NAME: ____________________________

VCE MATHEMATICAL METHODS (CAS)

Practice written examination 2

Reading time: 15 minutes **Writing time: 2 hours**

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 22 pages.
- Answer sheet for multiple choice questions.
- Formula booklet PROVIDED BY YOUR TEACHER.
- Working space is provided throughout the book.

Instructions

- All written responses must be in English.
- Write your student name in the space provided above on this page and on the answer sheet for multiple choice questions.

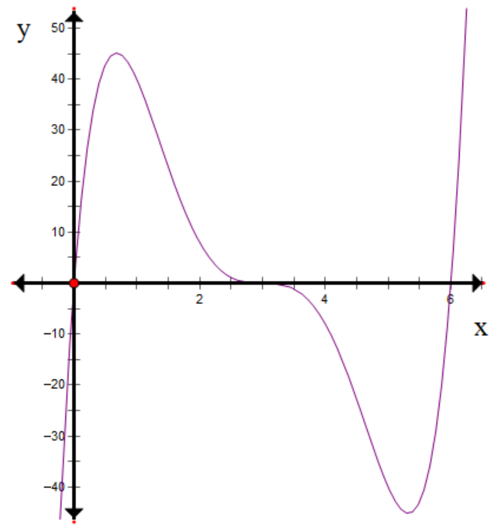
Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The equation of the curve shown could be:



A. $f(x) = x(x-3)(x-6)$
B. $f(x) = x(x-3)^2(x-6)$
C. $f(x) = x^2(x-3)(x-6)$
D. $f(x) = x(x-3)^3(x-6)$
E. $f(x) = x(x+3)^3(x+6)$

Question 2

For $f(x) = \log_e(x)$ and $g(x) = \frac{1}{x+2}$ $g(f(x))$ is:

A. $\frac{1}{\log_e(x)+2}, x \in R^+$

B. $\frac{1}{\log_e(x)+2}, x \in R^+\backslash\{2\}$

C. $\frac{1}{\log_e(x)+2},\ 0 \le x < \infty$

D. $\frac{1}{\log_e(x+2)},\ 0 \le x < \infty$

E. $\frac{1}{\log_e(x+2)}, x \in R^+\backslash\{2\}$

Question 3

Which of the following is true for $f(x) = e^{-3(x+2)} - 1$?

A. *f(x)* has a asymptote at x = -2

B. *f(x)* is decreasing for all *R*

C. *f(x)* has a y-intercept at (0, -1)

D. *f(x)* is reflected in the x-axis when compared to e^x

E. *f(x)* has been translated two units in the positive x direction when compared to e^x

Question 4

If $(x-3)^3 - 18x = x^3 - ax(x-1) - 27$ then the value of *a* is:

A. -9
B. 3
C. -3
D. 6
E. 9

Question 5

For the functions $f(x) = \sqrt{x^2 - 4}$ and $g(x) = \sqrt{36 - x^2}$ the domain of *f(x).g(x)* is:

A. $-\infty < x \le -2 \ \cup 2 \le x < \infty$
B. $-6 \le x \le 6$
C. $-2 \le x \le 2$
D. $-6 \le x \le -2 \ \cup 2 \le x \le 6$
E. $-\infty < x \le -6 \ \cup 6 \le x < \infty$

Question 6

If b = a^2, then $log_a x + \ log_b x$ equals:

A. $1.5\ log_a\ x$

B. $log_{a+b}x$

C. $log_{ab}x$

D. $(1+b)log_a x$

E. $log_a x(1+\ log_b x)$

Question 7

The solution to the matrix equation $\begin{bmatrix} 2 & 1 & 4 & 8 \\ 1 & 3 & 2 & 4 \\ 6 & 2 & 1 & 3 \\ 2 & 8 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 70 \\ 75 \\ 90 \\ 30 \end{bmatrix}$ is:

A. a = 815, b = 595, c = 750, d = 1040
B. a = 70, b = 75, c = 90, d = 30
C. 280
D. a = 3, b = 16, c = -44, d = 28
E. There is no solution

Question 8

The inverse of the function *f*: (-∞, 4) → R, $f(x) = \dfrac{3}{(x-4)} + 1$ is:

A. $f^{-1}\ (-\infty, 1] \to R, f^{-1}(x) = \frac{3}{(x-1)} - \ 4$

B. $f^{-1}\ (-\infty, 1) \to R, f^{-1}(x) = \frac{3}{(x-1)} + \ 4$

C. $f^{-1}\ (-\infty, 1] \to R, f^{-1}(x) = \frac{3}{(x-1)} + \ 4$

D. $f^{-1}\ (-\infty, 1) \to R, f^{-1}(x) = \frac{3}{(x-1)} - \ 4$

E. $f^{-1}\ (-\infty, -1) \to R, f^{-1}(x) = \frac{3}{(x-1)} + \ 4$

Question 9

The general solution to the equation $0 = 4\tan(5x) - 4$ is:

