MATHS METHODS 2012

UNIT 3 SAC 2 : TEST

PART 1 SOLUTIONS

1. (a) ignoring the translations

$y = 4\sin 2(x)$

$\Rightarrow y = 4\sin\left(\frac{x}{\frac{1}{2}}\right)$

dilation of factor 4 from x - axis (1)
dilation of factor $\frac{1}{2}$ from y - axis.

translation $\frac{2\pi}{3}$ units in positive x direction
translation 5 units in positive y direction (1)

(dilations must be stated before translations).

(b) range $= [5-4, 5+4]$
$= [1, 9]$ (1)

period $= \frac{2\pi}{2}$
$= \pi$ (1)

(c) $0 \leq x \leq \pi$

$-\frac{2\pi}{3} \leq x - \frac{2\pi}{3} \leq \frac{\pi}{3}$

$-\frac{4\pi}{3} \leq 2\left(x - \frac{2\pi}{3}\right) \leq \frac{2\pi}{3}$

(start to look for solutions between $-\frac{4\pi}{3}$ and $\frac{2\pi}{3}$)

$\sin 2\left(x - \frac{2\pi}{3}\right) = \frac{1}{2}$

$2\left(x - \frac{2\pi}{3}\right) = -\frac{7\pi}{6}, \frac{\pi}{6}$

(1) for $\frac{\pi}{6}$ as basic angle

$x - \frac{2\pi}{3} = -\frac{7\pi}{12}, \frac{\pi}{12}$

$x = -\frac{7\pi}{12} + \frac{8\pi}{12}, \frac{\pi}{12} + \frac{8\pi}{12}$

$x = \frac{\pi}{12}, \frac{9\pi}{12}$

$x = \frac{\pi}{12}, \frac{3\pi}{4}$ (1)

2. (a) vertical asymptote at $x = 0.75$

$\Rightarrow bx + c = 0$

$\frac{3}{4}b + c = 0$ $(\times 4)$

$3b + 4c = 0$

at $(1, 0)$

$0 = a\log_e(b + c)$

$0 = \log_e(b + c)$

$e^0 = b + c$

$b + c = 1$ $(\times -3)$

$3b + 4c = 0$

$-3b - 3c = -3$

$c = -3$

$\Rightarrow b = 4$

(b) $\log_e 9 = a\log_e(4 \times 1.5 - 3)$

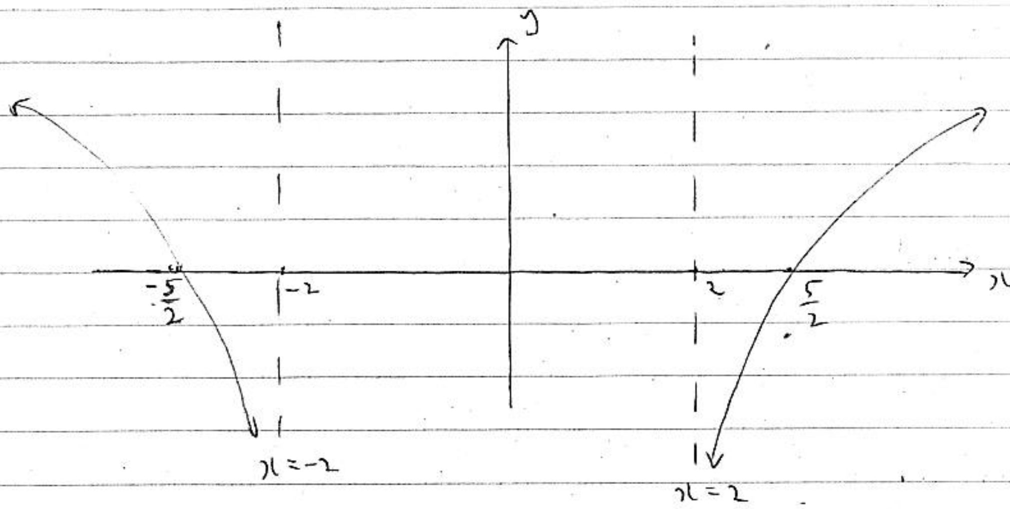
$\log_e 9 = a\log_e 3$

$2\log_e 3 = a\log_e 3$

$a = 2$ (1)

