Neap:

Trial Examination 2013

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
Question and answer booklet of 22 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
Answer sheet for multiple-choice questions.

Instructions
Detach the formula sheet from the centre of this book during reading time.
Write **your name** and **teacher's name** in the space provided above on this page.
All written responses must be in English.

At the end of the examination
Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2013 VCE Mathematical Methods Units 3 & 4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The number of solutions there are to the equation $\sin(x) = \sin^2(x)$ if $x \in [0, 2\pi]$ is

A. 2

B. 3

C. 4

D. 5

E. 6

Question 2

If $\int_{-2}^{4} f(x)dx = a$ and $\int_{3}^{4} f(x)dx = b$ then $\int_{3}^{-2} f(x)dx$ equals

A. $a + b$

B. $a - b$

C. $a + 2b$

D. $a - 2b$

E. $b - a$

Question 3

Consider the hybrid function *f* defined by

$$f(x)=\begin{cases}\dfrac{1}{x} & |x|<\dfrac{1}{2}\\[2mm] \dfrac{1}{x}+\dfrac{1}{2} & |x|\geq\dfrac{1}{2}\end{cases}$$

$f\left(f\left(-\frac{1}{3}\right)\right)$ equals

A. -3

B. $-\frac{5}{2}$

C. $\frac{1}{10}$

D. $-\frac{1}{6}$

E. $\frac{1}{6}$

Question 4

Let $h(x)=\begin{cases}x^2+Ax+B & x<1\\ -x^2+8x+4 & x\geq 1\end{cases}$

Given that *h* is differentiable at $x = 1$, the values of *A* and *B* respectively are

A. 4, 6

B. 4, 3

C. 6, 4

D. 8, 3

E. 8, 6

Question 5

Consider the following set of functions, each defined over its maximal domain.

$f(x)=x \qquad g(x)=\frac{4}{x} \qquad h(x)=\frac{x}{x-1} \qquad j(x)=\frac{x-2}{x}$

Which of these functions has the property that its inverse is identical to itself?

A. *f* only

B. *f* and *g* only

C. *f, g* and *h* only

D. all functions *f, g* , *h* and *j*

E. none of them

Question 6

A wheel rolling along the ground has a diameter of 18 cm and rotates once every 12 seconds. At time $t = 0$, a point P on the outside edge of the wheel is at its highest point.

The height, h cm, of point P above the ground at time t seconds is given by

A. $h(t) = 9\left(\sin\left(\frac{\pi}{12}(6-t)\right)+1\right)$

B. $h(t) = 9\left(\sin\left(\frac{\pi}{6}(3-t)\right)+1\right)$

C. $h(t) = 9\left(1-\cos\left(\frac{\pi t}{6}\right)\right)$

D. $h(t) = 9\left(1+\cos\left(\frac{\pi t}{12}\right)\right)$

E. $h(t) = 18\left(1+\cos\left(\frac{\pi t}{6}\right)\right)$

Question 7

Let $f(x) = \sqrt{x+9}$ and $g(x) = x - 6$.

The domain of $\frac{f}{g}(x)$ equals

A. $(-6, 9]$

B. $[-9, \infty)$

C. $[-9, 6)$

D. $[-9, 6) \cup (6, \infty)$

E. $(-\infty, -6) \cup (-6, 9]$

Question 8

Refer to the table below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	3	0	–4	3
1	5	2	–1	–3
2	1	5	6	–2
3	7	9	12	–1

If $h(x) = g(f(x))$, then the value of $h'(2)$ equals

A. –18

B. –12

C. –3

D. 2

E. 12

Question 9

Consider the relation $\{(x, y): \sqrt{x} + \sqrt{y} = 5\}$. A tangent and normal to the curve are drawn at the point (16,1). The tangent intersects the y-axis at (0, k) and the normal intersects the y-axis at (0, h).

The value of $k - h$ equals

A. 58

B. 68

C. 69

D. 70

E. 83

Question 10

The average rate of change of $f(x) = 3x^2 + 2x + k$ over the interval [0, 2] is equal to twice the average value of $f(x)$ over the same interval.

The value of k equals

A. –4

B. –2

C. 2

D. 4

E. 16

Question 11

An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls drawn are of the same colour is $\frac{11}{25}$.

