

INSIGHT YEAR 12 Trial Exam Paper

# 2013

# MATHEMATICAL METHODS (CAS) Written examination 2

Worked solutions

# This book presents:

- correct solutions with full working
- $\succ$  mark allocations
- ➤ tips

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# **SECTION 1**

#### **Question 1**

# Answer is C

# Worked solution

Period is  $\frac{2\pi}{n} = \frac{2\pi}{2} = \pi$  and the amplitude is the coefficient of the sine term, which is 1.



Tip

• Amplitude and period are never negative.

# **Question 2**

Answer is D

# Worked solution

Choose some sample values for *a* and *b*.

Using CAS, the graph of  $f(x) = |\log_e(-2+x)| + 5$  looks like

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This shows the lowest point that the graph reaches is 5, so the range is  $[5, \infty)$  or in the general sense  $[b, \infty)$ .

#### Answer is D

#### Worked solution

The amplitude is (-2-b), the period is 2 (so  $2 = \frac{2\pi}{n}$ , so  $n = \pi$ ), the graph has been reflected

across the *x*-axis, and shifted down 2 units. These values give an equation of  $y = -(-2-b)\sin(\pi x) - 2$ , which simplifies to give  $y = (b+2)\sin(\pi x) - 2$ .

# **Question 4**

#### Answer is E

#### Worked solution

The graph of  $y = \tan(ax)$  has a period of  $\frac{\pi}{a}$  and asymptotes at  $x = \pm \frac{\pi}{2a}$ .

So, in this case 
$$\frac{\pi}{2a} = \frac{1}{12}$$
  
 $12\pi = 2a$   
 $a = 6\pi$ 

# **Question 5**

Answer is B

Worked solution

$$z = 3$$
$$x - z = -5$$
$$x + y = 0$$

can be written as

0x + 0y + 1z = 31x + 0y - 1z = -51x + 1y + 0z = 0

Moving line 1 to the bottom gives 1x + 0y - 1z = -5 1x + 1y + 0z = 0 0x + 0y + 1z = 3In matrix form, this is  $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ 

matrix form, this is 
$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}$$

#### Answer is A

#### Worked solution

 $y = e^{2x+4} - 3$  can be written as  $y+3 = e^{2x+4}$  and therefore y = y'+3 and x = 2x'+4. So y' = y-3 and  $x' = \frac{1}{2}(x-4) = \frac{1}{2}x-2$ . The matrix equation that corresponds to these equations is **A**.

# **Question 7**

#### Answer is E

# Worked solution

Overall, the graph represents a negative polynomial of odd degree.

It has x-intercepts at x = a, x = 0, x = b. These give factors of (x-a), x, and (x-b). There is a stationary point of inflection at x = a, so the factor (x - a) becomes  $(x - a)^3$ .

This means the equation of the graph could be  $y = -x(x-a)^3(x-b)$  and a version of this is **E**.

# **Question 8**

#### Answer is D

#### Worked solution

Reflect the graph in the line y = x to get the graph of the inverse and then reflect in the y-axis to get the graph of  $y = f^{-1}(-x)$ .

# **Question 9**

# Answer is D Worked solution

The period has been halved, therefore there is a dilation of factor  $\frac{1}{2}$  from the y-axis. The graph has been reflected in the x-axis.

Answer is E

# Worked solution

Let b be any value, say b = 2.

Using CAS, the graph of  $y = e^{|x+2|}$  looks like



The graph is one to one for the domain  $[-b,\infty)$ .

# **Question 11**

Answer is D

Worked solution

$$g(f(x)) = \frac{1}{\sqrt{2e^x}}$$
 and with  $x = 0$ ,  $g(f(x)) = \frac{1}{\sqrt{2e^0}} = \frac{1}{\sqrt{2}}$ 

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# **Question 12**

# Answer is D

# Worked solution

Using the chain rule

$$\frac{dy}{dx} = \frac{1}{\sqrt{f(x)}} \times \frac{1}{2\sqrt{f(x)}} \times f'(x)$$
$$= \frac{f'(x)}{2f(x)}$$

# **Question 13**

Answer is D

# Worked solution

Using CAS



So the equation of the normal is  $y = \frac{x}{\pi} - 1$ . Rearranging, this gives  $(y+1)\pi = x$ .