A. $\frac{\pi}{20} + \frac{n\pi}{5}$

B. $\frac{\pi}{5} + \frac{n\pi}{4}$

C. $\frac{(n+1)\pi}{20}$

D. $\frac{\pi}{4} + \frac{n\pi}{5}$

E. $\frac{\pi}{5} + n\pi$

Question 10

The maximal domain for the function $f(x) = \frac{1}{x^3 - x^2 - 12x}$ is:

A. R
B. R\{-4, 0, 3}
C. R\{ 0, 3, 4}
D. (-3, 4)
E. R\{-3, 0, 4}

Question 11

The equation $3sin\left(\frac{\pi x}{2}\right) = 2\, log_e(x+1)$ has

A. No solutions
B. One solution
C. Two solutions
D. Three solutions
E. Four solutions

Question 12

$x^4 - 10x^2 + k = 0$ has four unique solutions when:

A. $k < 25$
B. $0 < k < 25$
C. $k > 0$
D. $k < 0 \cup k > 25$
E. $k = 25$

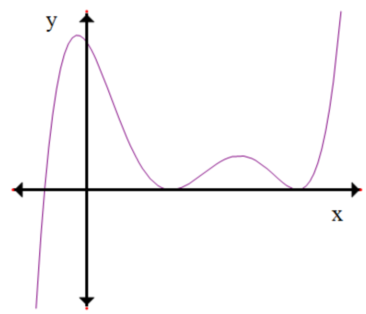
Question 13

The tangent to the curve $y = x^3 - x^2 - x - 1$ when x = 0, also cuts the curve at:

A. (-1, 0)
B. (2, 1)
C. (1, -2)
D. (-1, -2)
E. (1, 0)

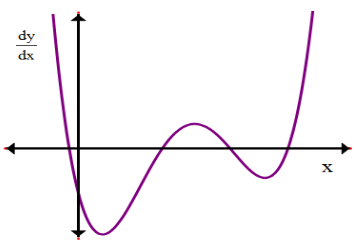
Question 14

A function *f* has a graph as shown.



The graph of the derivative function could be:

A.



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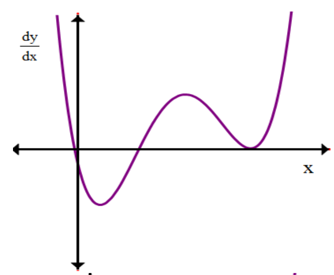

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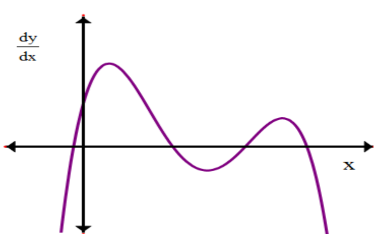

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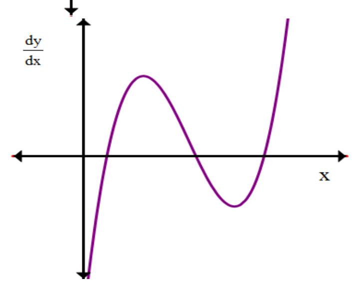

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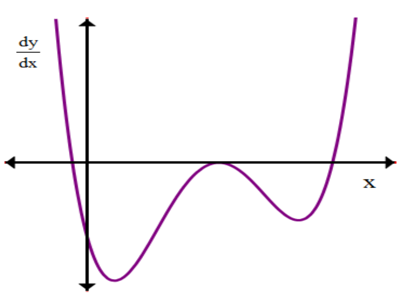

Question 15

The function $f(x) = \frac{x^3 - 7x^2 + 36}{x^2 - 3x}$ is'

A. Continuous for all R
B. Discontinuous at x = 3 only
C. Discontinuous at x = -2, 3 and 6
D. Discontinuous at x = -2 only
E. Discontinuous at x = 0 and 3

Question 16

The hybrid function $g(x) = \begin{cases} x^2 - 4x, & -\infty < x \le 3 \\ \frac{a}{(x-2)^2} - 2, & 3 < x < \infty \end{cases}$ is differentiable at x = 3 when:

A. a= 1
B. a = 2
C. a = -2
D. a = -1
E. a = -3

Question 17

The function $-x^3 + 12x^2 - 36x + 15$ is strictly increasing for:

A. $x \in R$
B. $2 < x < 6$
C. $-6 < x < 2$
D. $2 \le x \le 6$
E. $-6 \le x \le -2$

Question 18

For the function $f(x) = \frac{2}{x^2}$ the approximate change in *f* as x goes from 3 to 3.1 is given by:

A. $\frac{2}{9} + 0.1 \times \frac{-4}{27}$

B. $0.1 \times \frac{-4}{27}$

C. $\frac{2}{9} + 0.1 \times \frac{-4}{9}$

D. $0.1 \times \frac{-4}{9}$

E. $\frac{2}{27} + 0.1 \times \frac{-4}{27}$

Question 19

When evaluated, $\int_0^{2\pi} \left(e^{\sin(x)} - 1\right) dx$ is closest to:

A. 4.463
B. -2.791
C. 0
D. 1.395
E. 1.672

Question 20

The probability of a bus being on time on any given day is dependent only on whether it was on time the day before. If it was on time the first day, then the probability that it will be on time the next day is $\frac{3}{10}$. However, if the bus is late the first day then there is a $\frac{4}{5}$ chance it will be late the next day.

