VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2011

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

MATHEMATICAL METHODS (CAS)
Written examination 1

Tuesday 8 November 2011

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $\sqrt{4-x}$ with respect to x.

1 mark

b. If $g(x) = x^2\sin(2x)$, find $g'\left(\frac{\pi}{6}\right)$.

2 marks

Question 2

a. Find an antiderivative of $\dfrac{1}{3x-4}$ with respect to x.

1 mark

b. Solve the equation $4^x - 15 \times 2^x = 16$ for x.

3 marks

Question 3

a. State the range and period of the function

$$h: R \rightarrow R,\ h(x) = 4 + 3\cos\left(\frac{\pi x}{2}\right).$$

2 marks

b. Solve the equation

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \text{ for } x \in [0,\ \pi].$$

2 marks

Question 4

If the function f has the rule $f(x) = \sqrt{x^2 - 9}$ and the function g has the rule $g(x) = x + 5$

a. find integers c and d such that $f(g(x)) = \sqrt{(x + c)(x + d)}$

2 marks

b. state the maximal domain for which $f(g(x))$ is defined.

2 marks

Question 5

The probability distribution function for the continuous random variable X is given by

$$f(x) = \begin{cases} |3 - x| & if \quad 2 \le x \le 4 \\ 0 & elsewhere \end{cases}$$

a. Find $\Pr(X < 3.5)$.

2 marks

b. Find $\Pr(X < 2.5 \mid X < 3.5)$.

2 marks

Question 6

Consider the simultaneous linear equations

$$kx - 3y = k + 3$$
$$4x + (k + 7)y = 1$$

where k is a real constant.

a. Find the value of k for which there are infinitely many solutions.

3 marks

b. Find the values of k for which there is a unique solution.

1 mark

Question 7

A biased coin is tossed three times. The probability of a head from a toss of this coin is p.

a. Find, in terms of p, the probability of obtaining

i. three heads from the three tosses

ii. two heads and a tail from the three tosses.

1 + 1 = 2 marks

b. If the probability of obtaining three heads equals the probability of obtaining two heads and a tail, find p.

2 marks

Question 8

Two events, A and B, are such that $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{4}$.

If A' denotes the complement of A, calculate $\Pr(A' \cap B)$ when

a. $\Pr(A \cup B) = \frac{3}{4}$

2 marks

b. A and B are mutually exclusive.

1 mark

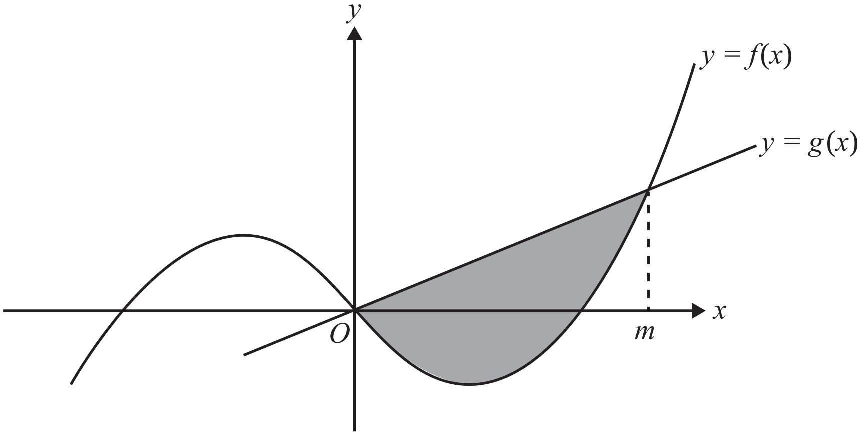
Question 9

Parts of the graphs of the functions

$$f: R \to R, f(x) = x^3 - ax \qquad a > 0$$
$$g: R \to R, g(x) = ax \qquad a > 0$$

are shown in the diagram below.

The graphs intersect when $x = 0$ and when $x = m$.



Question 9 – continued

The area of the shaded region is 64.

Find the value of a and the value of m.

4 marks

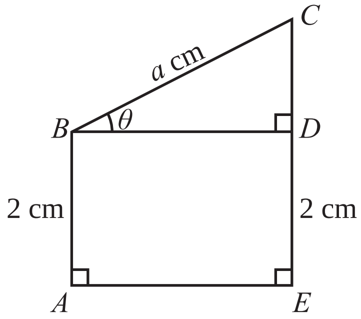
TURN OVER

Question 10

The figure shown represents a wire frame where *ABCE* is a convex quadrilateral. The point *D* is on line segment *EC* with *AB* = *ED* = 2 cm and *BC* = *a* cm, where *a* is a positive constant.

$$\angle BAE = \angle CEA = \frac{\pi}{2}$$

Let $\angle CBD = \theta$ where $0 < \theta < \frac{\pi}{2}$.



a. Find *BD* and *CD* in terms of *a* and *θ*.

2 marks

b. Find the length, *L* cm, of the wire in the frame, including length *BD,* in terms of *a* and *θ*.

1 mark

c. Find $\frac{dL}{d\theta}$, and **hence** show that $\frac{dL}{d\theta} = 0$ when $BD = 2CD$.

2 marks

d. Find the maximum value of L if $a = 3\sqrt{5}$.

1 mark

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x}\,dx = \log_e|x| + c$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

END OF FORMULA SHEET