Student Name:____

MATHEMATICAL METHODS (CAS)

Unit 3
Targeted Evaluation Task for School-assessed Coursework 3

TSSM™
Creating VCE Success

2012 Application task on Functions and Calculus for Outcomes 1, 2 & 3

Recommended writing time*: 250 minutes
Total number of marks available: 80 marks

TASK BOOK

*The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, bound summary booklet, approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 14 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.
- Show appropriate scales on the axes provided when sketching graphs.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

For questions worth more than 1 mark, relevant working must be shown.

SECTION 1

You are a civil engineer with a construction company that has been asked to redesign the track for a ride at an amusement park. In this application task you will attempt to derive mathematical models for the profile of the track so that it meets various constraints.

Question 1

The amusement park owners would like to follow the existing track profile as much as possible in the initial part of the ride.

The first part of the track has a parabolic shape that starts at a point 35 metres above the ground and has a minimum point that is 25 metres below the starting point and at a horizontal distance of 30 metres from the starting point. (The track is being built on horizontal ground.)

a. If you choose Cartesian axes so that the horizontal axis is at ground level and the vertical axis runs through the starting point, what are the coordinates of the starting point and minimum point of the first part of the track?

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2 marks

You decide to use a quadratic model for this first part of the track that starts at the original starting point and has a minimum at the same point and is of the form: $f_1(x) = A(x+B)^2 + C$.

b. Find the values of B and C, giving a reason for your answer.

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2 marks

c. Find the exact value of A.

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2 marks

d. Express $f_1(x)$ in the form $f_1(x) = ax^2 + bx + c$

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1 mark

e. What is the initial gradient of the track?

2 marks

Question 2

The owners of the amusement park decide that they need to change the initial gradient of the track to -2 to increase the speed more quickly in the early part of the ride.

a. Using a new function $f_2(x) = ax^2 + bx + c$ find the value of b required to obtain this initial gradient.

1 mark

b. Find the values of a and c, so that this function also has a minimum at the same point as the original track and hence write out the rule for $f_2(x)$.

3 marks

c. The first section of the track is to finish at the point B where the gradient is 1. Find the coordinates of B and hence write down the domain of $f_2(x)$.

3 marks

Question 3

A second section of the track is modelled by the function: $g(x)=ax^2+bx+c$ and must join smoothly with the first section at B and pass through the point C with coordinates (75, 25)

a. Find the exact values of the coefficients and hence the rule for $g(x)$ which fulfils these requirements.

5 marks

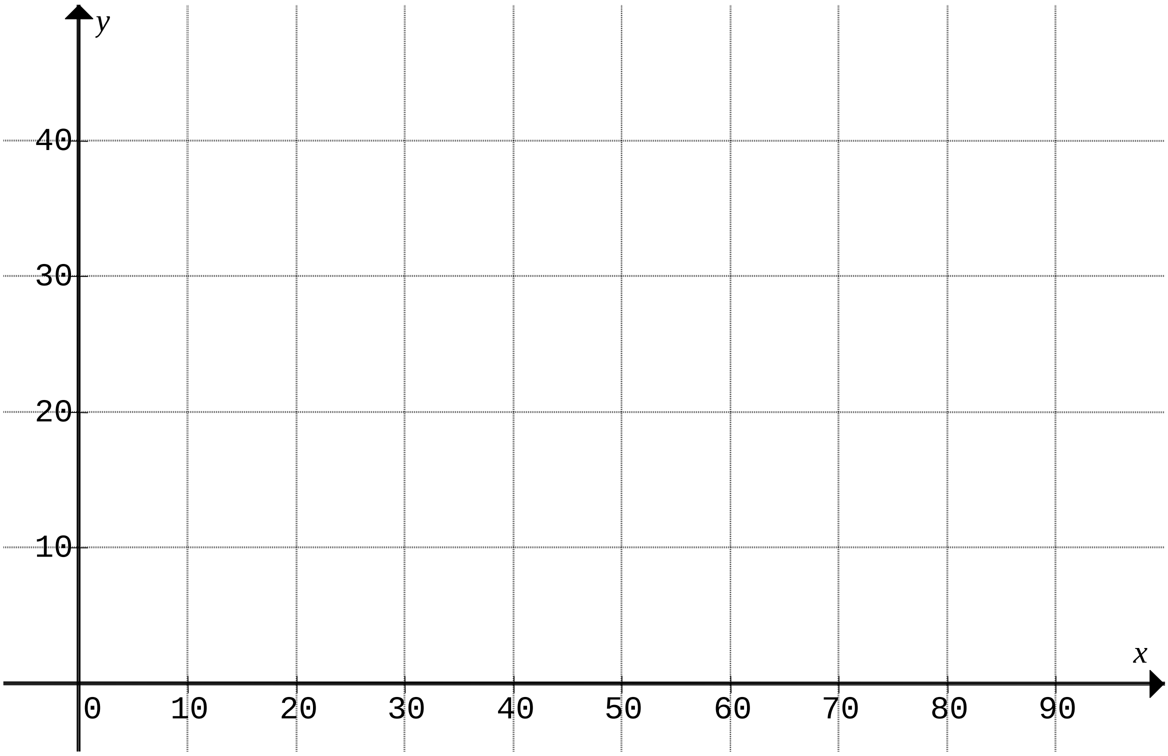
b. The second section of track is to end when the track is 5 metres above the ground. Use the quadratic rule to find this point and hence state the domain of $g(x)$.