The number of blue balls in the second urn is

A. 4

B. 20

C. 24

D. 44

E. 64

Question 12

Serena wants to improve her first serve at tennis. She trains each day before an upcoming tournament and gets her first serve in play 65% of the time. The day prior to the tournament Serena practises by having 180 first serves, one at a time.

Assuming the outcome of any one serve is independent of any other serve, the mean and standard deviation of the number of successful first serves is

A. $\mu = 117$ and $\sigma = \dfrac{3\sqrt{455}}{10}$

B. $\mu = 117$ and $\sigma = \dfrac{2\sqrt{35}}{13}$

C. $\mu = 63$ and $\sigma = \dfrac{3\sqrt{455}}{10}$

D. $\mu = 63$ and $\sigma = \dfrac{2\sqrt{35}}{13}$

E. $\mu = 117$ and $\sigma = \dfrac{819}{20}$

Question 13

The Melbourne Jets volleyball team is playing in a tournament. The probability that they will win their first match is 60%. Their coach has noticed that when they win a game, the probability that they will win their next game rises to 80%. If they lose a match, the probability that they win their next match falls to 25%.

The probability that the team will win its fourth match is found by using which of the following matrix products?

A. $\begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix}^4 \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$

B. $\begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix}^4 \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$

C. $\begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix}^3 \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$

D. $\begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix}^3 \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$

E. $\begin{bmatrix} 0.75 & 0.2 \\ 0.25 & 0.8 \end{bmatrix}^3 \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$

Question 14

At the school fair Hannah has a stand which sells "Aussie hats". Previous experience at other fairs has established that Hannah can sell 200 hats for $90 each. However, for every $1 increase in the price of a hat, 5 less hats will be sold.

How many hats should Hannah order to sell at the fair in order to maximise her profit, given that she pays $60 per hat to the manufacturer of the hats and all the hats she orders will be sold?

A. 5

B. 25

C. 165

D. 175

E. 185

Question 15

The area bounded by the line $y = x$ and the graph of the parabola $y = x^2 - x$ is cut in half by a line with equation $x = k$.

What is the value of k?

A. $\frac{1}{2}$

B. $\frac{3}{4}$

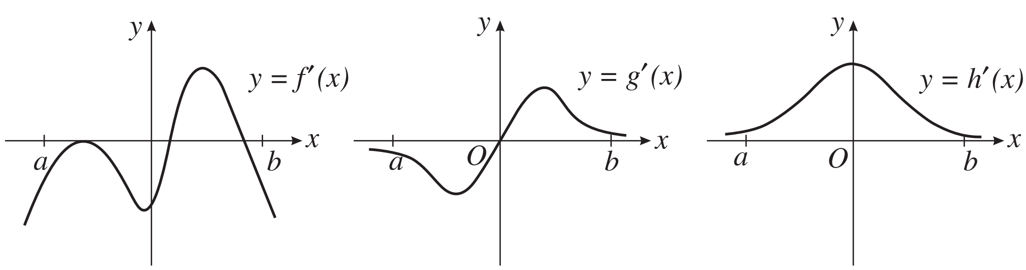
C. 1

D. $\frac{5}{4}$

E. $\frac{5}{3}$

Question 16

The graphs of the derivative of the functions f, g and h are shown below.



Which of the functions f, g or h have a local maximum on the domain $a < x < b$?

A. f only

B. g only

C. h only

D. f and g only

E. f, g and h

Question 17

$y = f(x)$ represents a function with range [–7, 5]. Consider the function $g(x) = 2|f(x) - 2| + 1$.
The range of g equals

A. [–7, 5]
B. [1, 7]
C. [1, 17]
D. [1, 19]
E. [–17, 7]

Question 18

One root of the equation $x^2 + kx + k = 0$ is $x = -\frac{1}{2}$.
The other root is

A. $-\frac{7}{8}$
B. $-\frac{1}{2}$
C. 1
D. $\frac{7}{8}$
E. $2k$

Question 19

Two straight lines, L_1 and L_2, with corresponding gradients, m_1 and m_2, where $m_1 > 0$ and $m_2 < 0$, are tangents to the graph of $y = x^2$. The points of contact of L_1 and L_2 with the graph, along with the intersection point of L_1 and L_2, are the 3 vertices of an equilateral triangle.