# Answer is D

# Worked solution

The average value of a function is calculated as  $\frac{1}{b-a}\int_a^b f(x) dx = \frac{1}{2-0}\int_0^2 2e(e^x-1) dx$ .

Using CAS, this gives



# **Question 15**

Answer is C

# Worked solution

This question represents the fundamental theorem of integral calculus with

$$\lim_{\delta x \to 0} \sum_{i=1}^{n} (4x_i \, \delta x) = \int_0^8 4x \, dx$$

Using	CAS,	this	is
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#### Answer is D

# Worked solution

For 
$$f'(x) = \frac{5}{\cos^2(kx)}$$
,  $f(x) = \int \frac{5}{\cos^2(kx)} dx$ 



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# **Question 17**

Answer is C

#### Worked solution

As q'(x) = p(x), this means that  $q(x) = \int p(x) dx$ .

So 
$$\int_{a}^{b} p(2x) dx = \left[\frac{1}{2}q(2x)\right]_{a}^{b} = \frac{1}{2}(q(2b) - q(2a)).$$

# Answer is D

# Worked solution

The graph of y = -2f(x) will give the gradient function of g. A quick sketch of y = -2f(x) gives



Over interval [a, b] the y-values are positive, therefore the gradient values are positive.

9

# **Question 19**

#### Answer is B

# Worked solution

Let the number of defectives in the box be *X*.

$$X \sim Bi(n, p)$$

We are given E(X) = 12 = np and Var(X) = 10 = npq.

So, 12q = 10,  $q = \frac{5}{6}$  and  $p = \frac{1}{6}$ .

The chance of not being defective is given by q and  $q = \frac{5}{6}$ .

# **Question 20**

Answer is D

# Worked solution

Only in a symmetrical distribution is the probability of being less than the mean equal to 0.5.

# Answer is B

#### Worked solution

$$\sum p(x) = a + b + 0.1 = 1$$
 and  $E(X) = 2a + 4b + 0.6 = 2.6$ 

Solving the simultaneous equations gives

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# **Question 22**

Answer is C

# Worked solution

 $X \sim N(\mu = 176, \sigma \text{ unknown})$ 

 $\Pr(X < 175) = 0.01$ 

This can be solved using CAS.

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# **SECTION 2**

# **Instructions for Section 2**

Answer all questions in the spaces provided.

A decimal answer will not be accepted if an **exact** answer is required to a question. For questions where more than one mark is available, appropriate working must be shown. Unless otherwise stated, diagrams are not drawn to scale.

# Question 1a.

#### **Worked solution**

The information given, together with the transition matrix, suggests

 $\begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \Pr(L_{i+1} \mid L_i) & \Pr(L_{i+1} \mid R_i) \\ \Pr(R_{i+1} \mid L_i) & \Pr(R_{i+1} \mid R_i) \end{bmatrix},$ 

where  $L_i$  is the event of buying lilies one week and  $R_i$  is the event of buying roses one week.

So the chance that Rebecca buys lilies one week, given that she bought roses the previous week is  $\frac{1}{3}$ .

# Mark allocation

• 1 mark for the correct answer.

# Question 1b.

# Worked solution

- i.
  - The fifth Saturday means that there will be four transitions. The probabilities will be

$$\begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}^4 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Using CAS, this gives



So, in 4 weeks' time, the probability that Rebecca buys lilies is  $\frac{7717}{16875}$ .

# Mark allocation: 1 + 1 = 2 marks

- 1 mark for setting the transition matrix to the power of 4.
- 1 mark for the correct answer.



• This needs to be the exact answer. A decimal approximation, regardless of the number of decimal places, is not acceptable.

# Question 1b.

# Worked solution

**ii.** This means that Rebecca buys lilies the first week, then roses for the next 3 weeks, and, finally, lilies the last week.

i.e. 
$$LRRRL = 1 \times \frac{2}{5} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{8}{135}$$

# Mark allocation

• 1 mark for the correct answer. (**Note:** Answer must be exact.)

# Question 1c.

# Worked solution

This can be calculated using the probabilities shown on the diagonals



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# Mark allocation: 1 + 1 = 2 marks

- 1 mark for method.
- 1 mark for the correct answer.



