Student Name:

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MATHEMATICAL METHODS (CAS)

Unit 4
Targeted Evaluation Task for School-assessed Coursework 1

2012 Analysis Problems Task on Logs & Exponentials for Outcomes 1, 2 & 3

Recommended writing time*: 100 minutes
Total number of marks available: 40 marks

TASK BOOK

*The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, bound summary booklet, approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.
- Show appropriate scales on the axes provided when sketching graphs.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

Any question worth more than 1 mark, relevant working must be shown.

Question 1

a. Find the exact value of x if $2^{x+1} = 8^{3x}$

2 marks

b. Find the value of x correct to 4 decimal places if $e^{-5x} - 12 = 0$

1 mark

Total 3 marks

Question 2

a. Find the exact value of x if $\log_5 2x = 3\log_5 4$.

1 mark

b. Show that $2\log_7 x + 3\log_7 x^{-2} - 4\log_7 3 = -4\log_7 3x$

2 marks

Total 3 marks

Question 3

If $f(x) = 16(1 - e^{-0.4x}),\ x \geq 0$

a. Find the rule for the inverse function, $f^{-1}(x)$.

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2 marks

b. State the domain and range of $f^{-1}(x)$.

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1mark

c. The graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect at two points. One of those points is (0, 0). Find the coordinates of the other intersection point correct to 3 decimal places.

_____

1 mark

d. If $f(x)$ gives the speed of a parachutist as a function of the time elapsed from when the parachutist jumped, what does $f^{-1}(x)$ give?

_____

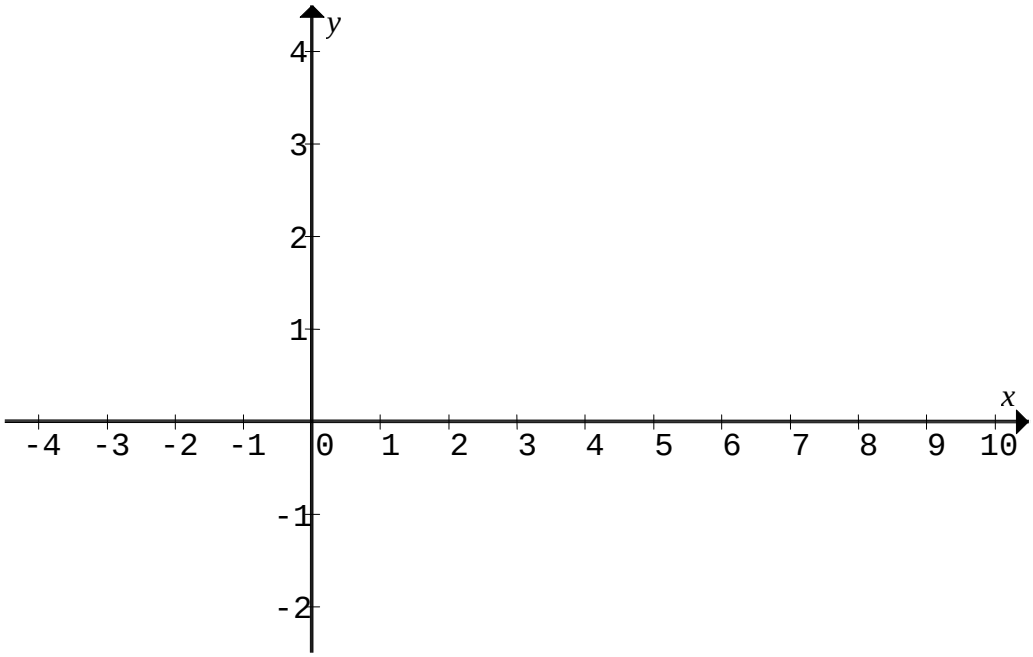
_____

1 mark

Total 5 marks

Question 4

Sketch a graph of $y = -\ln(x+3)+2$ clearly labeling any asymptotes and showing the exact axial intercepts.



4 marks

Question 5

Find the exact coordinates of the stationary points of $y = e^{x} - 6e^{-x} - 5x$.

3 marks

Question 6

The decay of radioactive isotopes can be modelled by the equation $m = Ae^{-kt}$ where m is the mass (in g) of the isotope remaining after time t (in hours).
Consider an isotope A which has a half-life of 20 hours, that is, after 20 hours half of the original mass remains.

a. Show that the value of k is 0.03466.