The area of this equilateral triangle is

A. $\frac{3}{2}$
B. $\frac{3\sqrt{3}}{2}$
C. $\frac{\sqrt{3}}{12}$
D. $\frac{3\sqrt{3}}{4}$
E. $3\sqrt{3}$

Question 20

An insurance company insures a large number of shops against damage from vandals. The insured value, V, in units of \$100 000, of a randomly selected shop is assumed to follow a probability distribution with density function

$$f(v) = \begin{cases} \dfrac{3}{v^4} & v > 1 \\ 0 & \text{otherwise} \end{cases}$$

Given that a randomly selected shop is insured for at least \$150 000, the probability that it is insured for under \$200 000 is

A. 0.578

B. 0.684

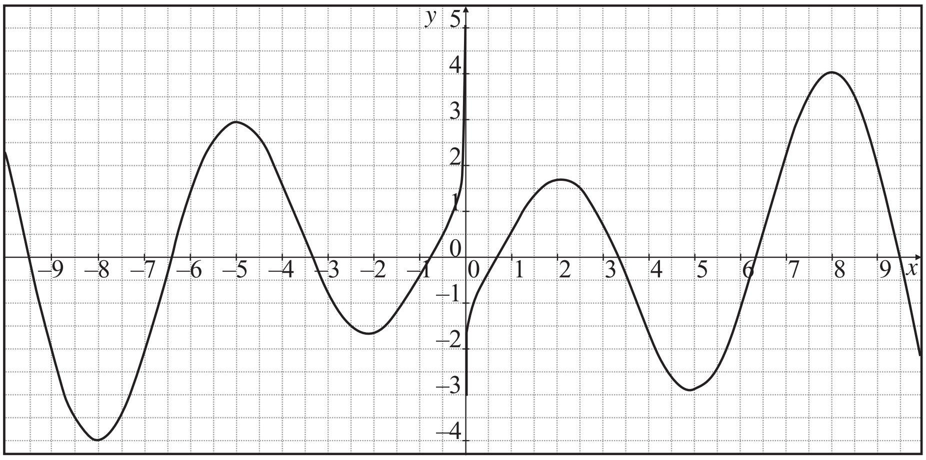
C. 0.704

D. 0.829

E. 0.875

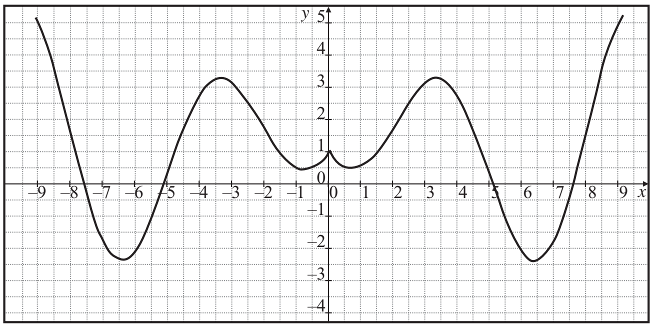
Question 21

The graph of the gradient function $y = f'(x)$ is shown below.

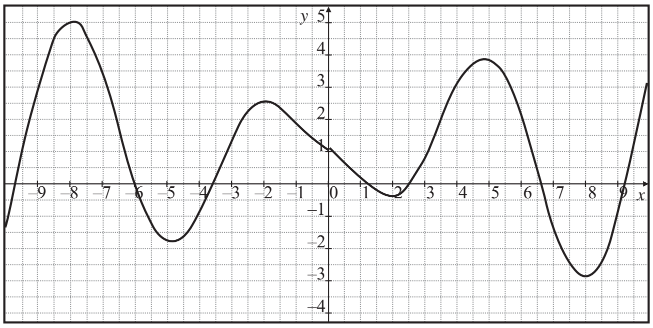


Which of the following could represent the graph of the function $f(x)$?

