

Year 2012

VCE

Mathematical Methods CAS

Trial Examination 1



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**Victorian Certificate of Education
2012**

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $\sqrt{16-x^3}$ with respect to x .

2 marks

b. Hence find $\int \frac{x^2}{\sqrt{16-x^3}} dx$

1 mark

Question 2

Find the general solution of $4\cos\left(\frac{\pi x}{3}\right) - 2 = 0$.

2 marks

Question 3

The function f has the rule $f(x) = \log_e(x+3)$ and the function g has the rule $g(x) = 5 + 2x - x^2$. State the maximal domain for which $f(g(x))$ is defined.

2 marks

Question 4

The image of the line $4x + y = 3$ under the transformation $T : R^2 \rightarrow R^2$ of the plane defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \end{bmatrix}$, is given by $ax + by = k$, find the values of a , b and k .

3 marks

Question 5

The distance q cm, that an image is from a certain lens in terms of p cm, the distance of the object from the lens, is given by $q = \frac{10p}{p-10}$.

- i. Show that the rate of change of distance that an image is from a certain lens, with respect to the distance of the object from the lens is given by $\frac{dq}{dp} = \frac{-100}{(p-10)^2}$

1 mark

- ii. Find using a linear approximation, the change in the distance of the image from the lens when the distance of the object from the lens changes from 15 to 15.1 cm.

2 marks

- iii. If the object distance is increasing at a rate 0.25 cm/sec, how fast is the image distance changing, when the distance from the object is 15 cm.

2 marks

Question 6

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} kx(3-x) & \text{for } 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

- i. Show that $k = \frac{2}{9}$.

1 mark

- ii. Find $\Pr(X > 2 | X > 1)$.

3 marks

Question 7

A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X = x)$	$\log_8(k+1)$	$\log_8(2k-4)$

i. Find the possible values of k .

3 marks

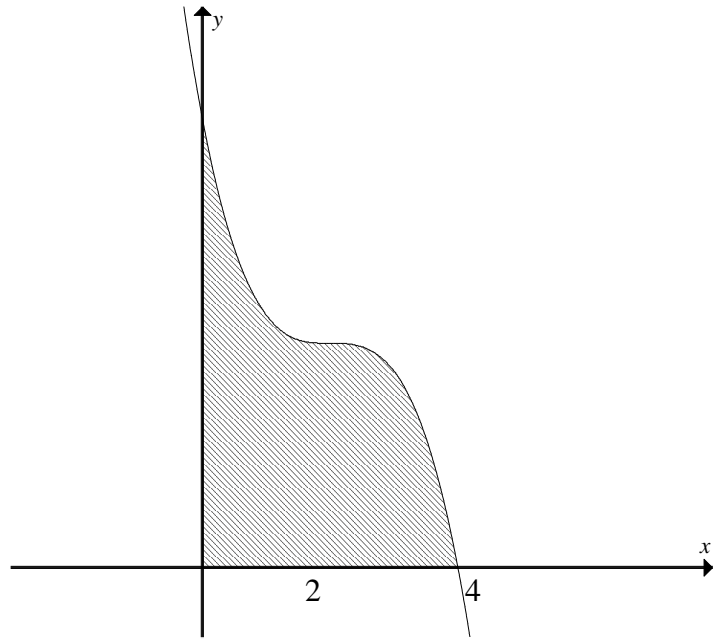
ii. Find $E(X)$, giving your answer as a fraction.

2 marks

Question 8

Part of the graph of $y = a(x - h)^3 + k$ is shown. The graph crosses the x -axis at $x = 4$, and has a stationary point at $x = 2$. The shaded area is the area bounded by the graph of $y = a(x - h)^3 + k$ and the coordinate axes, this area is equal to 64 units².

Find the values of a , h and k .



4 marks

Question 9

For the curve $y = 16 - x^3$, find the equation of the tangent to the curve, which passes through the origin. Hence find values of k , for which the equation $16 - x^3 = kx$ has

- i** exactly one real solution.
- ii** exactly two real solutions.
- iii** exactly three real solutions.

5 marks

Question 10

The speed v , in metres per second, of an object moving in a straight line is given by a function of time t , in seconds, where $v(t) = \frac{24}{\sqrt{4t+9}}$ where $t \geq 0$.

Find the distance travelled in metres by the object in the first 4 seconds.

2 marks

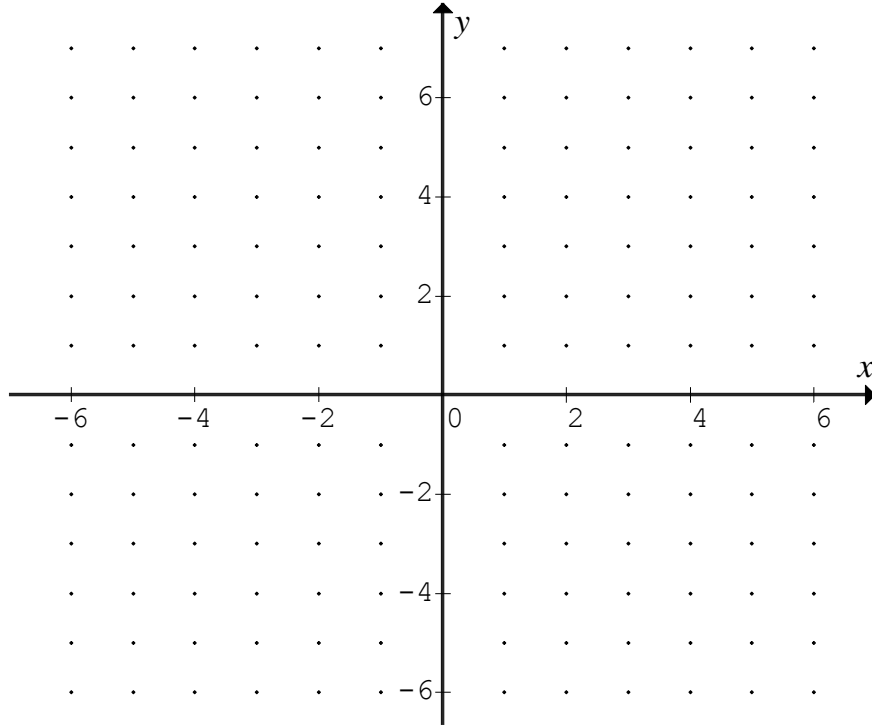
Question 11

Given the function $f : \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R}$ $f(x) = \frac{3x-5}{x-2}$.

i. Express $f(x)$ in the form $\frac{a}{x+b} + c$.

1 mark

- ii. Sketch the graph of the function $f : R \setminus \{2\} \rightarrow R$ $f(x) = \frac{3x-5}{x-2}$, stating the coordinates of all axial intercepts, and the equations of any asymptotes.



2 marks

- iii. Find the inverse function f^{-1} .

2 marks

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc \sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(e^{ax}) = ae^{ax}$ $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$ $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$ $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$ $\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$ $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ $\int \frac{1}{x} dx = \log_e x + c$ $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$ $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
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product rule:
$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Chain rule:
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

approximation:
$$f(x+h) \approx f(x) + h f'(x)$$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$