If there is equal chance of the bus being on time or late on the very first day of the year, what is the long term probability of it being on time on any given day?

A. $\frac{2}{9}$

B. $\frac{3}{11}$

C. $\frac{8}{11}$

D. $\frac{7}{9}$

E. $\frac{1}{2}$

Question 21

A probability of a biased coin coming up heads is $\frac{3}{5}$. The probability of a run length of 8 heads in a row is closest to:

A. 0.0168
B. 0.0004
C. 0.2400
D. 0.6000
E. 0.0067

Question 22

The lengths of plastic straws is found to be normally distributed with a mean of 15.2 cm. If quality control says that 95% of all straws must be longer than 14.9 cm, then the variance of the distribution is closest to:

A. 0.1824
B. 0.0333
C. 0.4271
D. 0.1
E. Cannot be determined from the given information.

MATHEMATICAL METHODS
PRACTICE EXAMINATION 2
MULTIPLE CHOICE ANSWER SHEET

NAME:____

Indicate your answer by filling in one option for each question only.

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
14	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
15	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
16	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
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19	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
20	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
21	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
22	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the function: $f(x) = \sin\left(2x - \frac{\pi}{2}\right) + 1$

a) Write down the period of the function.

1 mark

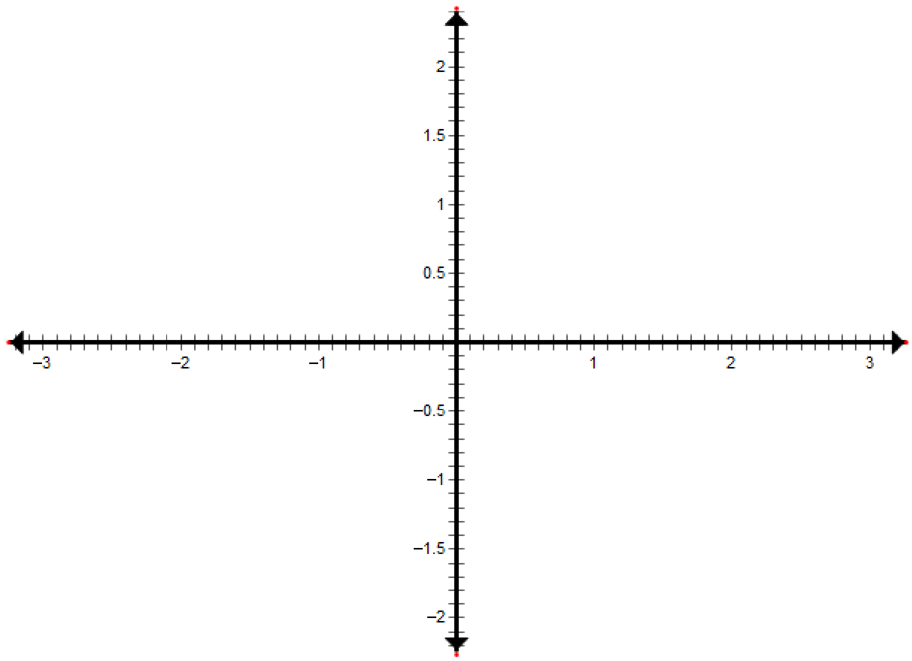
b) What is the range of *f(x)*?

1 mark

c) Describe the ***two translations*** used to transform sin(x) into the function *f(x)*.

2 marks

d) Sketch the curve of *f(x)* for the domain $-\pi \leq x \leq \pi$. Include the coordinates of all stationary points, intercepts and endpoints.



4 marks

e) The function $f(x) = \sin\left(2x - \frac{\pi}{2}\right) + 1$ can also be expressed as

$$f(x) = sin^2(x) - cos^2(x) + 1.$$

Explain why.

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1 mark

Now consider the function $g(x) = -\cos(x) + 1$ over the same domain.

f) Use algebra to determine the x-values of the points of intersection between f(x) and g(x) over the domain $-\pi \leq x \leq \pi$.

4 marks

g) Use calculus to differentiate *f(x)*.

1 mark

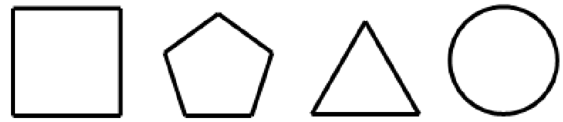
h) Use calculus to differentiate *g(x)*.

1 mark

i) Now show that at $x = 0$, the functions $f(x)$ and $g(x)$ are tangent to each other.

3 marks

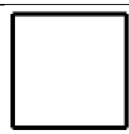
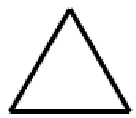
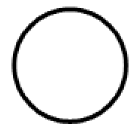
Question 2



A new game app for the MyPhone has just been released called Cookie Crush. Four different shaped Cookies are arranged randomly in eight by eight grids.

