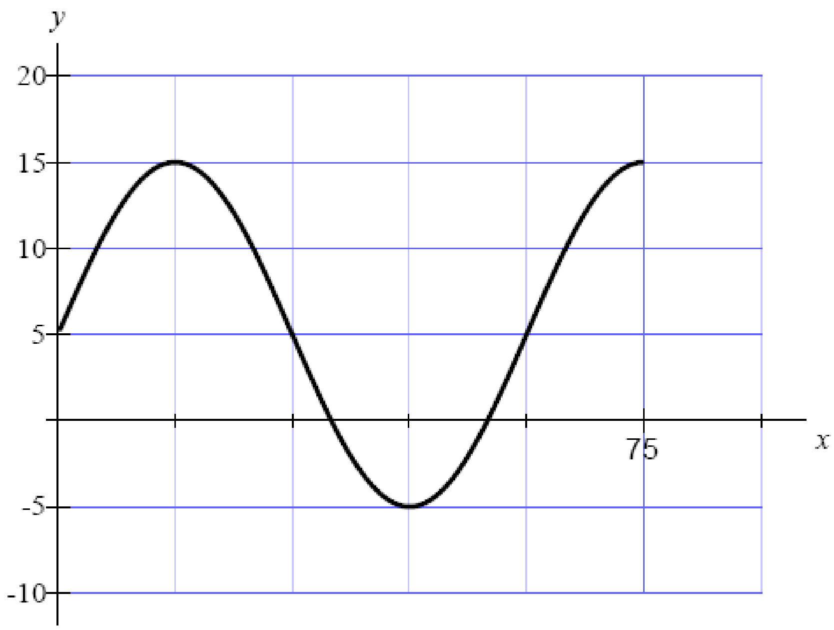
2 + 1 + 1 + 1 + 3 = 8 marks

d. The volume of mousse in the bowl is given by $V = \frac{\pi}{3125} h^4$, where h is the height of the mousse, in cm.
Chocolate mousse is added to the bowl as a liquid and is poured into the bowl at a rate of 5 cm^3 per minute. Find the rate at which the height of the chocolate mousse is increasing when the volume is $\frac{\pi}{625}$ cm^3.

3 marks

Total = 14 marks

Question 3
Let $f:[0,75]\rightarrow R,\ f(x)=5+a\sin(bx)$.
The graph of $y=f(x)$ is shown below.



a. **i.** State the value of *a*.

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ii State the period and, hence, show that $b=\dfrac{\pi}{30}$.

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1 + 2 = 3 marks

b. State the interval of values for which the graph of f is strictly **decreasing**.

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1 mark

c. Let $g : R \to R,\ g(x) = |x|$

i. Find the rule for $g(f(x))$.

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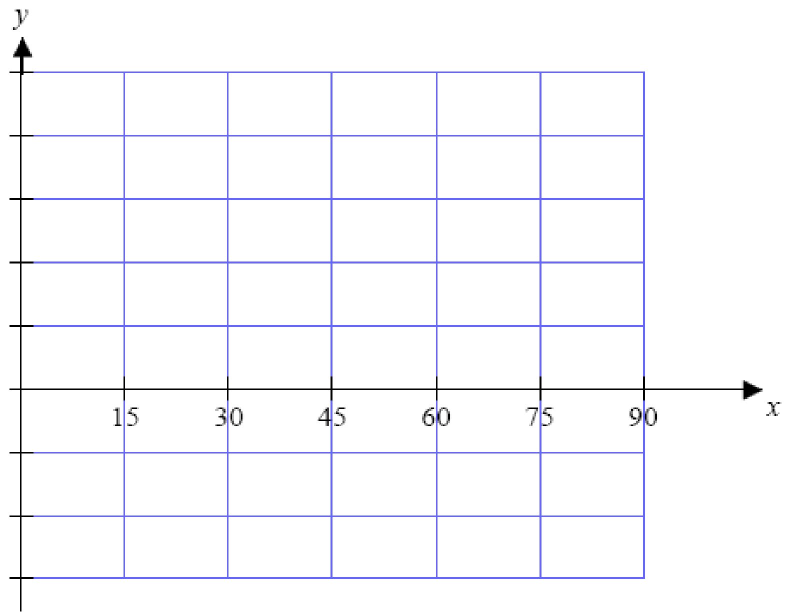
ii. On the axes of the previous graph, sketch the graph of $y = g(f(x))$, labelling endpoints as coordinates.

Let $h(x)$ be the derivative of $g(f(x))$ with respect to *x*.