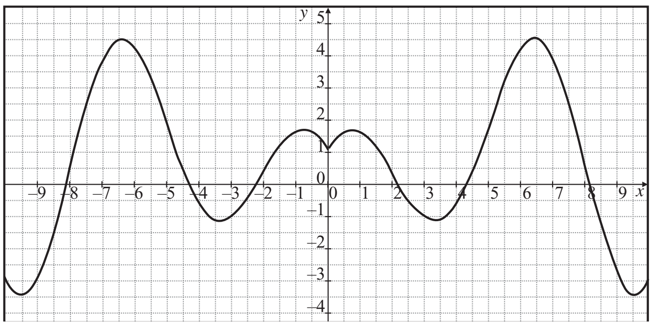
A.



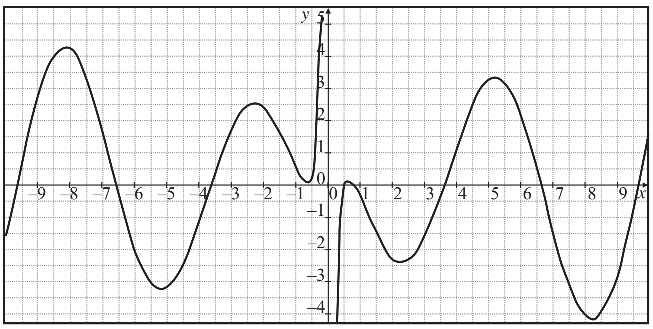
B.



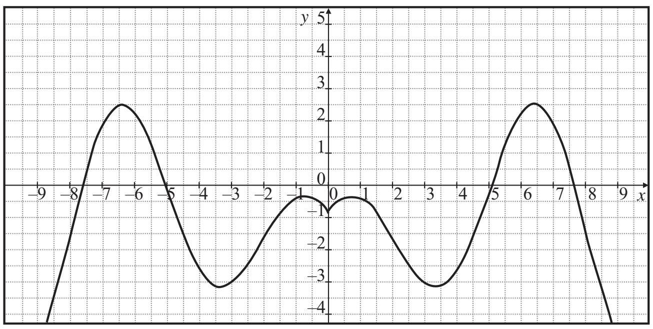
C.



D.



E .



Question 22

If $x = \log_9(2)$ and $y = \log_5(4)$, then $\log_6(15)$, in terms of x and y, is given by

A. $\dfrac{2x+y}{xy}$

B. $\dfrac{2x+y}{(1+4x)y}$

C. $\dfrac{4x+y}{(1+2x)y}$

D. $\dfrac{4x+y}{2}$

E. $\dfrac{x(4x+y)}{(1+y)}$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

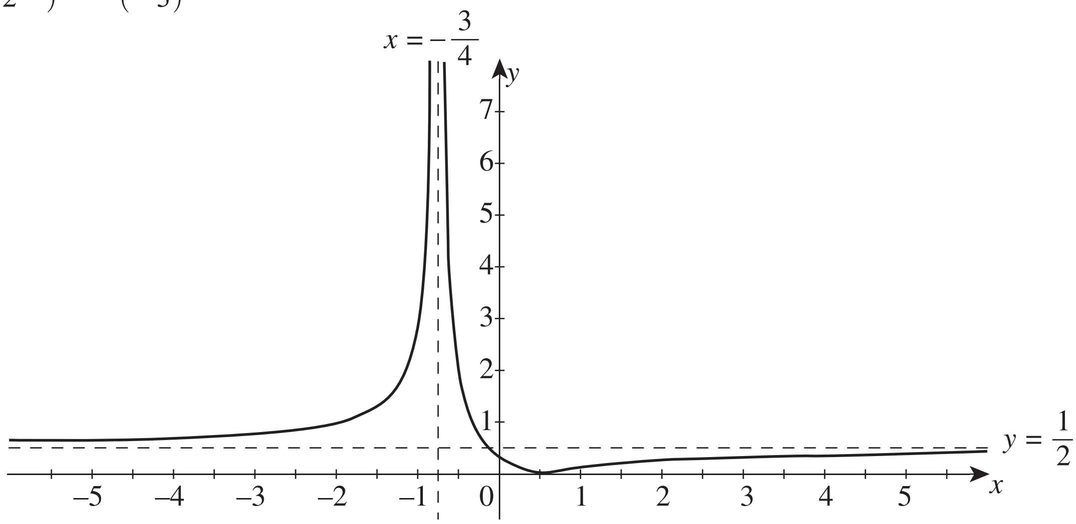
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (15 marks)

A function f, is given by the rule $f(x)=\left|\dfrac{ax+b}{cx+d}\right|$.

