THE HEFFERNAN GROUP

P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025
info@theheffernangroup.com.au
www.theheffernangroup.com.au

Student Name……………………………………

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

TRIAL EXAMINATION 1

2012

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 11 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 11 of this exam.

Question 1

a. For $y = \dfrac{e^{2x}}{x^2 + 3}$ find $\dfrac{dy}{dx}$.

1 mark

b. Let $f(x) = 2x\tan\left(\dfrac{x}{3}\right)$. Find $f'(\pi)$.

2 marks

Question 2

a. Find an antiderivative of $\sin(3x-1)$ with respect to *x*.

1 mark

b. Evaluate $\displaystyle\int_0^2 \frac{1}{2x+1}dx$.

2 marks

Question 3

Let $f: R^{+} \to R, f(x) = \log_e(x)$.

a. Show that $f\left(\frac{1}{u}\right) = -f(u)$ where $u \in R^{+}$.

1 mark

b. If $2f(u) = f(2v) + f(3v)$ where $u, v \in R^{+}$, then express u in terms of v.

2 marks

Question 4

a. Find the period and the range of the function $g: R \to R, g(x) = 3 + \sin\left(\frac{2x}{3}\right)$.

2 marks

b. Solve the equation $\cos(3x) = \frac{\sqrt{3}}{2}$ for $x \in R$.

3 marks

Question 5

The discrete random variable X has a probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.2	0.1	0.4	p

a. Find the value of p.

1 mark

b. Find $\Pr(X \le 1 | X < 3)$.

2 marks

c. Find $E(X)$, the mean of X.

2 marks

Question 6

Let $f: R \to R, f(x) = 1 - e^{2(x-1)}$.
Find f^{-1}, the inverse function of f.

3 marks

Question 7

A transformation $T : R^2 \to R^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

The curve with equation $y = \frac{1}{x}$ is transformed according to *T* to produce the image equation $y = \frac{a}{x+b} + c$.

Find the values of *a*, *b* and *c*.

3 marks

Question 8

For two events A and B, $\Pr(A)=\frac{1}{3}$ and $\Pr(B)=\frac{1}{2}$.
Find $\Pr(A \cup B')$ if

a. A and B are mutually exclusive.

2 marks

b. A and B are independent events.

2 marks

Question 9

June shops either at the market or locally every week.
If she shops at the market one week the probability that she shops there the following week is 0.6. If she shops locally one week then the probability that she shops at the market the following week is 0.7.

This week June shopped at the market.

What is the probability that June will shop locally exactly once in the coming three weeks?

3 marks

Question 10

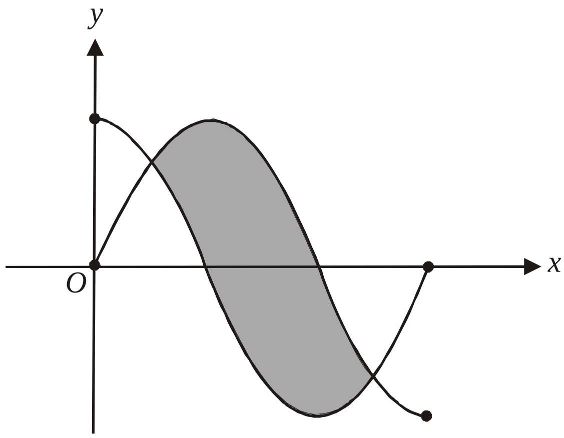
A normal to the graph of $y = x^{\frac{3}{5}} + c$ at the point where $x = 1$ has an x-intercept at $(a, 0)$ where a and c are both non-zero real constants.
Find an expression for a in terms of c.

3 marks

Question 11

Let $f:\left[0,\frac{3\pi}{2}\right]\to R,\ f(x)=2\sin(x)$ and $g:\left[0,\frac{3\pi}{2}\right]\to R,\ g(x)=2\cos(x)$.

The graphs of *f* and *g* are shown below.



Find the area of the shaded region.

5 marks

Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$