4 marks

c. Use calculus to find the coordinates of the maximum point of the second section of track.

2 marks

d. On the set of axes below sketch a graph of $f_2(x)$ and $g(x)$ clearly labelling key points.



4 marks

SECTION 2

Question 1

The chief engineer of the company suggests that instead of two quadratic functions you try a cubic model of the form $h_1(x) = ax^3 + bx^2 + cx + d$ for the track. You decide to do so with the following constraints:
The gradient at the start of the track must be -2.
There must be a minimum point at (30, 10)
The track must end at (95, 5)

a. Show that the value of *c* is -2.

1 mark

b. Write down the 3other simultaneous equations in *a*, *b* and *d* which represent this information.

3 marks

c. Express these 3 equations as a matrix equation in the form AX = B.

2 marks

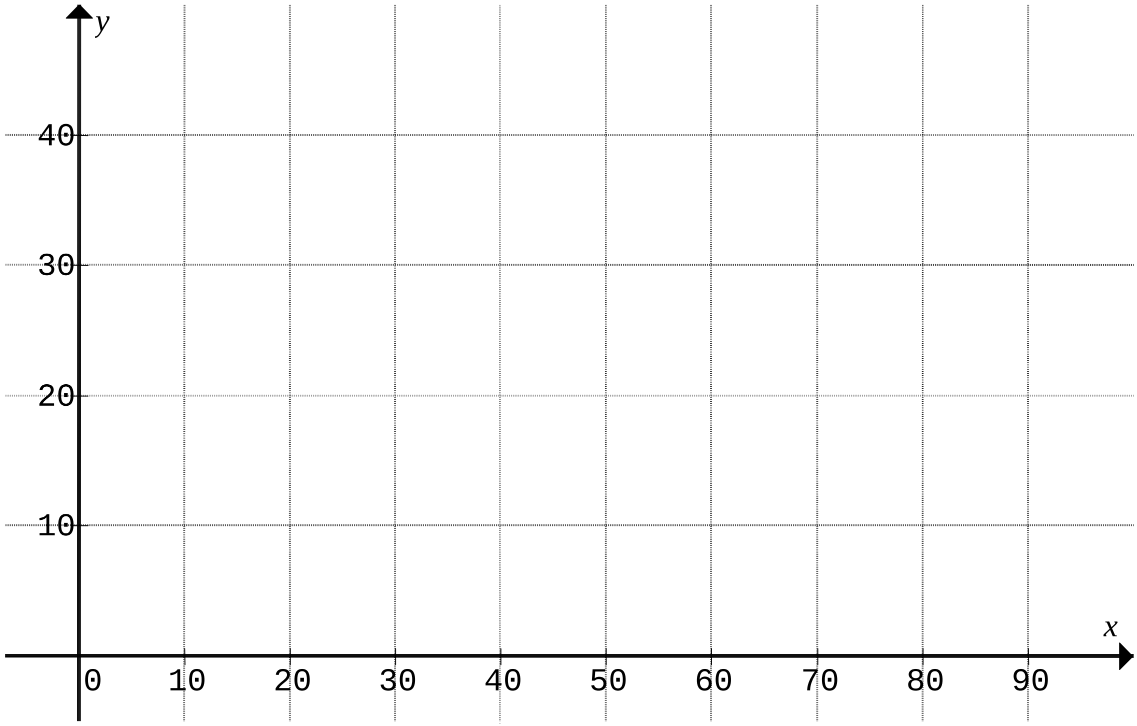
d. Express X in terms of A and B

1 mark

e. Use your graphics calculator to solve the matrix equation and hence write out the rule for $h_1(x)$. Express *a*, *b* and *d* to 4 significant figures.