In the long run squares are twice as likely to appear as triangles and four times as likely as either circles or pentagons.

a) Complete the table of probabilities for the shapes used in this game.

				
Pr(X=x)				$\frac{1}{8}$

2 marks

To score points in the game, you have to “crush” cookies of a given shape when they turn red.
Crushing a square is worth one point
a triangle is worth two points
a circle is worth three points
a pentagon is worth four points.

b) What is the expected point value (average) of a cookie being crushed?

2 marks

c) Sean is interested in spotting all of the pentagons that are in the grid from move to move. In terms of the mean and standard deviation write down what Sean should find if he keeps track of the pentagons over a long period of time? Give your answer to two decimal places where appropriate.

2 marks

The probability of someone playing Cookie Crush somewhere around the world at any time of the day can be modelled by a continuous random variable with the rule:

$$f(t) = \begin{cases} -\frac{3}{100}cos\left(\frac{\pi\, t}{4}\right) + \frac{1}{24}, & 0 \le t \le 24 \\ 0 & , \quad elsewhere \end{cases}$$

Where t is time in hours after midnight.

d) What is the probability that a person chosen at random from the population will be playing Cookie Crush during business hours, between 9am and 5pm?

2 marks

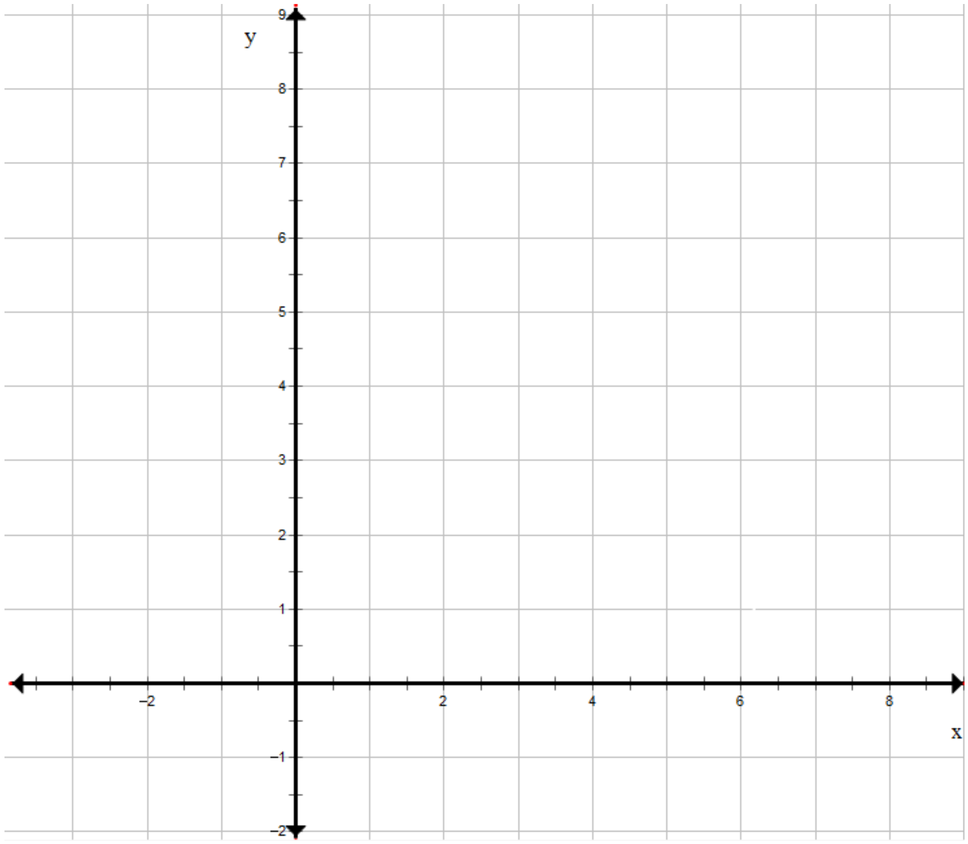
e) How long after midnight on a given day will it be before 75% of people have played Cookie Crush? Give your answer to 3 decimal places.

2 marks

Question 3

Consider the function $f(x) = \ln\left(\sqrt{x}\right)$.

a) Sketch the curve for the domain, $0 < x \leq 9$. Include the coordinates of the endpoint where $x = 9$.



3 marks

b) Now determine the inverse of $f(x)$.

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2 marks

Let the $f^{-1}(x) = g(x)$

c) Sketch on the same axes, the curve of $g(x)$.

2 marks

d) Use calculus to find the integral of $g(x)$ between $-\infty$ and 0.

2 marks

e) The result in **d)** is also the area bounded by $f(x)$ between which two x-values?

1 mark

f) Given that $h(x) = x \ln(\sqrt{x})$ find $h'(x)$.

1 mark

g) Hence show that an antiderivative for $f(x)$ is $x \ln\left(\sqrt{x}\right) - \frac{x}{2}$.

2 marks

You now have two methods for determining the area bounded by *f(x)* and the x-axis.

h) Use the result from g) to find $\int_1^9 f(x)dx$.

