Neap:

Trial Examination 2013

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

Question and answer booklet of 10 pages, with detachable sheet of miscellaneous formulas in the centerfold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this book during reading time.
Write your **name** and **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2013 VCE Mathematical Methods Units 3 & 4 Written Examination 1.

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Copyright © 2013 Neap ABN 49 910 906 643 96–106 Pelham St Carlton VIC 3053 Tel: (03) 8341 8341 Fax: (03) 8341 8300

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (3 marks)

a. If $y = \dfrac{2}{1-5x}$, find $\dfrac{dy}{dx}$. 1 mark

b. If $f(x) = x\cos^2\left(\dfrac{\pi x}{2}\right)$, find $f'\left(\dfrac{1}{2}\right)$. 2 marks

Copyright © 2013 Neap

Question 2 (2 marks)

The gradient of a curve is given by $\frac{dy}{dx} = \frac{6}{\sqrt{x^3}}$.

Given that the point $\left(\frac{1}{4}, 4\right)$ lies on the curve, find the equation of the curve.

Question 3 (2 marks)

Consider the functions f and g with rules $f(x) = a\log_e(x-1)$ and $g(x) = 1 + e^{\frac{x}{a}}$, each defined on their maximal domain, where $a \in R \setminus \{0\}$.

By finding $f(g(x))$, show that the functions f and g are inverses.

Copyright © 2013 Neap

Question 4 (5 marks)

Airlines regularly overbook flights because they expect some passengers to not arrive for their flight. An airline runs a commuter flight from Melbourne to Mildura on a plane that seats 40 passengers.

Past experience has shown that if 43 tickets are sold for the flight, then the probability distribution for the number who actually turn up for the flight is given by:

Number who actually turn up	38	39	40	41	42	43
Probability	0.1	0.45	0.35	0.05	0.04	0.01

Assume 43 tickets are sold for each flight.

a. Determine the probability that all passengers who show up for this flight will get a seat. 1 mark

b. Calculate the expected number of passengers who do not show up for this flight. 2 marks

c. Given that not all passenger seats are filled for this flight, find the probability that only 38 passengers showed up for the flight. 2 marks

