Year 2012

VCE

Mathematical Methods CAS

Trial Examination 1

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Victorian Certificate of Education
2012

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical anser is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $\sqrt{16-x^3}$ with respect to x.

2 marks

b. Hence find $\int \frac{x^2}{\sqrt{16-x^3}}\,dx$

1 mark

Question 2

Find the general solution of $4\cos\left(\frac{\pi x}{3}\right)-2=0$.

2 marks

Question 3

The function f has the rule $f(x) = \log_e(x+3)$ and the function g has the rule $g(x) = 5 + 2x - x^2$. State the maximal domain for which $f(g(x))$ is defined.

2 marks

Question 4

The image of the line $4x + y = 3$ under the transformation $T : R^2 \to R^2$ of the plane defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \end{bmatrix}$, is given by $ax + by = k$, find the values of *a*, *b* and *k*.

3 marks

Question 5

The distance q cm, that an image is from a certain lens in terms of p cm, the distance of the object from the lens, is given by $q = \dfrac{10p}{p-10}$.

i. Show that the rate of change of distance that an image is from a certain lens, with respect to the distance of the object from the lens is given by $\dfrac{dq}{dp} = \dfrac{-100}{(p-10)^2}$

1 mark

ii. Find using a linear approximation, the change in the distance of the image from the lens when the distance of the object from the lens changes from 15 to 15.1 cm.

2 marks

iii. If the object distance is increasing at a rate 0.25 cm/sec, how fast is the image distance changing, when the distance from the object is 15 cm.

2 marks

Question 6

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} k\,x(3-x) & \text{for} \quad 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

i. Show that $k = \frac{2}{9}$.

1 mark

ii. Find $\Pr(X > 2 \mid X > 1)$.

3 marks

Question 7

A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X = x)$	$\log_8(k+1)$	$\log_8(2k-4)$

i. Find the possible values of k.

3 marks

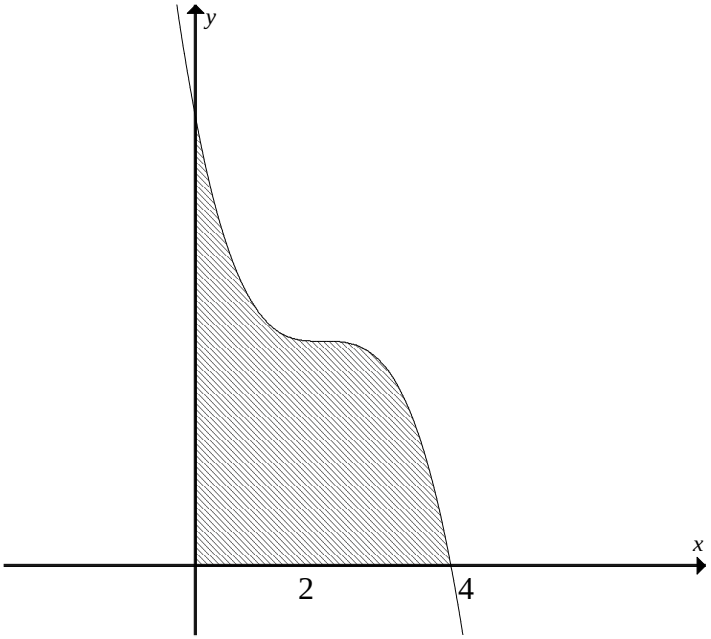
ii. Find $E(X)$, giving your answer as a fraction.

2 marks

Question 8

Part of the graph of $y = a(x-h)^3 + k$ is shown. The graph crosses the *x*-axis at $x = 4$, and has a stationary point at $x = 2$. The shaded area is the area bounded by the graph of $y = a(x-h)^3 + k$ and the coordinate axes, this area is equal to 64 units^2.

Find the values of *a*, *h* and *k*.



4 marks

Question 9

For the curve $y = 16 - x^3$, find the equation of the tangent to the curve, which passes through the origin. Hence find values of *k*, for which the equation $16 - x^3 = kx$ has

i exactly one real solution.

ii exactly two real solutions.

iii exactly three real solutions.

5 marks

Question 10

The speed *v*, in metres per second, of an object moving in a straight line is given by a function of time *t*, in seconds, where $v(t) = \frac{24}{\sqrt{4t+9}}$ where $t \geq 0$.

Find the distance travelled in metres by the object in the first 4 seconds.

2 marks

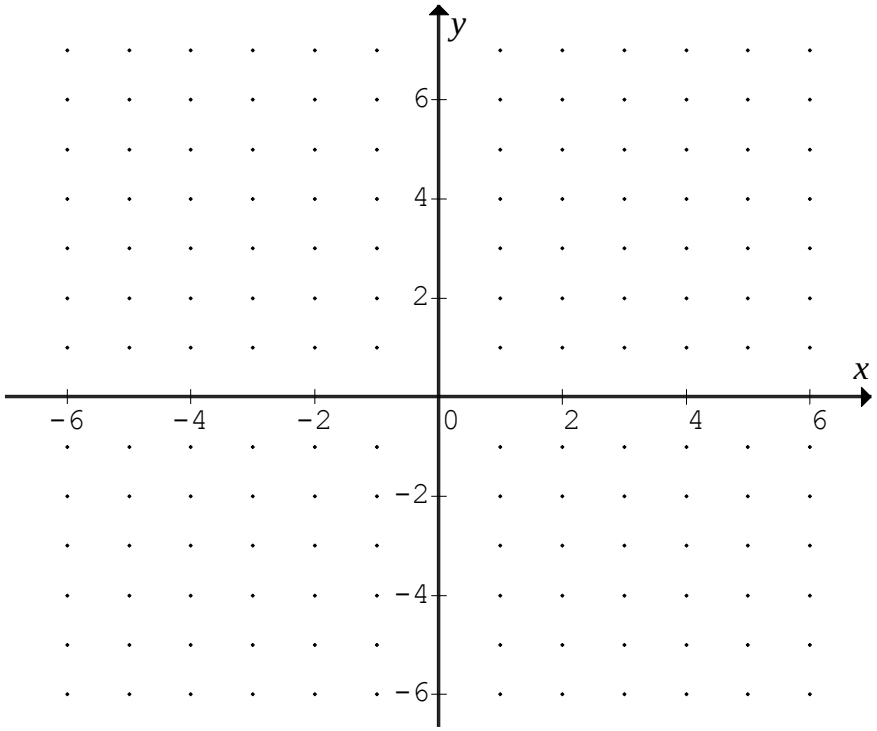
Question 11

Given the function $f : R \setminus \{2\} \to R \quad f(x) = \frac{3x-5}{x-2}$.

i. Express $f(x)$ in the form $\frac{a}{x+b} + c$.

1 mark

ii. Sketch the graph of the function $f : R \setminus \{2\} \to R \quad f(x) = \dfrac{3x-5}{x-2}$, stating the coordinates of all axial intercepts, and the equations of any asymptotes.



2 marks

iii. Find the inverse function f^{-1}.

2 marks

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c \ , \ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\,f(x)\,dx$