• An alternative method is to raise the power of the transition matrix to a sufficiently large power and then evaluate.

# Question 1d.

# Worked solution

As the function is symmetrical around x = 6, the mean, median (and mode) are concurrent and occur at x = 6.

- 1 mark for the mean.
- 1 mark for the median.

# Question 1e.

# Worked solution

This is a conditional probability question; i.e.  $\Pr(T_R > 9 | T_R > 6)$ .

$$\Pr(T_{R} > 9 | T_{R} > 6) = \frac{\Pr(T_{R} > 9)}{\Pr(T_{R} > 6)}$$

The probability of lasting longer than 9 days =  $\int_{9}^{12} \frac{1}{288} t(12-t) dt = \frac{5}{32}$ .



$$\Pr(T_R > 9 \mid T_R > 6) = \frac{\frac{5}{32}}{0.5} = \frac{5}{16}$$

- 1 mark for finding  $\Pr(T_R \ge 9) = \frac{5}{32}$ .
- 1 mark for the correct answer.

# Question 1f.

# Worked solution

For roses,  $\Pr(T_R \le 9) = \frac{27}{32} = 0.8438$ , correct to 4 decimal places. For lilies,  $\Pr(T_L \le 9) = 0.9641$ , correct to 4 decimal places.

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# Mark allocation: 1 + 1 = 2 marks

• 1 mark for each correct answer.



• Answers must be in decimal form for this question;  $\frac{27}{32}$  is not acceptable.

# Question 1g.

# Worked solution

For the probabilities to be equal,  $\int_{k}^{12} \frac{1}{288} t(12-t) dt = \text{normCDf}(k, 100000, \sigma = 1, \mu = 7.2).$ 



Check whether to let k = 7.77 or 7.76.

If k = 7.77 then correct to 2 decimal places the integral probability is 0.29, whereas the normal distribution probability is 0.28 (so the two are not equal at the 2 decimal places level).





So, *k* = 7.76.

# Mark allocation: 1 + 1 = 2 marks

- 1 mark for method.
- 1 mark for the correct answer.

However if k = 7.76, the integral probability gives 0.29 and the normal distribution

# Question 2a.

# Worked solution

f is strictly increasing for  $x: f'(x) \ge 0$ . The minimum turning point occurs at (4, -1).



So *f* is strictly increasing for  $[4, \infty)$ .

# Mark allocation: 1 + 1 = 2 marks

- 1 mark for finding the turning point.
- 1 mark for the correct answer.

Note: Must have square bracket at 4.

# Question 2b.

# Worked solution



- 1 mark for the correct shape.
- 1 mark for co-ordinates labelled correctly.

# Question 2c.

# Worked solution

Let the width of the rectangle be w, so  $\left|\int_{1}^{9} x - 4\sqrt{x} + 3 dx\right| = w \times (9-1)$ .



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- 1 mark for method.
- 1 mark for the correct answer.

# Question 2d.

# Worked solution

The shaded area is  $\frac{16}{3}$ , therefore half the area is  $\frac{8}{3}$ .

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The graph of y = f(x) intersects the line y = p at  $x = p - 4\sqrt{p+1} + 5$  and  $x = p + 4\sqrt{p+1} + 5$ .

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So the integral is  $\int_{p-4\sqrt{p+1+5}}^{p+4\sqrt{p+1+5}} (p-(x-4\sqrt{x}+3)) dx = \frac{8}{3}$  and we need to find *p*.

Using the graph screen of CAS, this is found to be p = -0.370.



Mark allocation: 1 + 1 + 1 = 3 marks

- 1 mark for stating the area as  $\frac{16}{6} = \frac{8}{3}$ .
- 1 mark for determining the points of intersection of the line y = p with the graph y = f(x).
- 1 mark for finding the value of *p*.

# Question 2e.

# Worked solution

i. 
$$f(g(x)) = x^2 - 4\sqrt{x^2} + 3$$
  
=  $x^2 - 4|x| + 3$ 

# Mark allocation: 1 + 1 = 2 marks

- 1 mark for method.
- 1 mark for the correct answer.

Note: Must show a working step and not just the answer.