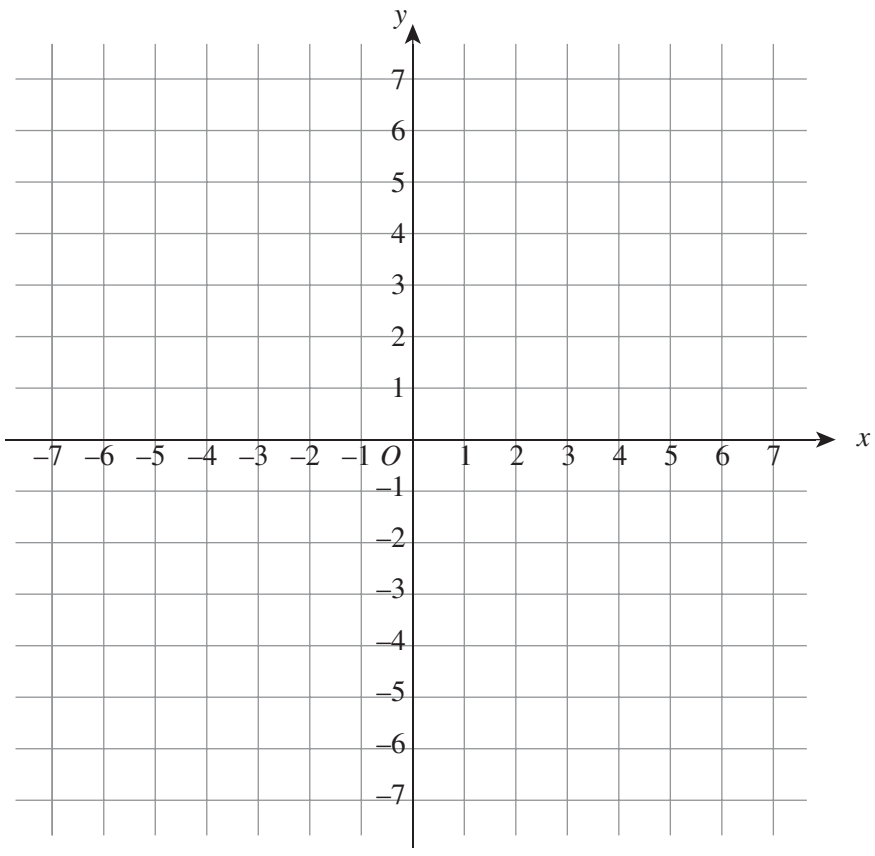
Copyright © 2013 Neap

Question 5 (7 marks)

A function of the form $f(x) = \dfrac{a}{(x+b)^2}$ is defined over its maximal domain. It has a vertical asymptote with equation $x = 2$ and it passes through $(0, -\dfrac{1}{2})$.

a. Find *a* and *b*. 2 marks

b. On the axes provided below, sketch the graph of $y = \left|\dfrac{3}{2} + f(x)\right|$. Label carefully the axes intercepts and give the equations of any asymptotes. (You may make use of the result $\dfrac{1}{\sqrt{3}} \approx 0.6$.) 3 marks



Copyright © 2013 Neap

c. Find the equation of the image of the graph of f under a dilation of scale factor 2 from the y-axis followed by a translation of 3 units in the negative direction of the x-axis. Express your answer in the form $y = \dfrac{c}{(x+d)^2}$ where c and d are integers. 2 marks

Copyright © 2013 Neap

Question 6 (4 marks)

Consider the functions *f* and *g* defined below:

$f : (0, \infty) \rightarrow R,\ f(x) = \log_e(x^2)$

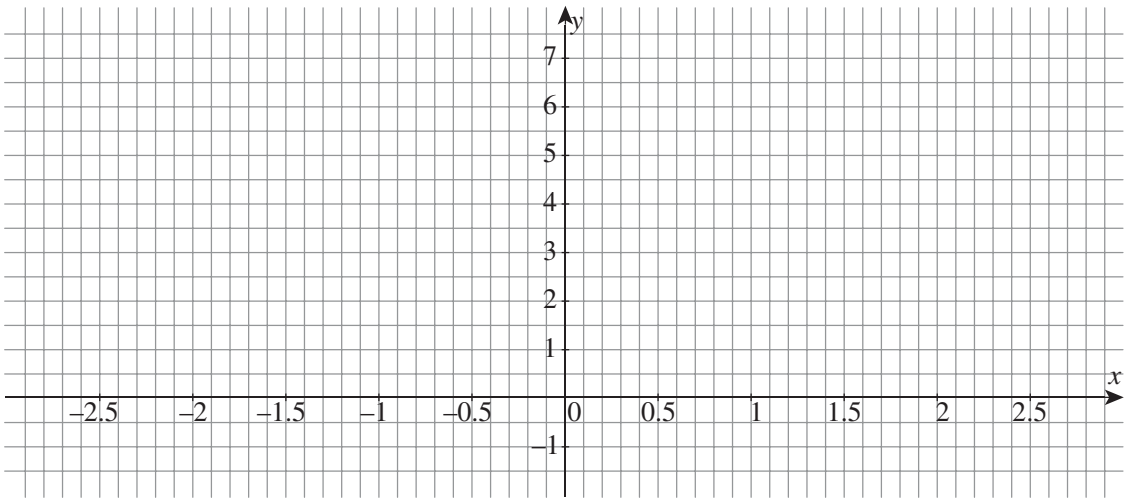
$g: R \rightarrow R,\ g(x) = e^{2x}$

Let *h* be defined as the composition of *f* with *g,* that is $h(x) = f(g(x))$.

Let *k* be defined as the composition of *g* with *f,* that is $k(x) = g(f(x))$.

a. Find simplified rules for *h* and *k.* 2 marks

b. On the axes below, sketch graphs of *h* and *k,* marking the coordinates of any points of intersection. 2 marks



Copyright © 2013 Neap

Question 7 (3 marks)

Solve the equation $2\log_3(x+4)-\log_3(-x)=2$ for allowable values of x.