Its graph, shown below, has asymptotes with equations $x=-\dfrac{3}{4}$ and $y=\dfrac{1}{2}$ and it passes through the points $\left(\dfrac{1}{2},0\right)$ and $\left(0,\dfrac{1}{3}\right)$.



a. State the maximal domain over which f is defined and give its range. 2 marks

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b. Given that $f(x)=\frac{1}{2}\left|1-\frac{5}{4x+3}\right|$, find the value of *a, b, c* and *d*. 2 marks

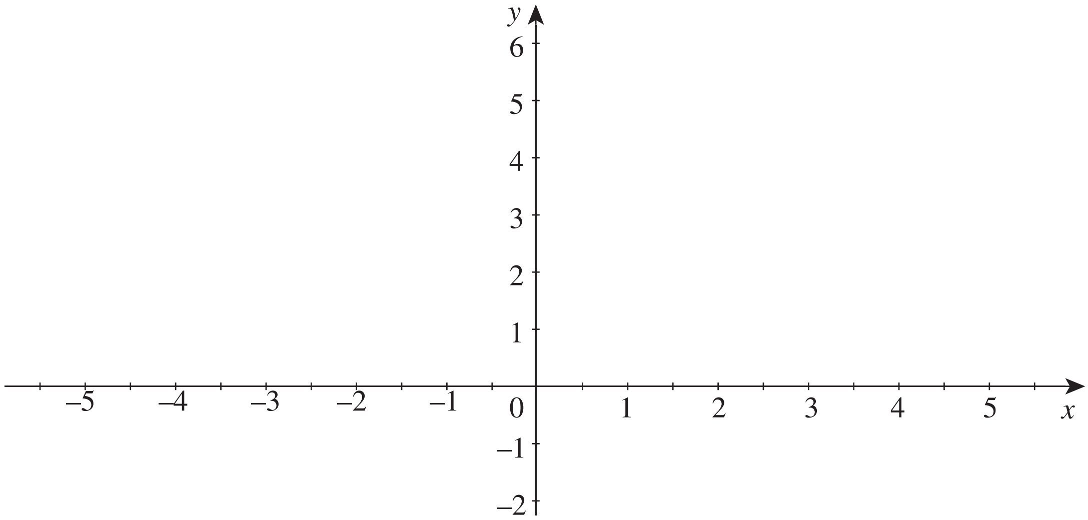
c. Consider the function $g:(m,n]\rightarrow R,\quad g(x)=f(x)$. Given that g^{-1} exists and g has the same range as f,

i. state the value of m and n. 2 marks

ii. determine the rule for g^{-1} and state its domain. 2 marks

iii. On the axes below sketch the graph of g and g^{-1}. Clearly show any asymptotes and label the axis intercepts with their exact values.