2 marks

f. Sketch a graph of $h_1(x)$ on the set of axes below.



2 marks

g. Find the coordinates of the maximum turning point correct to 1 decimal place.

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1 mark

Question 2

a. While keeping *b*, *c* and *d* constant, investigate the effect of varying *a* slightly. What effect does increasing the magnitude of *a* have upon the graph of $h_1(x)$?

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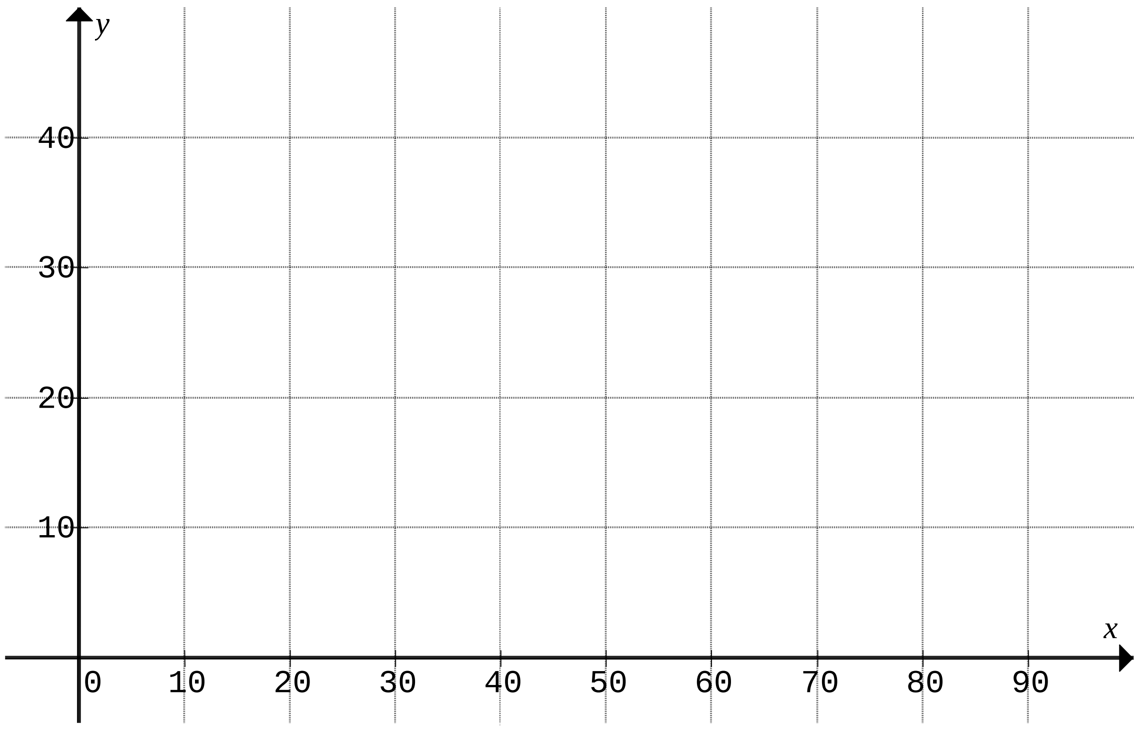
3 marks

b. While keeping *a*, *c* and *d* constant investigate the effect of varying *b* slightly. What effect does increasing the magnitude of *b* have upon the graph of $h_1(x)$?

3 marks

c. Hence find values of *a* and *b* to form a new cubic function $h_2(x)$ (with the same values of *c* and *d* that you used for $h_1(x)$) that has a maximum value at about (70,25) while still ending as close to (95,5) as possible. Write your values of *a* and *b* below and sketch a graph of $h_2(x)$ on the set of axes below.