1 mark

i) Explain briefly why using the inverse to find this area would require more effort in this particular situation.

1 mark

Question 4

A ball of dry ice is evaporating is evaporating in such a way that its radius is defined by the equation:

$$r(t) = 10 - \frac{t^2}{20}\text{ cm}$$

a) What is the initial radius of the ball?

1 mark

b) At what rate is the radius changing with respect to time?

2 marks

c) How long does it take for the ball to evaporate completely? Quote your answer to 3 decimal places.

2 marks

d) Using the equation given for radius, show that the equation for the surface area of the ball at any time t, is

$$A(t) = \pi\left(\frac{t^4}{100} - 4t^2 + 400\right)$$

2 marks

e) What is the initial area of the ball?

1 mark

f) Using integration, determine a formula the volume of the ball, *V(t)*. Note, $V(t) = \int A(t)dt$.

1 mark

g) Now, using the initial radius of the ball from a) and the formula for volume, $= \frac{4}{3}\pi r^3$, show that the value of the constant C is $\frac{400\pi}{3}$ cm^3.

1 mark

h) What is the average rate of decrease of volume with time? Give your answer correct to 2 decimal places.

1 mark

i) At what time does the instantaneous decrease in volume equal the average rate of change?

4 marks

END OF QUESTION AND ANSWER BOOKLET

Solution Pathway

SECTION 1

Question 1

The answer is

D $f(x) = x(x-3)^3(x-6)$

x-intercepts occur at x = 0, x = 3 and x = 6. At x = 3 there is a stationary point of inflection indicating a triple solution or root.

Question 2

The answer is

A $\frac{1}{log_e(x)+2}, x \in R^+$

Replace x in *g(x)* with $log_e x$. Note that *g(x)* is not defined when x = -2 but this does not effect the result as the domain of *f(x)* is $x \geq 0$.

Question 3

The answer is

B *f(x)* is decreasing for all *R*

Exponential functions always increase or decrease over their entire domain and this function is reflected in the y-axis (note the -3) so it is decreasing for all R.

Question 4

The answer is

E 9

If $(x-3)^3 - 18x = x^3 - 9x^2 + 27x - 27 - 18x = x^3 - 9x^2 + 9x - 27$

Take out a common factor of -9 from the x and x^2 terms to find a = 9.

Question 5

The answer is

D $-6 \leq x \leq -2 \;\cup 2 \leq x \leq 6$
This is the intersection of the domains of the two parent functions.

Question 6

The answer is

A $1.5 \log_a x$

$\log_b x = \frac{\log_a x}{\log_a b}$ so $\log_a x + \log_b x = \log_a x + \frac{\log_a x}{\log_a b}$

$= \log_a x\left(1 + \frac{1}{\log_a b}\right)$

Now, b = a^2 so $= \log_a x\left(1 + \frac{1}{\log_a a^2}\right)$

$= \log_a x\left(1 + \frac{1}{2\log_a a}\right)$

$= \log_a x\left(1 + \frac{1}{2}\right) = 1.5 \log_a x$

Question 7

The answer is

D a = 3, b = 16, c = -44, d = 28

Using a calculator is the only way this can be done, as stipulated in the study design.

Question 8

The answer is

B $f^{-1}: (-\infty, 1) \to R, f^{-1}(x) = \frac{3}{(x-1)} + 4$

Let y = *f(x)*

$y = \frac{3}{(x-1)} + 4$

Function	Domain	Range
$f(x)$	$(-\infty, 4)$	$(-\infty, 1)$
$f^{-1}(x)$	$(-\infty, 1)$	$(-\infty, 4)$

$y - 4 = \frac{3}{(x-1)}$

$x - 1 = \frac{3}{(y-4)}$

$x = \frac{3}{(y-4)} + 1$

Question 9

The answer is

A $\frac{\pi}{20} + \frac{n\pi}{5}$

Use the formula $\theta = n\pi + tan^{-1}\left(\frac{4}{4}\right)$ where $\theta = 5x$

Then divide through by 5.

Question 10

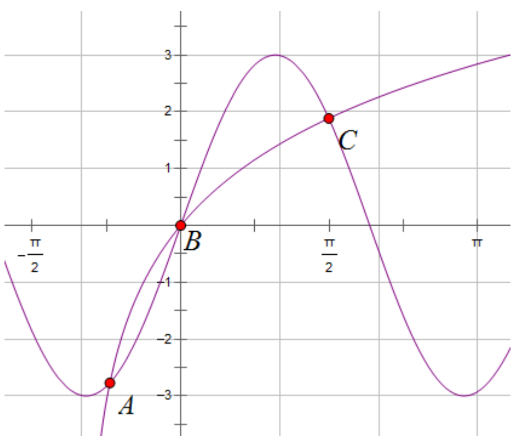
The answer is

E R\{-3, 0, 4}
Factorise the denominator to get x(x + 3)(x – 4). The solutions to the cubic become asymptotes in the reciprocal.

Question 11

The answer is

D Three solutions
Sketch the curves on the same set of axes. Using solve alone on the calculator may not identify all three solutions.