# Question 2e.

Worked solution

ii. 
$$f(g(x)) = x^{2} - 4\sqrt{x^{2}} + 3$$
$$= x^{2} - 4|x| + 3$$
$$= \begin{cases} x^{2} - 4x + 3 & x \ge 0\\ x^{2} + 4x + 3, & x < 0 \end{cases}$$
So 
$$\frac{d}{dx}(f(g(x))) = \begin{cases} 2x - 4, & x > 0\\ 2x + 4, & x < 0 \end{cases}$$

- 1 mark for setting up the hybrid function of f(g(x)).
- 1 mark for the correct derivative with domains specified.



- 1 mark for y = 2x + 4, x < 0 section.
- 1 mark for y = 2x 4, x > 0 section.
- 1 mark for open circles and correctly labelled intercepts.

# Question 3a.

Worked solution

Using CAS,  $f'(\pi) = 0.5$  and  $f'(3\pi) = 0.5$ 

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# Mark allocation: 1 + 1 = 2 marks

• 1 mark for each correct answer.

# Question 3b.

# Worked solution

i. Using CAS gives 
$$\frac{d}{dx}(2\cos^2\left(\frac{x}{4}\right)+1) = -\cos\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)$$
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# Mark allocation

• 1 mark for the correct answer.

# Question 3b.

# Worked solution

ii. Gradient of normal is 2, therefore gradient of tangent is  $-\frac{1}{2}$ .

$$-\cos\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right) = -\frac{1}{2}$$

Using CAS to solve over the domain  $[-\pi, 6\pi]$  gives



So there are *two* points on the curve where the gradient of the normal is 2; i.e. at  $(\pi, 2)$  and  $(5\pi, 2)$ .

- 1 mark for gradient of the tangent equals  $-\frac{1}{2}$ .
- 1 mark for giving answer as co-ordinates.

# **Question 3c.**

# Worked solution

# Using CAS gives

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$$\therefore y = \frac{x}{2} - \frac{3\pi}{2} + 2$$
 and  $y = -\frac{x}{2} + \frac{5\pi}{2} + 2$ 

# Mark allocation: 1 + 1 = 2 marks

• 1 mark for each correct answer.

# Question 3d.

# Worked solution

The tangent lines intersect at  $x = 4\pi$ ,  $y = \frac{\pi}{2} + 2$ .



So if the graph of y = f(x) is shifted down by  $\frac{\pi}{2} + 2$  units, then the tangents will intersect at

the x-axis, therefore  $b = -\frac{\pi}{2} - 2$ .

- 1 mark for finding the point of intersection.
- 1 mark for the correct answer.

# CONTINUES OVER PAGE

# Question 3e.

# Worked solution

Using CAS to simplify gives  $2\cos^2\left(\frac{x}{4}\right) + 1 = \cos\left(\frac{x}{2}\right) + 2$ .

So 
$$a = \frac{1}{2}, b = 2.$$
  
  
**Y** Edit Action Interactive [X]  
  
 $a = \frac{1}{2}, b = 2.$   
  
  
 $a = \frac{1}{2}, b = 2.$ 

- 1 mark for finding *a* and *b* values.
- 1 mark for showing period.

# Question 3f.

# Worked solution

i. Need the lowest point in the cycle to occur at  $x = 6\pi$ , so half the period needs to be  $6\pi$ .

Therefore, the period of the transformed graph is  $12\pi$ . So  $\frac{2\pi}{n} = 12\pi$ ,  $n = \frac{1}{6}$ .

Using 
$$f(kx) = 2\cos^2\left(\frac{kx}{4}\right) + 1 = \cos\left(\frac{kx}{2}\right) + 2$$
, so  $\frac{k}{2} = \frac{1}{6}$ ,  $k = \frac{1}{3}$ .

Using a graph to verify this shows



# Mark allocation

• 1 mark for the correct answer.

# Question 3f.

# Worked solution

ii. The equation has *one* solution at the end of the domain (at  $x = 6\pi$ ) for  $k = \frac{1}{3}$ ;

i.e. 
$$f(kx) = 2\cos^2\left(\frac{x}{12}\right) + 1 = \cos\left(\frac{x}{6}\right) + 2$$

The equation has *two* solutions in the domain, with one at  $x = 6\pi$ , when the period of the graph is  $4\pi$ .

