



# 2011 Trial Examination

## STUDENT NUMBER

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# MATHEMATICAL METHODS (CAS)

## Units 3 & 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

### QUESTION AND ANSWER BOOK

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

#### Materials supplied

- Question and answer book of 8 pages.
- Working space is provided throughout the book.

#### Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

- a. Differentiate  $f(x) = \log_e(2 - x)^4$  with respect to  $x$ .

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2 marks

- b. Evaluate  $f^{-1}(0)$  where  $f^{-1}$  is the inverse function of  $f$ .

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3 marks

- c. For  $g(x) = x^2 e^{\sin(x)}$ , find  $g'\left(\frac{\pi}{2}\right)$

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2 marks

**Question 2**

a. Find an anti-derivative of  $\sqrt{x} - 2 \sin\left(\frac{\pi x}{4}\right)$  with respect to  $x$ .

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2 marks

b. Find  $m$  given that  $\int_0^2 (2x - 1)^3 dx = e^{\log_e m}$

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2 marks

**TURN OVER**

**Question 3**

- a. Write down the amplitude and the period of the function  $f: R \rightarrow R, f(x) = -3 \cos\left(\frac{\pi-x}{5}\right)$ .

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2 marks

- b. Solve the equation  $-\cos(2x) = \sin(2x)$  for  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

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3 marks

- c. Sketch the graph of  $y = \tan(2x)$  for  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , showing the exact values of the axial intercepts and asymptotes.

2 marks

**Question 4**

The transformations  $T: R^2 \rightarrow R^2$  is defined by  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ . The image of the curve  $y = 2 \log_e(4x) + 1$  under the transformation  $T$  has equation  $y = a \log_e(bx + c) + d$ . Find the values of  $a, b, c,$  and  $d$ .

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4 marks

**Question 5**

Soft drink is leaking from a bottle at a constant rate to form a circular puddle on the bench. The soft drink is added to the puddle at a rate of  $12 \text{ mm}^3$  per minute causing the puddle to spread out evenly, which has a height that is a sixth of its radius.

Find the exact rate at which the radius of the puddle is increasing when the radius is 20 mm.

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3 marks

**TURN OVER**

**Question 6**

A square piece of paper is cut from a larger piece of paper. Use the relationship  $f(x + h) \approx f(x) + hf'(x)$  for a small positive value of  $h$ , to find an approximate value for the side length of the square if the area of the square piece of paper is 9.01 cm.

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4 marks

**Question 7**

The continuous random variable  $X$  has a distribution with probability density function given by

$$f(x) = \begin{cases} m \cos(x) & x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \\ 0 & \text{elsewhere} \end{cases}$$

where  $m$  is a constant, find

- a. the value of  $m$ .

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3 marks

b.  $Pr(X > \pi)$

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2 marks

**Question 8**

Let  $X$  be a normally distributed random variable with mean 11 and variance 4 and let  $Z$  be the random variable with the standard normal distribution. Find

a.  $Pr(X < 7)$

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1 mark

b.  $m$  such that  $Pr(X < 8) = Pr(Z > m)$

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2 marks

**TURN OVER**

**Question 9**

a. Show that  $\frac{3-2x}{x+2} = \frac{7}{x+2} - 2$

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2 marks

b. Hence, find  $\int \frac{3-2x}{x+2} dx$ .

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1 mark

**END OF QUESTION AND ANSWER BOOK**