Question 12

The answer is

B *0 < k < 25*
Let A = x^2

Use the quadratic formula: $A = \frac{10 \pm \sqrt{100-4k}}{2}$

So $x^2 = \frac{10 \pm \sqrt{100-4k}}{2}$

Therefore $x = \sqrt{\frac{10 \pm \sqrt{100-4k}}{2}}$

This only has solutions if $100 - 4k \geq 0$

So $k \leq 25$.

Also, for $10 \pm \sqrt{100-4k}$ the $\sqrt{100-4k} \leq 10$, so $k \geq 0$.

Hence the answer is B

Question 13

The answer is

C (1, -2)
Differentiate to get $\frac{dy}{dx} = 3x^2 - 2x - 1$ and find the gradient at x = 0.

M = -1. Hence the equation of the tangent is y = -x – 1.

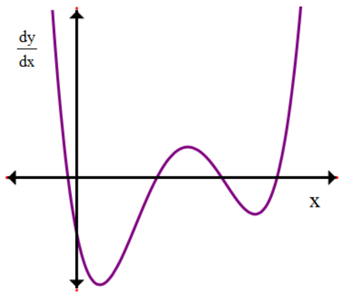
Now solve $-x - 1 = x^3 - x^2 - x - 1$ to get $0 = x^3 - x^2$

So x = 0 or 1.

The second point is (1, -2)

Question 14

The answer is **A**



Four stationary points in the original become 4 intercepts in the derivative. The original is a positive odd degree function so the derivative will be a positive even degree function.

Question 15

The answer is

E Discontinuous at x = 0 and 3
When x = 0 or 3, the denominator is zero and hence the entire function is undefined and therefore discontinuous.

Question 16

The answer is

D a = -1
To be differentiable at the boundary the two sides of the function must have the same y-value and the same gradient. The gradient from left hand side is 2 and the y value is -3.

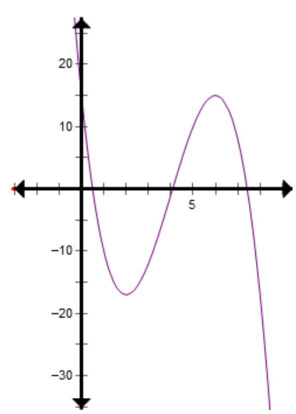
From the right hand side this gradient is only satisfied when a = -1

Question 17

The answer is

D $2 \leq x \leq 6$

Differentiate and find the stationary points. These occur at x = 2 and x = 6



Question 18

The answer is

B $0.1 \times \frac{-4}{27}$

h = 0.1 , $f'(x) = \frac{-4}{x^3}$ so $f'(3) = \frac{-4}{27}$

Question 19

The answer is

E 1.672

This cannot be integrated formally so numerical techniques must be employed. Use the calculator.

Question 20

The answer is

A $\frac{2}{9}$

The transition matrix is $\begin{bmatrix} \frac{3}{10} & \frac{1}{5} \\ \frac{7}{10} & \frac{4}{5} \end{bmatrix}$. The initial state matrix is $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$.

Question 21

The answer is

E 0.0067

This is a binomial distribution. Run length is given by $p^n q = \frac{3}{5}^8 \times \frac{2}{5} = 0.0067$.

Question 22

The answer is

B 0.0333

The percentage below 14.9 = 5% or 0.05

Invnorm(0.05, 0, 1) = -1.64485

Use z-score: $z = \frac{x - \mu}{\sigma}$

$-1.64485 = \frac{14.9 - 15.2}{\sigma}$

$\sigma = \frac{14.9 - 15.2}{-1.64485} = 0.182387$

Variance is $\sigma^2 = 0.0333$

SECTION 2

Question 1

Consider the function: $f(x) = \sin\left(2x - \frac{\pi}{2}\right) + 1$

a) Period = $\frac{2\pi}{2} = \pi$

(1 mark)

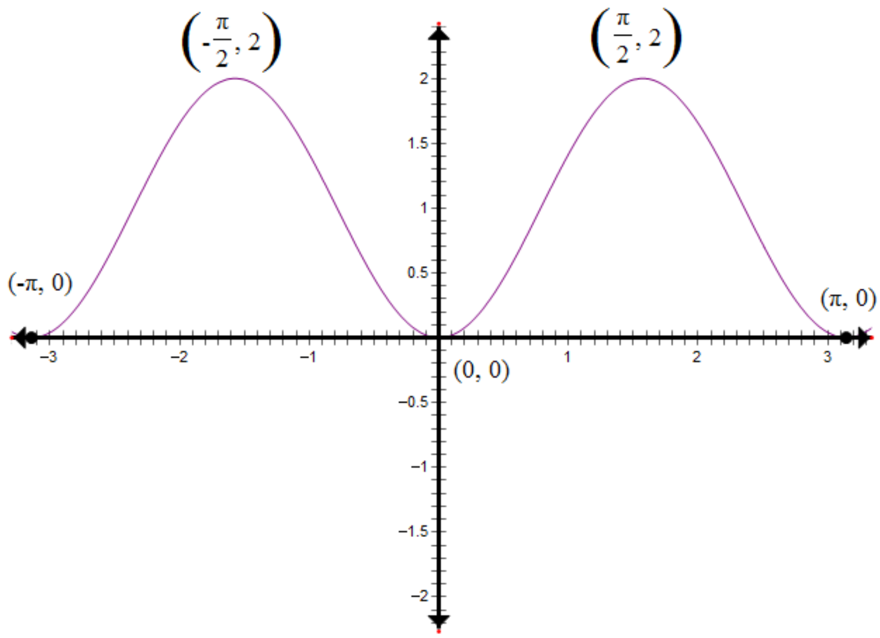
b) $0 \leq y \leq 2$

(1 mark)