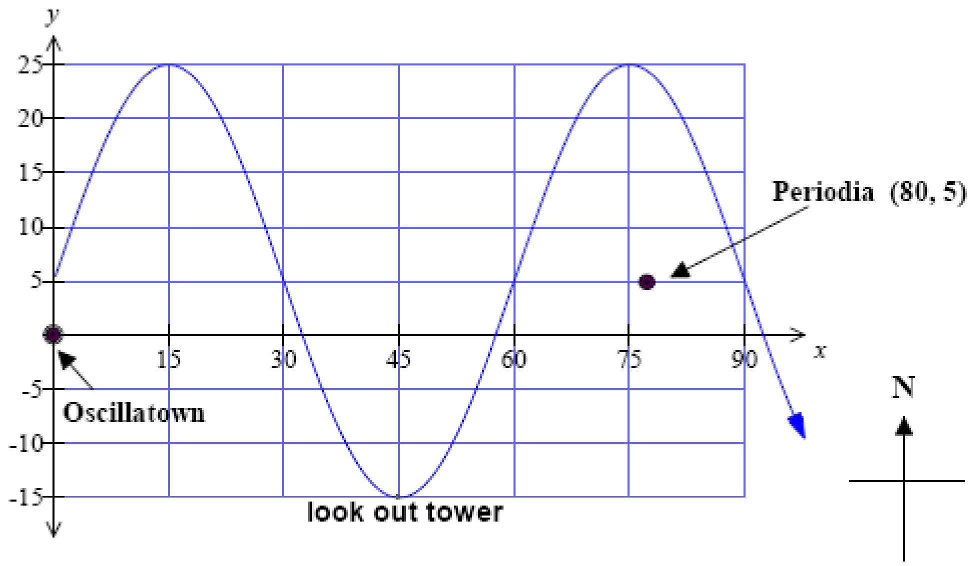
iii. Sketch the graph of $h(x)$ on the axes provided below. Label endpoints as coordinates, correct to 2 decimal places.



1 + 2 + 2 = 5 marks

The newly elected State government is proposing to build a man-made river through a national park. One proposal from an architectural firm has the man-made river weaving a path as shown in the diagram below. The equation of the path of the river is given by $y = 5 + 20\sin\left(\frac{\pi x}{30}\right)$.

Two towns, Oscillatown and Periodia, are marked on the map. All distances are in kilometres.



d. The plan includes the idea of building a straight road that must pass through Oscillatown and Periodia. What will be the length of the road, which will be built to the north of the river between Oscillatown and Periodia? Give your answer to 2 decimal places.

2 marks

e. The State government has rejected this proposal, stating that sightseers will not get to see the river adequately. Instead, it requests that the river follows a similar model but that it weave through at least three complete cycles before passing by the lookout tower at point (45, –15).

The rule for the equation of the river's path in the new proposal is $y = 5 + 20\sin(kx)$.

Find the smallest value of k that meets the new requirements.

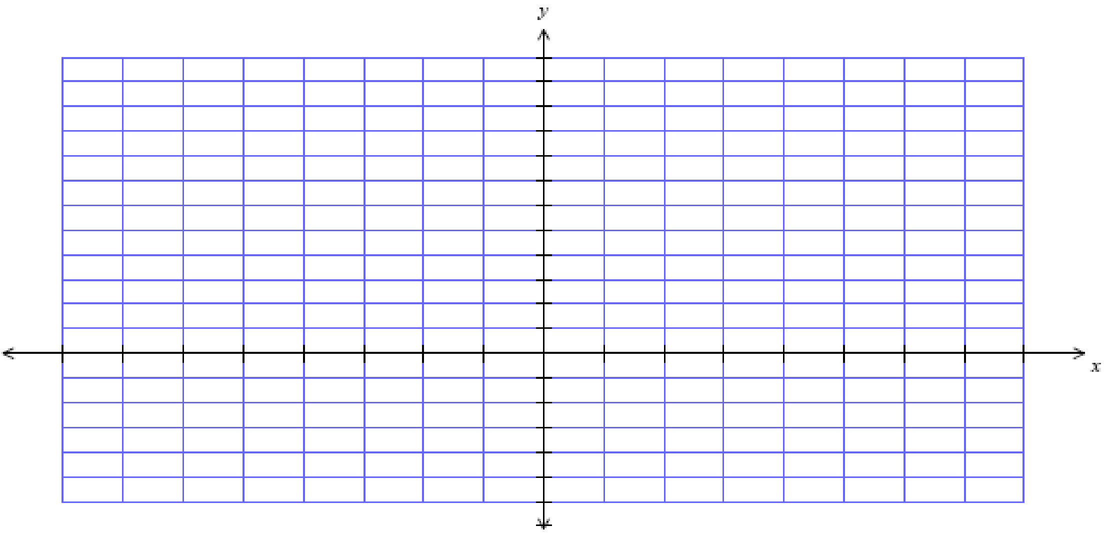
2 marks

Total = 13 marks

Question 4

Let $f:[0,5] \to R$, where $f(x) = 2e^{0.3x} - 1$.

