The Mathematical Association of Victoria
Trial Examination 2011
Maths Methods CAS Examination 1 - SOLUTIONS

Question 1

a. $\frac{d}{dx}(x\tan(x))$

Using the product rule **1M**

$=\tan(x)+x\sec^2(x)$ **1A**

b. i. $\frac{d}{dx}\left(e^{2x}+2x\right)$

$=2e^{2x}+2$ **1A**

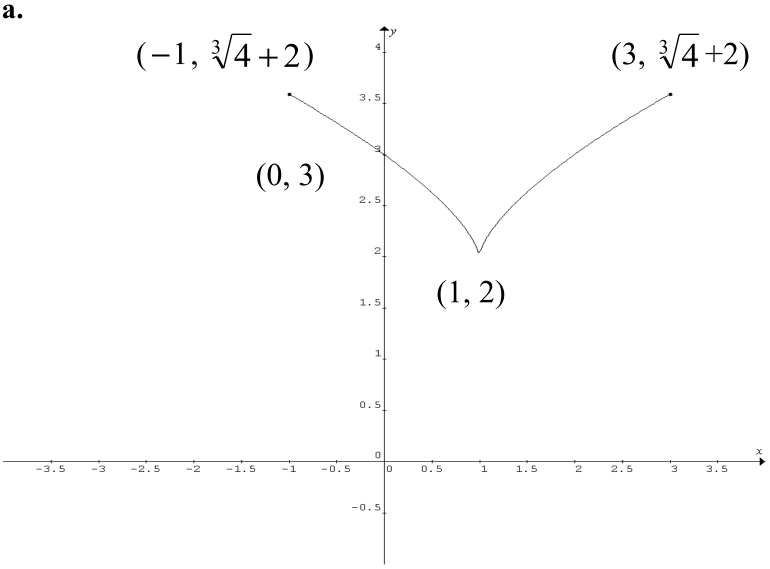
ii. $\int\left(\frac{4(e^{2x}+1)}{e^{2x}+2x}\right)dx$

$=2\int\left(\frac{2e^{2x}+2}{e^{2x}+2x}\right)dx$

$=2\log_e\left(\left|e^{2x}+2x\right|\right)$ **1A**

Question 2

a.



Shape and closed circles for endpoints **1A**

All coordinates correct **1A**

b. Average value $=\frac{1}{2-0}\int_0^2\left((x-1)^{\frac{2}{3}}+2\right)dx$ **1A**

$=\int_0^1\left((x-1)^{\frac{2}{3}}+2\right)dx$ due to symmetry

$=\left[\frac{3}{5}(x-1)^{\frac{5}{3}}+2x\right]_0^1$ **1A**

$=(0+2)-\left(-\frac{3}{5}-0\right)$

$=2.6$ **1A**

Question 3

$kx+2y=6$

$3x+(k-1)y=6$

For no solution the lines are parallel

$\begin{vmatrix} k & 2 \\ 3 & k-1 \end{vmatrix}=0$ or using ratios $\frac{3}{k}=\frac{k-1}{2}$ or using gradients $\frac{-k}{2}=\frac{-3}{k-1}$ **1M**

$k(k-1)-6=0$

$k^2-k-6=0$

$(k-3)(k+2)=0$

$k=3$ or $k=-2$ **1A**

Check to make sure they are not the same line

Using ratios $\frac{3}{k}\neq\frac{6}{6}$ or $\frac{k-1}{2}\neq\frac{6}{6}$ or using the y-intercept $\frac{6}{2}\neq\frac{6}{k-1}$

When $k=3$, $\frac{3}{3}=\frac{6}{6}=1$, hence the same line

When $k=-2$, $\frac{3}{-2}\neq\frac{6}{6}$, hence parallel lines, no solution **1A**