c) The translations are: $\frac{\pi}{4}$ units in the positive x-direction *and* 1 unit in the positive y-direction. (2 marks)

d)

(4 marks)



e) Recognise that the original formulation of *f(x)* has an argument of 2x while the new one is only x. This means that a double angle formula is involved and that $\sin\left(2x - \frac{\pi}{2}\right) = sin^2(x) - cos^2(x)$

(1 mark)

f) Let $sin^2(x) - cos^2(x) + 1 = -\cos(x) + 1$

$$sin^2(x) = cos^2(x) - \cos(x)$$ (1 mark)

Using $sin^2(x) + cos^2(x) = 1$ and rearranging for $sin^2(x) = 1 - cos^2(x)$

We get $1 - cos^2(x) = cos^2(x) - \cos(x)$ (1 mark)

$$0 = 2cos^2(x) - \cos(x) - 1$$

Let A = cos(x): $0 = 2A^2 - A - 1$

So $A = \frac{1 \pm \sqrt{1^2 - 4 \times 2 \times -1}}{2 \times 2} = x = \frac{1 \pm \sqrt{9}}{4} = 1 \; or \; \frac{-1}{2}$ (or use factorization) (1 mark)

Hence $\cos(x) = 1 \; or \; \frac{-1}{2}$

So x = 0 or $\pm \frac{2\pi}{3}$ (1 mark)

g) $f'(x) = 2\cos\left(2x - \frac{\pi}{2}\right)$ using chain rule (1 mark)

h)

$g'(x) = \sin(x)$ (1 mark)

i) This requires that the curves pass through the same point and have the same gradient.

When x = 0 $f(0) = \sin\left(\frac{-\pi}{2}\right) + 1 = -1 + 1 = 0$

And $g(x) = -\cos(0) + 1 = -1 + 1 = 0$

So both functions pass through the point (0, 0) (1 mark)

$f'(0) = 2cos\left(\frac{-\pi}{2}\right) = 0$

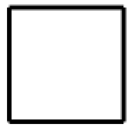
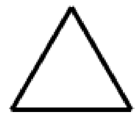
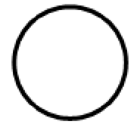
$g'(x) = \sin(0) = 0$ (1 mark)

Both curves have the same gradient at x = 0. Hence they are tangent to one another at x = 0.

(1 mark)

Question 2

a)

				
Pr(X=x)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(3 marks)

b)

X	1	2	3	4
Pr(X=x)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
X.Pr(x)	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$

Method – 1 mark

E(x) = $\frac{15}{8} = 1.875$ Answer – 1 mark

(2 marks)

c) This can be worked out using binomial distribution with n = 64 and $p = \frac{1}{8}$ (both given)

Mean = 8 and standard deviation = $\sqrt{64 \times \frac{1}{8} \times \frac{7}{8}} = \sqrt{7} \approx 2.65$ (1 mark)

Sean should expect an average of 8 pentagons on the screen at a time with an standard deviation of 2.65 pentagons. (1 mark)

d) $\int_9^{17} -\frac{3}{100} cos\left(\frac{\pi\, t}{4}\right) + \frac{1}{24}\, dt = \left[-\frac{12}{100\pi}\sin\left(\frac{\pi\, t}{4}\right) + \frac{t}{24}\right]_9^{17}$ (1 mark)

$$= \left(-\frac{12}{100\pi}\sin\left(\frac{\pi \times 17}{4}\right) + \frac{17}{24}\right) - \left(-\frac{12}{100\pi}\sin\left(\frac{\pi \times 9}{4}\right) + \frac{9}{24}\right)$$

$$= \frac{1}{3}$$ (1 mark)

e)

$$0.75 = \int_0^Q -\frac{3}{100} cos\left(\frac{\pi\, t}{4}\right) + \frac{1}{24}\, dt \qquad (1)$$