a. Sketch the graph of $y = f(x)$ on the axes provided below. Label any asymptotes with their equations and the endpoints as exact coordinates.

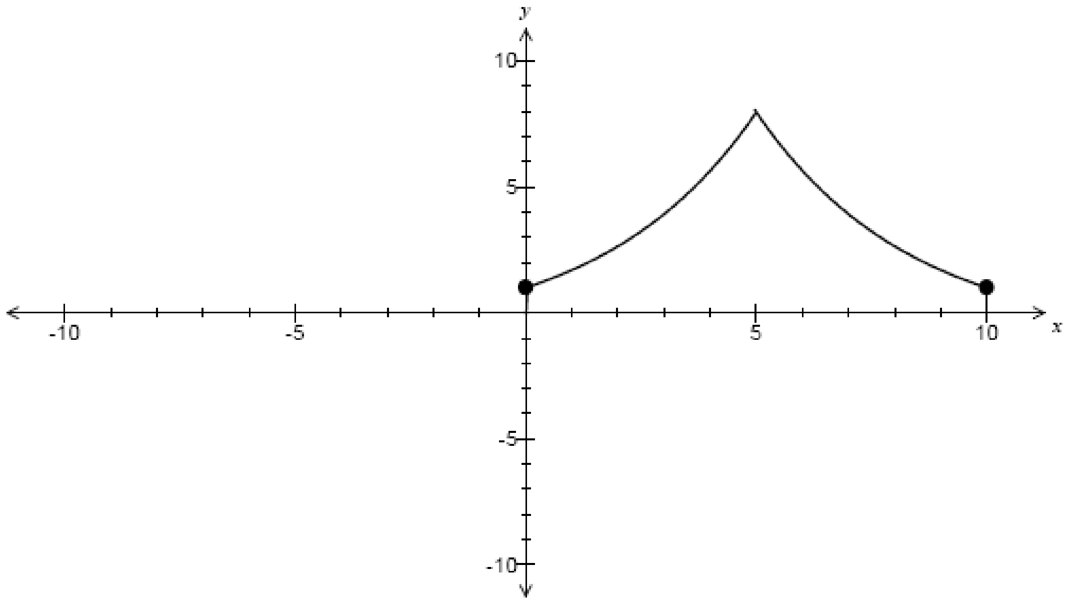


2 marks

b. State the range of the function $f(x)$.

1 mark

The function $f(x)$ forms part of an obstacle in a computer game. The aim of the computer game is to jump 'Bruno', the character in the game, over the obstacles and off to safety before 'Borat', the enemy character, can attack him. The complete obstacle is illustrated below. All measurements are in centimetres.



The obstacle is symmetrical about the line $x = 5$, with the equation of $y = g(x)$ being a 'mirror image' of $f(x)$.

c. State the transformations to produce $y = g(x)$ from $y = f(x)$.

2 marks

When Bruno jumps, he jumps according to the rule $y = 20 - ax^2$.
Find the values of a, correct to 4 decimal places, that would see Bruno

d. land on the left side of the obstacle.

2 marks

e. jump completely over the obstacle.

2 marks

When the computer game player hits the 'random' button, Bruno jumps so that his basic equation $y = 20 - x^2$ is transformed according to a randomly selected matrix. On one particular occasion, the matrix is $\begin{bmatrix} 0.2 & 0 \\ 0 & 0.9 \end{bmatrix}$.

f. Find the equation of the transformed jump.

2 marks

In general, the random matrix is described by $\begin{bmatrix} k & 0 \\ 0 & 0.9 \end{bmatrix}$, $k > 0$. This matrix transforms the basic equation $y = 20 - x^2$ whenever the random button is pressed.

g. Bruno needs to clear the obstacle and must start his jump from the left side of the obstacle. Find the closest distance he can stand to commence his jump and clear the obstacle when affected by the general random matrix.

2 marks

Total = 13 marks

END OF QUESTION AND ANSWER BOOK