4 marks

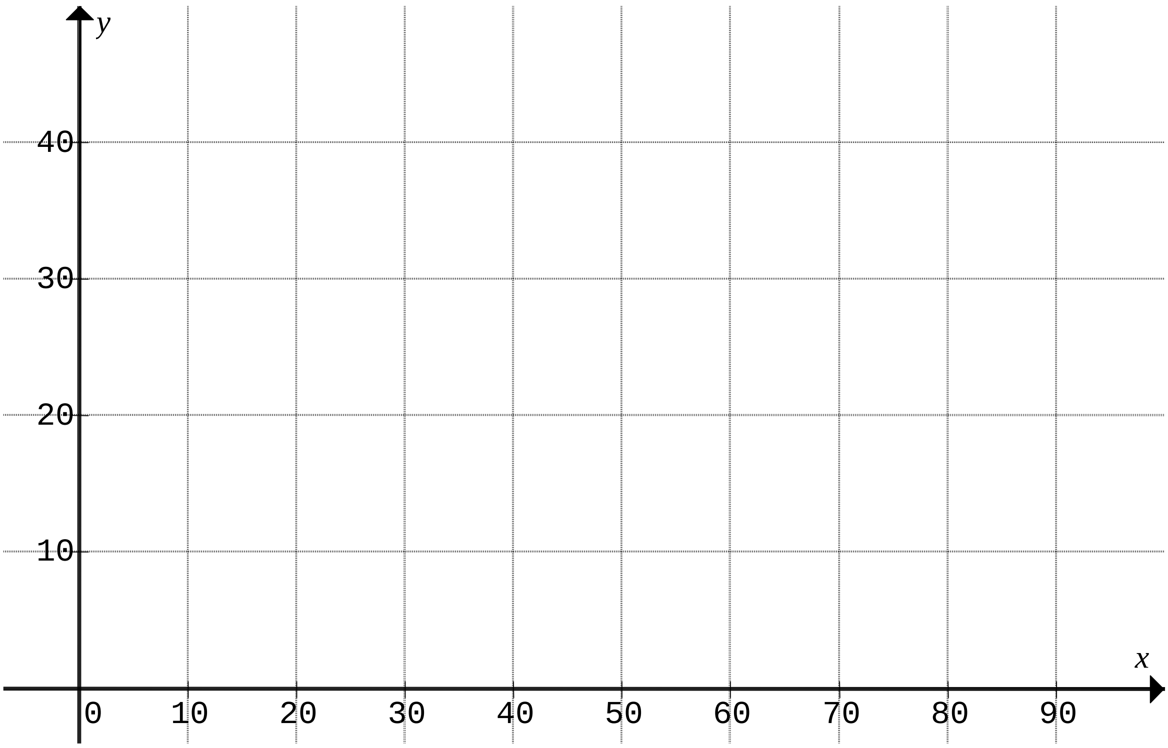
d. What effect would varying *d* have on the graph of $h(x)$?

1 mark

SECTION 3

One further suggestion from the chief engineer is to investigate the use of a function of the form $j(x) = a\cos^2(bx)e^{-cx}$ to model the track. a, b and c are positive real numbers.

Question 1
As an initial attempt use $a = 30$, $b = 0.06$ and $c = 0.01$. On the set of axes below sketch a graph of $j(x)$ using these values over the domain (0, 95). Label the coordinates of the turning points correct to 2 decimal places.



3 marks

Question 2

a. Find the derivative of $j(x)$ in terms of a, b, and c. Factorise your final answer as much as possible.

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3 marks

b. For $x \geq 0$ find, in terms of b and c, expressions for the x-values of the first two minima and the first two maxima. State which of the solutions give the minima and which give the maxima giving reasons for your answer.

9 marks

After consultation with the client, it is decided that for this model the track should start from a height of 40 m and the second minimum should be at ground level at a horizontal distance of 95 m from the start.

Question 3

a. What value of a is required to meet these criteria?

1 mark

b. Use your results from **Q2b** to find, in terms of π, the value of b which would be necessary to meet the second criterion.

2 marks

c. Using these values of a and b, experiment, with the aid of a graphics calculator, to find a value for c which produces a value of 25 m above ground for the height of the first maximum after the start. Write down this value of c (correct to 4 decimal places) and the x-coordinate of the maximum (correct to 2 decimal places).

2 marks

Question 4

a. What advantage does a model using $j(x)$ have over the other models investigated if at some point in the future the owners want to extend the track and make a longer ride?

2 marks

b. How could $j(x)$ be adapted to give a track that starts from a height of 35 m above the ground and has minimum points that are 10 m above the ground?

2 marks

c. Will the changes made in part **b.** above affect the horizontal distances of the turning points of the track from the start of the track? Give reasons for your answer.

2 marks

END OF TASK BOOK