3. $y = \log_e (2|x| - 4)$

(is a reflection in the y-axis of $y = \log_e(2x-4)$ as well as the graph of $y = \log_e(2x-4)$).



x-intercept, $0 = \log_e(2x-4)$

$$e^0 = 2x - 4$$
$$2x = 5$$
$$x = \frac{5}{2}$$

vertical asymptote, where $2x - 4 = 0$

$$x = 2$$

4. Let $a = 2^x$

$2 \times a - 5 = \frac{12}{a}$ $(\times a)$

$2a^2 - 5a = 12$

$2a^2 - 5a - 12 = 0$ (1)

$(2a+3)(a-4) = 0$

$a = -\frac{3}{2}$ or $a = 4$

$2^x = -\frac{3}{2}$ or $2^x = 4$

no solution $\therefore x = 2$

as $2^x > 0$ (1) (1)

5.

	f	g
dom	R	$(0, \infty)$
ran	$[-1,1]$	R

ran $g \subseteq$ dom f

$\therefore f(g(x))$ defined (1)

dom $f(g(x))$ = dom $g(x)$

$= (0, \infty)$ (1)

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PART 2 SOLUTIONS

MULTI. CHOICE

1. period $= \pi$

$\pi = \frac{2\pi}{n}$

$n = 2$

(not A or B)

AMPLITUDE $= 3$

when $x = 0$

C, D, E are true

BY INSPECTION

it's a cos graph translated

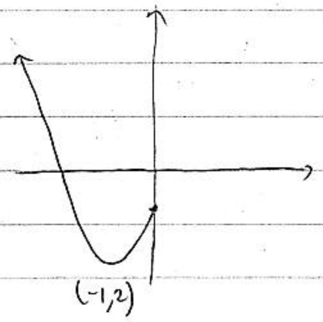
$\frac{\pi}{4}$ left, $\therefore$ 'D' D

(or a sin graph $\frac{\pi}{2}$ RIGHT) or C

2. must be one-to-one

A. is one to one

B.



B

not one-to-one

3. $0 = \frac{1}{2}\log_e(x-1) + 3$

use solve,

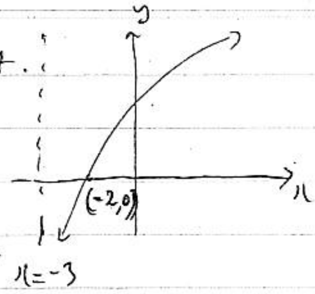
or

$-6 = \log_e(x-1)$

$e^{-6} = x - 1$

$x = \frac{1}{e^6} + 1$ A

4.



y values change,

x values stay the same

but $(-2,0)$ would stay as $(-2,0)$

and asymptote would not change E

5. $y = \log_e(x+2) + 3$

then

$y = \log_e(x+2) + 3 + 1$

then

$y = -\left(\log_e(x+2) + 4\right)$

$y = -\log_e(x+2) - 4$ D

6. period $= \frac{\pi}{\frac{2\pi}{5}}$

$= \frac{\pi}{1} \times \frac{5}{2\pi}$

$= \frac{5}{2}$ C

7. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -x \\ 2y \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} -x-2 \\ 2y+2 \end{bmatrix}$

$x' = -x - 2$ and $y' = 2y + 2$

$x = -x' - 2$ and $y = \frac{y' - 2}{2}$

equation of image

$\frac{y'-2}{2} = e^{-x'-2}$

$y' - 2 = 2e^{-x'-2}$

$y = 2e^{-x-2} + 2$ (C)

8. on calculator,
solve $(10\cos(3x) = 5, x) \mid 0 \le x \le \pi$

$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

$\frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} = \frac{13\pi}{9}$ (D)

9. inverse
$x = 2\log_e(y+2) - 1$

solve for y on calculator,

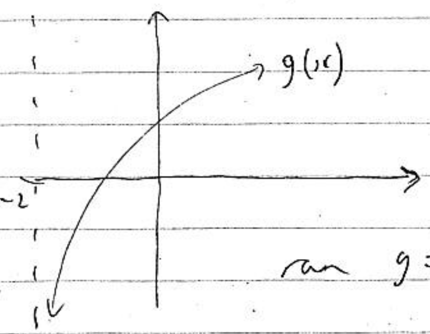
OR

$\frac{x+1}{2} = \log_e(y+2)$

$y + 2 = e^{\frac{x+1}{2}}$

$y = e^{\frac{x+1}{2}} - 2$

dom g^{-1} = ran g



ran $g = R$

$\therefore g^{-1}: R \to R, g^{-1}(x) = e^{\frac{x+1}{2}} - 2$

(D)

