**SACRED HEART GIRLS’ COLLEGE**

**OAKLEIGH**



**Mathematical Methods CAS 2012**

**Unit 4 SAC 4: ANALYSIS TASK**

**PART B**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

 **Teacher (please circle)**: Ms Gates Mr Smith

**Part B:**

**Section 1: 5 short answer questions**

**(summary notes and CAS calculator permitted)**

**Reading: 5 minutes**

**Writing: Section 1: 35 minutes**

**Marks: 18**

**SECTION 1: SHORT ANSWER QUESTIONS**

**Instructions:**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this test are **not** drawn to scale.

Question 1

1. Find the gradient of the normal to the curve $y=\frac{cos⁡(x)}{e^{x}}$ at $x=π.$

2 marks

1. Find the derivative of $log\_{e}(2f(x))$with respect to *x*.

2 marks

Question 2

Find the average value of the function or $y=cos⁡(x)$ over the interval $\left[0,\frac{π}{2}\right]$.

2 marks

Question 3

The acceleration of a car is given by $a=-2t+3$ where $t\geq 0$.

1. Find the rule for the velocity if the initial velocity is 1 ms-1.

2 marks

1. At what time, correct to 3 decimal places, is the velocity zero?

1 mark

Question 4

1. Find the *x* intercept for $f\left(x\right)=2sin\left(x-\frac{π}{2}\right)$ for $x\in \left[0,\frac{3π}{2}\right]$

1 mark

1. Write the definite integral for the area of the region enclosed by the curve

 $f\left(x\right)=2sin\left(x-\frac{π}{2}\right)$, the *x* axis and the lines $x=0$ and $x=\frac{3π}{2}$.

2 marks

1. Evaluate the definite integral

1 mark

Question5

The graph of $f:R\rightarrow R, f\left(x\right)=e^{\frac{x}{2}}+1$ is shown. The normal to the graph of *f* where it crosses the *y* axis is also shown.



1. Show that the equation to the normal to the graph of *f* where it crosses the *y* axis is

 $y=-2x+2.$

2 marks

1. Find the exact area of the shaded region.

3 marks

END OF SECTION 1

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**PART B**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

 **Teacher (please circle)**: Ms Gates Mr Smith

**Part B:**

**Section 2: One extended response question**

**(summary notes and CAS calculator permitted)**

**Reading: 5 minutes**

**Writing: Section 2: 40 minutes**

**Marks: 18**

**Instructions:**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this test are **not** drawn to scale.

Dorothy Smart, the florist who sells flowers at the Oakleigh Market, grows water lilies in a small pond in her backyard. The pond is bounded by her neighbour’s property, two garden beds that are at right angles to each other and the lawn modelled by the curve

$y=4log\_{e}\left(x+e\right),x\in [0,e^{2}-e]$.

All measurements are in metres.



1. **B** has coordinates $(e^{2}-e, 0)$. Find the exact coordinates of the points **A** and **C**.

2 marks

Dorothy wants to calculate the area of the pond. She investigates using different methods.

1. Method 1: Dorothy divides the length OB into three intervals of width $1.56 $metres and uses rectangle approximation to find the approximate area of the pond.
	* 1. Use left endpoint estimate to find the approximate area of the pond in square metres correct to two decimal places.



2 marks

* + 1. Use right endpoint estimate to find the approximate area of the pond in square metres correct to two decimal places.



2 marks

1. Method 2: Dorothy attempts to find the exact area of the surface OACB by first differentiating the function $y=(x+e)log\_{e}(x+e)$
	* 1. Differentiate $y=(x+e)log\_{e}(x+e)$

1 mark

* + 1. Hence, find an antiderivative for $log\_{e}(x+e)$.

3 marks

* + 1. Using your answer in part ii, find the exact area of the pond.

2 marks

1. Method 3: Dorothy’s cousin, Daisy, suggests a different approach for finding the area of the pond. Daisy’s method uses the inverse function of $y=4log\_{e}(x+e)$.
	* 1. If $f:\left[0,e^{2}-e\right]\rightarrow R, f\left(x\right)=4log\_{e}(x+e)$ find $f^{-1}(x)$.

2 marks

* + 1. Sketch $f^{-1}(x)$ on the axes below labelling axes intercepts with coordinates.



2 marks

* + 1. Write the definite integral to find the exact area enclosed by the inverse function, the *x* axis and the straight line $x=8$.

1 mark

* + 1. Hence, find the area described in part iii.

1 mark

END OF SAC