Question 4

$2\log_2(x-1)+\log_2(x+1)=0$

$\log_2(x-1)^2+\log_2(x+1)=0$

$\log_2(x-1)^2(x+1)=0$ **1M**

$(x-1)^2(x+1)=1$ **1A**

$(x^2-2x+1)(x+1)-1=0$

$x^3-x^2-x=0$ **1A**

$x(x^2-x-1)=0$

$x\neq 0$, $x\neq\frac{1-\sqrt{5}}{2}$

$x=\frac{1+\sqrt{5}}{2}$ **1A**

Question 5

a. $f(x)=1-e^{-x}$

Let $y=1-e^{-x}$

Inverse: swap x and y

$x=1-e^{-y}$ **1M**

$e^{-y}=1-x$

$y=-\log_e(1-x)$ **1A**

The domain is $(-\infty,1)$ **1A**

OR **OR**

$f^{-1}:(-\infty,1)\to R,$ where $f^{-1}(x)=-\log_e(1-x)$ **2A**

b. (0, 0) **1A**

Question 6

$h(x) = g(f(x)) = \left|x^3 + 3 - 1\right| = \left|x^3 + 2\right|$ **1A**

$h(x) = \begin{cases} 2 + x^3, & x \geq \sqrt[3]{-2} \\ -2 - x^3, & x < \sqrt[3]{-2} \end{cases}$ **1A**

$h'(x) = \begin{cases} 3x^2, & x > \sqrt[3]{-2} \\ -3x^2, & x < \sqrt[3]{-2} \end{cases}$ **1A**

Question 7

a. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$

$x' = -2(x+1),\ y' = 3(y-1)$ **1M**

$x = -\frac{x'}{2} - 1,\ y = \frac{y'}{3} + 1$

Substitute into $y = \frac{2}{x+1} - 1$

$\frac{y'}{3} + 1 = -\frac{4}{x'} - 1$ **1H**

$y' = -\frac{12}{x'} - 6$

The equation of the image is $y = -\frac{12}{x} - 6$ **1A**

b. Translation one unit to the right **1A**
Reflection in the y-axis **1A**
Dilation by a factor of six from the x-axis **1A**
OR **OR**
There are other solutions such as
Reflect in the x-axis **1A**
Dilate by a factor of 6 from the x-axis **1A**
Translate by 1 unit in the positive x direction **1A** both translations correct
Translate by 12 units in the negative y-direction

The order must be correct

Question 8

a. Using sum of probabilities is 1 **1M**

$p + 3p + q + 0.03 + 0.01 = 1$

$4p + q = 0.96$

Since

$q = 2p \Rightarrow 4p + 2p = 0.96$

$6p = 0.96$ **1A**

$p = 0.16$

b.

$$\Pr(X < 2 \mid X < 3) = \frac{\Pr(X < 2 \cap X < 3)}{\Pr(X < 3)}$$

$$= \frac{\Pr(X < 2)}{\Pr(X < 3)} = \frac{0.64}{0.96} = \frac{2}{3}$$ **1A**

Question 9

a.

$$\frac{k}{2}\int_0^3\left(\cos\left(\frac{\pi}{3}t\right)+1\right)dt=1$$ **1M**

$$\frac{k}{2}\left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}t\right)+t\right]_0^3=1$$

$$\frac{k}{2}\left\{\left[\frac{3}{\pi}\sin(\pi)+3\right]-\left[\frac{3}{\pi}\sin(0)+0\right]\right\}=1$$

$$\frac{k}{2}\times 3=1 \Rightarrow k=\frac{2}{3}$$ **1A**

b.

$$\Pr(T>2)=\frac{1}{3}\int_2^3 f(t)\,dt$$

$$=\frac{1}{3}\left\{\left[\frac{3}{\pi}\sin(\pi)+3\right]-\left[\frac{3}{\pi}\sin\left(\frac{2\pi}{3}\right)+2\right]\right\}$$ **1A**

$$=\frac{1}{3}\left\{3-\frac{3}{\pi}\times\frac{\sqrt{3}}{2}-2\right\}$$

$$=\frac{1}{3}\left(1-\frac{3\sqrt{3}}{2\pi}\right) \quad \textbf{OR} \quad \frac{1}{3}-\frac{\sqrt{3}}{2\pi}$$ **1A**

Question 10

$$\frac{d}{dx}\sqrt{\left(1+\cos^3(2x)\right)}=0$$

Using the chain rule **1M**

$$\frac{-6\cos^2(2x)\sin(2x)}{2\sqrt{\left(1+\cos^3(2x)\right)}}=0$$

The denominator cannot equal zero

Hence solve $\cos^2(2x)\sin(2x)=0$ **1M**

$\cos(2x)=0$ or $\sin(2x)=0$ **1M**

$2x=\frac{\pi}{2},\frac{3\pi}{2}...$ or $2x=0,\pi,2\pi...$

$x=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\pi$ but $x\neq\frac{\pi}{2}$ because $\cos^3(2x)\neq -1$

$x=0,\frac{\pi}{4},\frac{3\pi}{4},\pi$ **1A**

END OF SOLUTIONS