**SACRED HEART GIRLS’ COLLEGE**

**OAKLEIGH**



**Mathematical Methods CAS 2012**

**Unit 4 SAC 5: Probability TASK**

**PART B**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

 **Teacher (please circle)**: Ms Gates Mr Smith

**Part B:**

**6 item analysis questions**

 **(notes and calculator permitted)**

**Reading: 5 minutes**

**Writing: 40 minutes**

**Marks: 26**

**Instructions:**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this test are **not** drawn to scale.

Question 1

Three fair coins are tossed simultaneously. Let X represent the number of heads that come up. The probability distribution for X is given by

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A. | X | 0 | 1 | 2 | 3 |
|  | Pr(X=x) | $$\frac{1}{4}$$ | $$\frac{1}{4}$$ | $$\frac{1}{4}$$ | $$\frac{1}{4}$$ |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| B. | X | 0 | 1 | 2 | 3 |
|  | Pr(X=x) | $$\frac{1}{8}$$ | $$\frac{2}{8}$$ | $$\frac{4}{8}$$ | $$\frac{1}{8}$$ |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| C. | X | 0 | 1 | 2 | 3 |
|  | Pr(X=x) | $$\frac{1}{8}$$ | $$\frac{3}{8}$$ | $$\frac{3}{8}$$ | $$\frac{1}{8}$$ |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| D. | X | 0 | 1 | 2 | 3 |
|  | Pr(X=x) | $$\frac{1}{12}$$ | $$\frac{5}{12}$$ | $$\frac{5}{12}$$ | $$\frac{1}{12}$$ |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| E. | X | 0 | 1 | 2 | 3 |
|  | Pr(X=x) | $$\frac{2}{12}$$ | $$\frac{4}{12}$$ | $$\frac{5}{12}$$ | $$\frac{1}{12}$$ |

1. Find the correct response

1 mark

1. What is the probability of obtaining either two heads and a tail or two tails and a head?

1 mark

1. Why is alternative D incorrect?

1 mark

1. Why is alternative E incorrect?

1 mark

Question 2

Every Sunday Jessica has take-away food for lunch. She only ever buys sushi or pho. If she has sushi one Sunday the probability that she has sushi the next Sunday is 0.4. If she has pho one Sunday the probability that she has pho the next Sunday is 0.25. In the long run, the proportion of Sundays that Jessica has sushi for lunch is

1. $\frac{4}{9}$
2. $\frac{5}{9}$
3. $\frac{1}{2}$
4. $\frac{11}{12}$
5. $\frac{7}{9}$
6. Find the correct response.

2 marks

1. What error would lead to alternative A?

1 mark

Question 3

In a country town, 500 people go to see a film each week at one of the two cinemas, Palais or Kina. In the past year it has been found that 70% of those who go to the Palais cinema in any one week will return there the following week. 40% of those who visit Kina will go to Palais the following week.

The Palais cinema has been closed for renovations for a few weeks so everyone has been going to Kina. Palais cinema has just reopened.

The number of people going to the Palais cinema in the second week after it has reopened will be

1. 195
2. 200
3. 210
4. 240
5. 260
6. Write the initial state matrix and the transition matrix for this situation.

2 marks

1. Hence, find the correct alternative.

2 marks

1. What error would lead to alternative D

1 mark

Question 4

X is a random variable with a probability density function given by

$$f\left(x\right)=\left\{ \begin{array}{c}\frac{x}{2}-\frac{1}{2} 1\leq x\leq 3\\ 0 elsewhere\end{array}\right.$$

The median of X is

1. $1-\sqrt{2}$
2. $\frac{7}{3}$
3. $\sqrt{2}$
4. $2$
5. $1+\sqrt{2}$
6. Find the correct response.

2 marks

1. What error would lead to alternative B?

1 mark

1. What error would lead to alternative D?

1 mark

1. Explain why alternative A can be a correct mathematical solution to this question even though it is not correct within the context of the actual function given.

1 mark

Question 5

X is a random variable with a probability density function given by

$$f\left(x\right)=\left\{ \begin{array}{c}\frac{4xsin^{2}x}{π^{2}} 0\leq x\leq π\\ 0 elsewhere\end{array}\right.$$

The mean of X is

1. 0.5295
2. 1.5708
3. 1.7761
4. 1.7927
5. 1.8366
6. Find the correct response.

2 marks

1. Show that alternative A is the standard deviation of X.

2 marks

1. What feature of the graph of the probability density function is at x=1.8366.

1 mark

Question 6

Spark-O batteries have a mean life of 30 months. If the life of the batteries is normally distributed and the probability that a Spark-O battery lasts longer than 36 months is 0.05 the standard deviation of the batteries is closest to

1. 1.65 months
2. 3 months
3. 3.65 months
4. 6.32 months
5. 9.87 months
6. Find the correct response showing working.

2 marks

1. What is a possible reason for alternative B being chosen?

1 mark

1. What change would have to be made to the probability of a battery lasting longer than 36 months for alternative D to be chosen?

1 mark

END OF SAC