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_____

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2 marks

b. If 9.48 g of the isotope remained after 6.8 hours show that the initial mass of the isotope was 12 g.

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1 mark

Total 3 marks

Question 7

The isotope A described in **Question 6** decays to form another isotope B.

a. If B is not radioactive (that is it does not decay) use the information given in **Question 6** to express the mass of isotope B, in the form $m_B = p(1-e^{-qt})$

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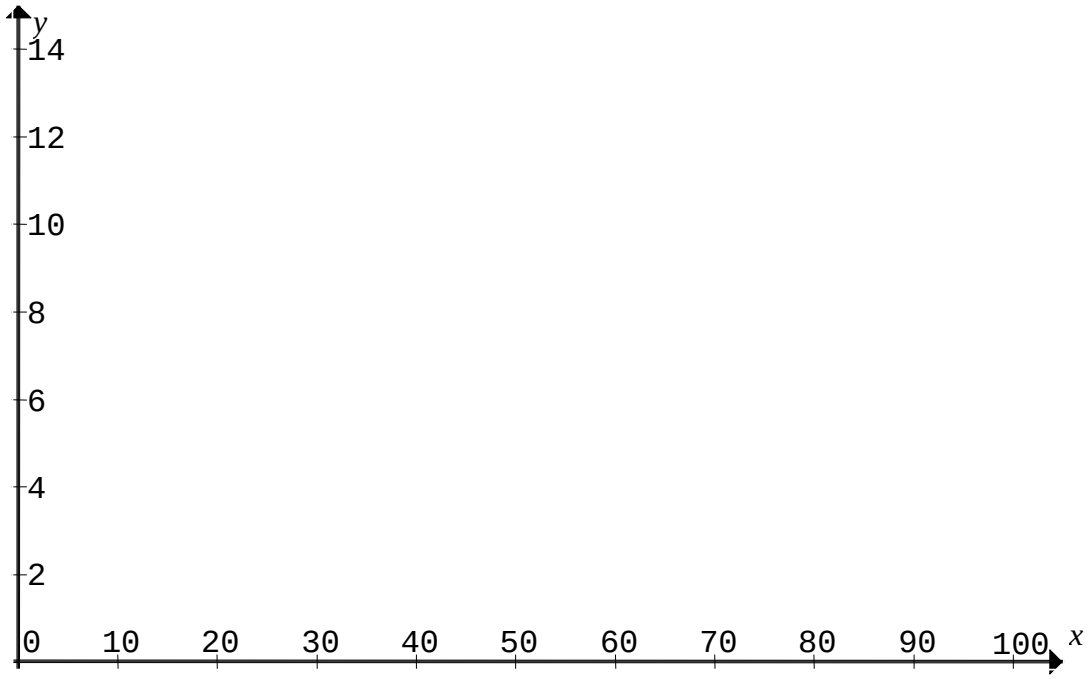
1 mark

b. Sketch a graph of m_B against t, labelling any asymptotes.



2 marks

c. If isotope B was radioactive as well (and would start to decay once it was formed) and had a somewhat longer half-life than A, describe what the graph of m_B against t would now look like.

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2 marks
Total 5 marks

Question 8

Atmospheric pressure, P, varies with the height, h, above the Earth's surface according to the equation:

$$P = P_0 e^{-kh} \text{ where } P_0 \text{ is the pressure at ground level and } k \text{ is a real constant.}$$

If the pressure is 250 millibars at a height of 10 km and 15 millibars at a height of 30 km.

a. Show that the value of k (correct to 4 decimal places) is 0.1407.

2 marks

b. Show that the value of P_0 (correct to the nearest millibar) is 1021.

2 marks

c. At what height, correct to the nearest metre, would the pressure be 65 millibars?

2 marks

Total 6 marks

Question 9

For the graph of $y = 2xe^{-0.08x} + 0.0017x^2$

a. Find the coordinates of the maximum and minimum points correct to 4 decimal places.

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_____

2 marks

b. Find the range of x – values for which $y > 7.8$. Give your answers correct to 4 decimal places.

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3 marks
Total 5 marks

Question 10

If $f(x) = (x-a)^2 e^{-bx}$

a. Find $f'(x)$ in factorised form.

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2 marks

b. Find the x – values of the turning points of $y = f(x)$ in terms of a and b.

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1 mark
Total 3 marks

END OF TASK BOOK