3 marks

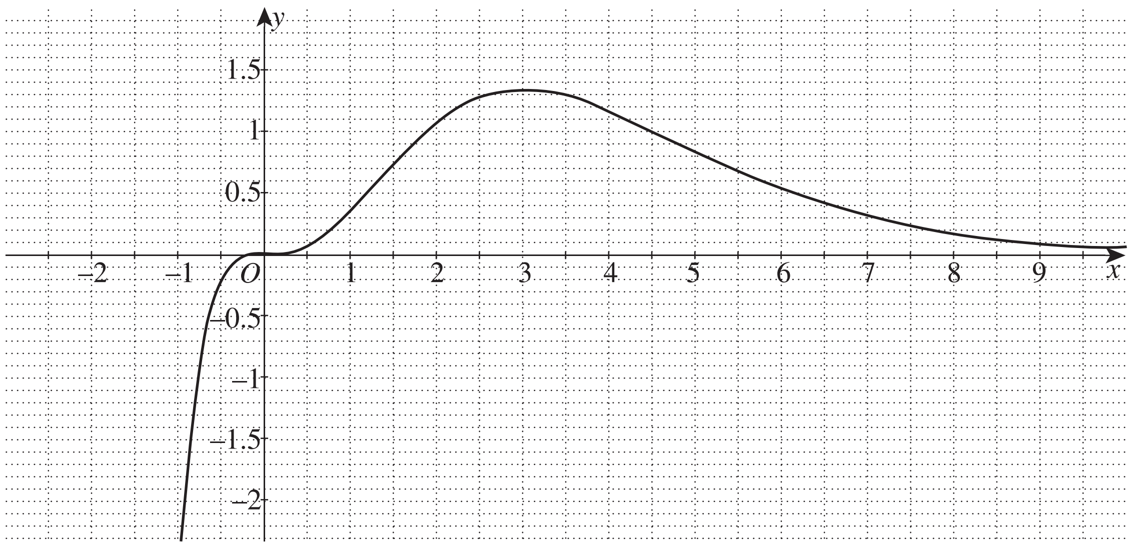


iv. Solve the equation $g^{-1}(x) - g(x) = 0$. 1 mark

d. The tangent to the graph of f at its y intercept meets the graph again at (p, q). Determine values for p and q, correct to three decimal places. 3 marks

Question 2 (15 marks)

The diagram shows a sketch of the curve $y = \dfrac{x^3}{e^x}$.



a. Find $\dfrac{dy}{dx}$, expressing your answer in the form $\dfrac{dy}{dx} = \dfrac{(ax^2 + bx^3)}{e^x}$, where a and b are integers. 1 mark

b. Determine the nature and coordinates of any stationary points on the graph of $y = \dfrac{x^3}{e^x}$. 3 marks

c. Hence show that $3\log_e(x) \le x + 3\log_e(3) - 3$. 3 marks

d. **i.** Sketch the graph of the gradient function on the set of axes on page 15, showing the coordinates of any axis intercepts and the equation of any asymptotes.

2 marks

ii. For $x \geq 0$, use calculus to find the maximum and minimum values of $\frac{dy}{dx}$ and the corresponding x values for which they occur.

2 marks

iii. Define the steepness, S, of the graph of $y = \frac{x^3}{e^x}$ by $S = \left|\frac{dy}{dx}\right|$.

For $x \geq 0$, give the largest interval for which the steepness of the graph is strictly decreasing. Answer correct to two decimal places.

2 marks

e. Another curve has equation $y^2 = k(x-3),\ k \neq 0$.

Determine the range of values of k for which the equation $\frac{x^6}{e^{2x}} = k(x-3)$ has two real solutions.

2 marks

Question 3 (18 marks)

A hotel in country Victoria is very popular for both its accommodation and first class restaurant.

Accommodation enquiries may be made at the hotel either by phone, through an internet booking agency or through an email via the hotel's website. Whichever way, enquiries occur randomly, with resulting bookings also occurring randomly.

An analysis of enquiries and bookings was carried out. The table below indicates some conclusions from this analysis.

Type of enquiry	Probability of receiving this type of enquiry	Probability that a booking results from this type of enquiry
Phone	0.4	k
Internet booking agency	0.5	k^2
Email via hotel website	0.1	k^3

Consider a week where the hotel received 100 enquiries. Assume all enquiries occur independently.

a. **i.** Given that the number of enquiries that came through the phone follows a binomial distribution, calculate the mean and standard deviation of the number of these enquiries. Answer to two decimal places where necessary.

2 marks

ii. Calculate the probability that at least 30 of these 100 enquiries came through the phone. Give your answer correct to four decimal places.

1 mark

b. **i.** In terms of k, find an expression that gives the probability that the eighth phone call resulted in the first booking from phone enquiries on that day.

1 mark

ii. In terms of k, find an expression that gives the probability that the eighth phone call resulted in the fourth booking from phone enquiries on that day.

2 marks

iii. What value of k results in the maximum probability that the 8th phone call resulted in the fourth booking from phone enquiries on that day? State this maximum probability correct to four decimal places.

2 marks

c. It is found that the 42% of overall enquiries do not result in a booking.

Find k, correct to two decimal places. 2 marks

d. Suppose that when a booking resulted from an enquiry there was a 25% chance it was by the internet booking agency.