10. solve $\sin(2x) = -1$
on calculator

$x = \frac{(4n-1)\pi}{4}, n \in Z$

$x = \frac{4n\pi}{4} - \frac{\pi}{4}$

$= n\pi - \frac{\pi}{4}$ (A)

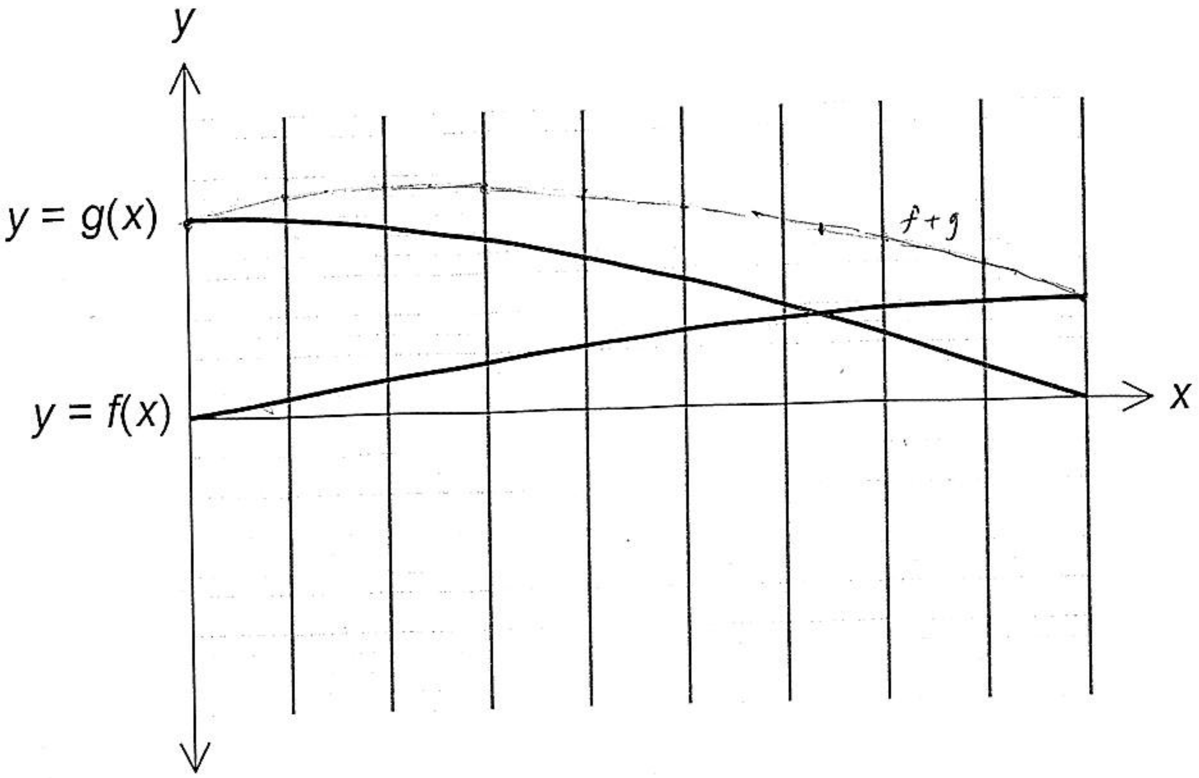
11. period $= \frac{2\pi}{2}$

use calculator to solve
for e.g. $|2\cos(2x)| = 2$
for given domain
gives 9 solutions
(or draw or picture graph)

(E)

12. $R \setminus \{2\}$ (C)

1.



2 marks

1 mark for endpoints placed correctly

1 mark for correctly placed shape

2. (a) $h(0) = -1.0$ m.

(b) maximum height = 8.2 m.
at t = 25.4 seconds

(c) $4 = -0.5 + e^{0.07t} \sin\left(\frac{\pi t - 554}{2}\right) + 0.11t$

$t = 16.9$ s.