 $\therefore$  Period =  $4\pi = \frac{2\pi}{n}$ ,  $n = \frac{1}{2}$ , so k = 1.



Mark allocation: 1 + 1 = 2 marks

- 1 mark for identifying k < 1.
- 1 mark for the correct answer.

# Question 3f.

#### Worked solution

iii. The graph of  $y = \cos\left(\frac{5x}{6}\right) + 2$  shows that there would be *three* points in the domain when y = 1, so  $k = \frac{5}{3}$ .



The graph of y = cos(x) + 2 has *four* points in the domain where y = 1, so k = 2.



- 1 mark for obtaining either  $\frac{5}{3}$  or 2.
- 1 mark for the correct answer.

#### Question 4a.

#### Worked solution

i. 
$$f(u) = e^{au}$$
 and  $f(-u) = e^{-au}$ , so  $f(u)f(-u) = e^{au} \times e^{-au} = e^{au-au} = e^0 = 1$ 

#### Mark allocation

• 1 mark for obtaining  $e^0 = 1$ .

#### Question 4a.

#### Worked solution

ii.  $f(u+v) = e^{a(u+v)} = e^{au+av} = e^{au} \cdot e^{av} = f(u)f(v)$ 

# Mark allocation: 1 + 1 = 2 marks

- 1 mark for finding f(u+v).
- 1 mark for forming f(u).f(v).

# Question 4b.

#### Worked solution

Using CAS gives



So the character intersects the obstacle when  $9 - e^{0.2x} = 3$ ; i.e. at  $x = 5\log_e 6$ , y = 3.  $\Rightarrow (5\log_e 6, 3)$ 

- 1 mark for setting  $9 e^{0.2x} = 3$ .
- 1 mark for the correct answer.

# **Question 4c.**

At (5, 3),

# Worked solution

To land on the horizontal top section, he needs to land between (5, 3) and (10, 3).

$$y = 9 - e^{ax} \implies 3 = 9 - e^{5a}$$
$$e^{5a} = 6$$
$$a = \frac{1}{5} \log_e 6$$

At (10, 3), 
$$y = 9 - e^{ax} \implies 3 = 9 - e^{10a}$$
  
 $e^{10a} = 6$   
 $a = \frac{1}{10} \log_e 6$ 

$$\therefore \frac{1}{10}\log_e 6 \le a \le \frac{1}{5}\log_e 6$$

# Mark allocation: 1 + 1 = 2 marks

- 1 mark for finding either endpoint.
- 1 mark for the correct interval values.

#### Question 4d.

#### Worked solution

To clear the obstacle, he must jump beyond (13, 0).

$$9 - e^{13a} = 0, e^{13a} = 9$$
  
 $a = \frac{1}{13} \log_e 9$ 

So,  $0 < a < \frac{1}{13} \log_e 9$  to clear the obstacle.

# Mark allocation

• 1 mark for correct interval values.

# **Question 4e.**

#### Worked solution

$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} 2 & 0\\0 & 3 \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix}$$
$$\begin{cases} x' = 2x\\y' = 3y \end{bmatrix} \implies x = \frac{x'}{2}, \ y = \frac{y'}{3}$$

So the equation  $y = 9 - e^{0.2x}$  becomes  $\frac{y}{3} = 9 - e^{\frac{0.2x}{2}} \implies y = 27 - 3e^{0.1x}$ .

# Mark allocation: 1 + 1 = 2 marks

- 1 mark for getting  $x = \frac{x'}{2}$ ,  $y = \frac{y'}{3}$ .
- 1 mark for finding new equation.

#### **Question 4f.**

#### Worked solution

The graph of  $y = 9 - e^{0.1x}$  shows that Super Marius will clear the base of the obstacle, so just the height of the obstacle will pose a problem.



The rising section extends from x = 5 to x = 10. At x = 10 the graph has a y value of 9 - e, so to clear the obstacle the horizontal top section has to be less than 9 - e units high.

$$\therefore 0 \le p < 9 - e$$

#### Mark allocation: 1 + 1 = 2 marks

- 1 mark for finding 9 e.
- 1 mark for correct interval values.

#### **END OF SOLUTIONS BOOK**