Question 8 (5 marks)

a. A continuous random variable X has a probability density function given by:

$$p(x)=\begin{cases} kx(4-x^2) & 0\le x\le 2 \\ 0 & \text{otherwise} \end{cases}$$

i. Find the value of k. 1 mark

ii. Show that m, the median value of X, satisfies the equation $m^4-8m^2+8=0$. 2 marks

b. The mean and the standard deviation on Mrs Tuff's last Year 12 Maths Methods test were 72 and 8 respectively. She thought these were too high. Mrs Tuff is going to transform the scores so that the new mean is 65 with a standard deviation of 5.

If the random variable X represents the original scores and the random variable Y represents the transformed scores, then $Y=aX+b$, where a and b are positive constants.

Find a and b 2 marks

Copyright © 2013 Neap

Question 9 (3 marks)

In a certain town the **rate of deaths** due to a particular disease has been found to closely fit the mathematical model $\frac{dN}{dt} = \frac{10\,000}{(t+3)^3}$, where t is the time in months after the disease was first detected and N is the number of deaths.

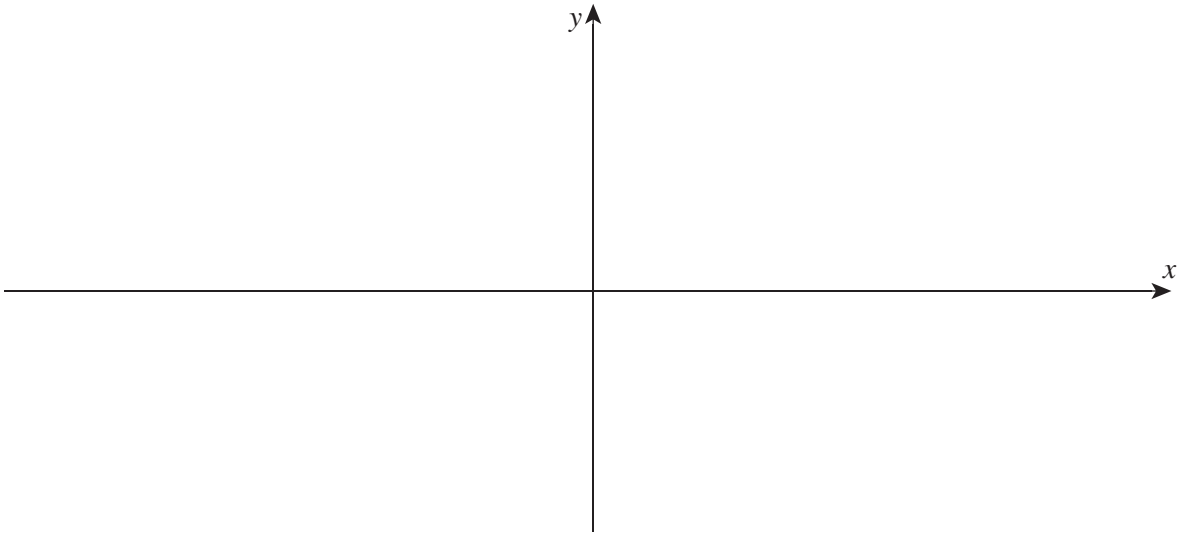
Calculate the total number of deaths predicted by the model, giving your answer to the nearest whole number of deaths.

Copyright © 2013 Neap

Question 10 (7 marks)

Let $f(x) = a\cos(ax)$ where a is a positive constant.

a. State the values of the closest x-intercepts either side of the y-axis. Hence sketch the graph of f for x values between these two intercepts. 2 marks



b. Tangents to the graph of f are constructed at these two x intercepts.
Find the equation of each tangent. 3 marks

c. Show that the area of the region enclosed by these tangents and the graph of f is independent of the value of a. 2 marks

END OF QUESTION AND ANSWER BOOKLET

Copyright © 2013 Neap