Find the value of k, correct to 2 decimal places. 2 marks

The hotel restaurant is popular with guests but is very expensive. Alternatives are the hotel cafe or other restaurants nearby. Assume where a guest chooses to dine each night depends only on where they dined the previous night. If a guest dines in the hotel restaurant one night, then the probability of dining in the hotel restaurant the following night is $\frac{3}{5}$. The transition matrix for the probabilities of the guest dining in the restaurant or dining elsewhere is $\begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}$.

e. **i.** Suppose a guest dines in the hotel restaurant on Sunday night. What is the probability that they will dine in the hotel restaurant on Wednesday night of the same week?

2 marks

ii. In the long term what percentage of nights can the hotel assume guests will dine in the hotel restaurant? Give your answer to the nearest percent.

1 mark

The restaurant is regarded for its lobster dishes. It purchases its lobster directly from the local seafood supplier who is not always reliable with the weight.

It is found that the weights of lobsters purchased follow a normal distribution with mean μ kg and standard deviation σ kg. It is known that 25% of the lobsters have weights which differ from μ by at most m kg.

f. Find the probability that a randomly chosen lobster has a weight which exceeds μ kg by at most $3m$ kg. Give your answer correct to four decimal places.

3 marks

Question 4 (10 marks)

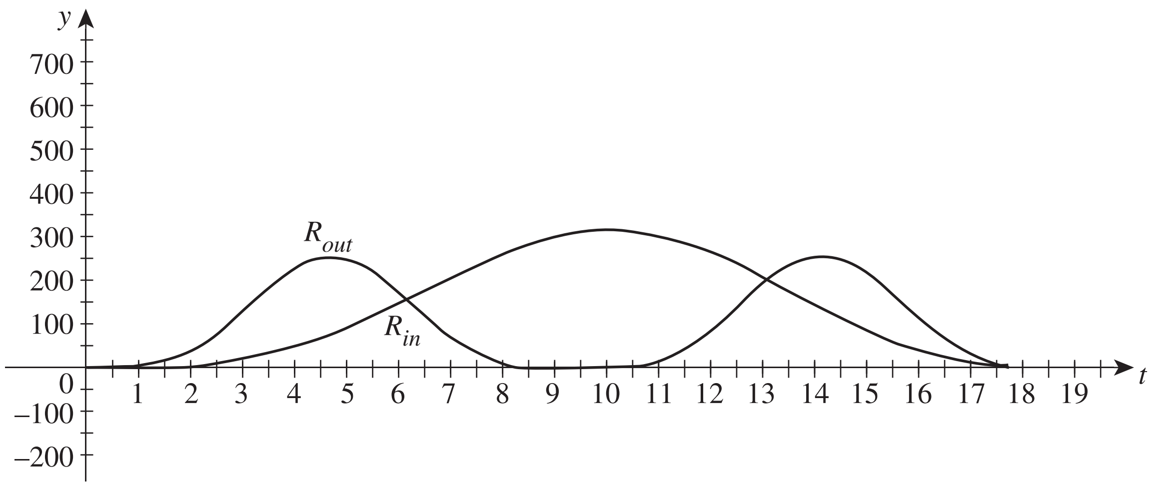
A chemical factory flushes one of its large tanks which initially holds 1200 litres of liquid. During an 18-hour-time interval, water is pumped into the tank at the rate of

$$R_{in} = 100\sqrt{t}\sin^3\left(\frac{t}{6}\right) \text{ litres per hour}$$

During the same time interval, liquid is removed from the tank at the rate of

$$R_{out} = 250\sin^4\left(\frac{t}{3}\right) \text{ litres per hour}$$

The graphs of R_{in} and R_{out} are shown below:



a. On the axes above sketch the graph of $y = R_{in} - R_{out}$ 2 marks

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b. Over what time interval is the liquid remaining in the tank increasing? Answer correct to three decimal places. 1 mark

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c. How many litres of liquid will the tank contain at $t = 15$? Give your answer correct to the nearest litre. 2 marks

d. Calculate how much liquid is in the tank at the times when the inflow rate equals the outflow rate and hence, determine when, during the 18-hour-period, is the liquid in the tank is at an absolute minimum? 3 marks

e. For $t > 18$, no water is pumped into the tank, but the liquid continues to be removed at the same rate as before. This continues until the tank is emptied. Let the time at which the tank becomes empty be $t = T$.

Write an equation involving an integral expression which can be used to find T and hence, determine T correct to the nearest minute. 2 marks

END OF QUESTION AND ANSWER BOOKLET