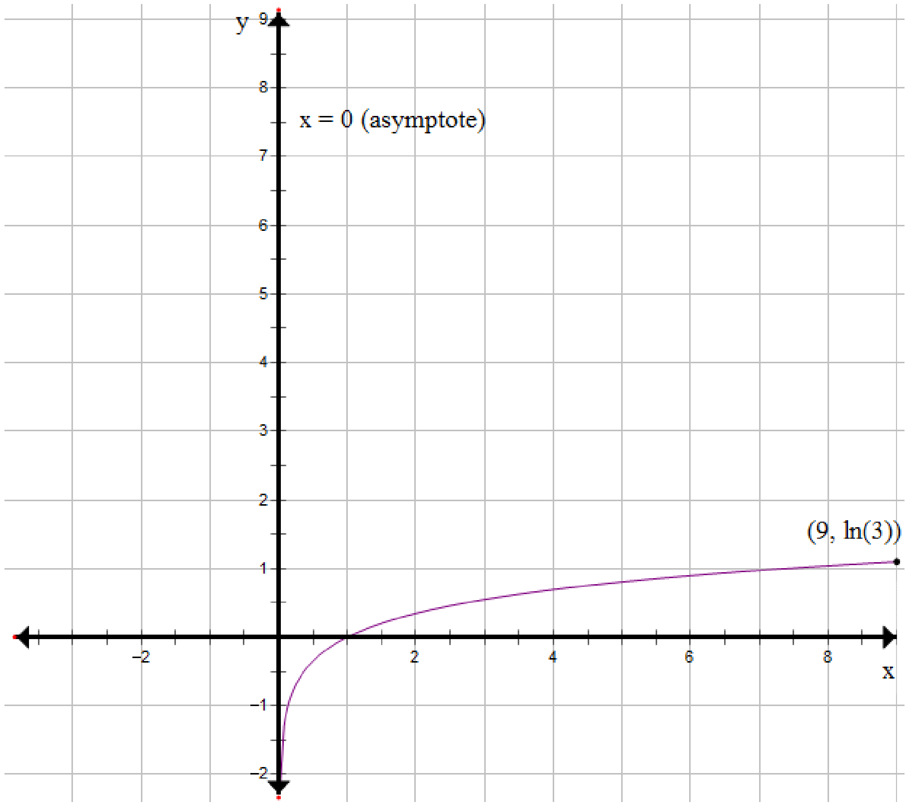
$$solve(0.75 \;=\; \left(-\frac{12}{100\pi} \sin\left(\frac{\pi \times Q}{4}\right) + \frac{Q}{24}\right), Q)$$

$Q = 18.759$ hrs (1)

(2 marks)

Question 3

a)



(3 marks)

b) Let $f(x)$ = y

Then, $y = log_x(\sqrt{x})$

Swap x and y

$x = log_x(\sqrt{y})$

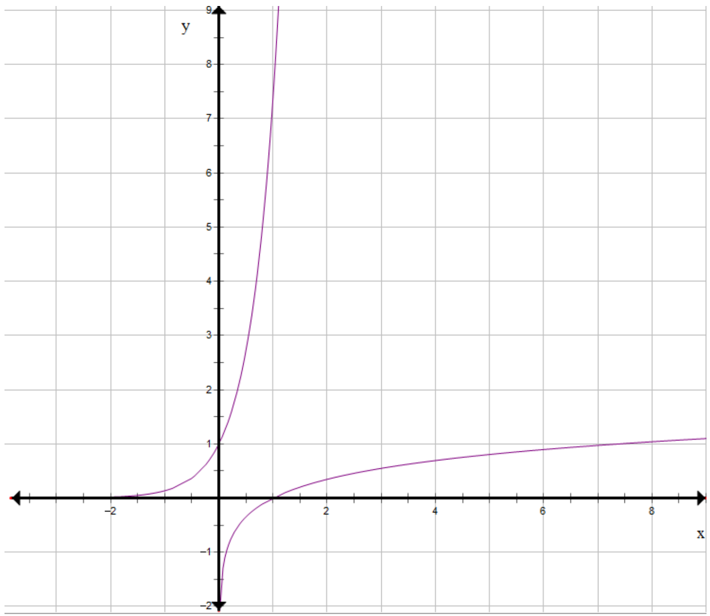
Remove the log: $e^x = \sqrt{y}$ (1)

Square both sides: $(e^x)^2 = y$

Expand the bracket: $f^{-1}(x) = e^{2x}$ (1)

(2 marks)

c)



Shape – 1 Position - 1

(2 marks)

d)

$$\int_{-\infty}^{0} e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_{-\infty}^{0}$$ (1)

$$= \left[\frac{e^{0}}{2} - \frac{e^{-2\infty}}{2}\right]$$

$= \frac{1}{2}$ square units (1)

(2 marks)

e) Between 0 and 1. (1 mark)

f)

$h(x) = \frac{x\ln(x)}{2}$

So $h'(x) = \frac{\ln(x)}{2} + \frac{1}{2}$ (1 mark)

g)

$f(x) = \ln\left(\sqrt{x}\right)$

$= \frac{\ln(x)}{2}$

$= \frac{\ln(x)}{2} + \frac{1}{2} - \frac{1}{2}$

$= h'(x) - \frac{1}{2}$ (1)

Therefore $F(x) = \int h'(x) - \frac{1}{2}\,dx$

$$F(x) = x\ln\left(\sqrt{x}\right) - \frac{x}{2} + C \qquad (1)$$

(2 marks)

h)

$$\int_1^9 f(x)dx = \left[x\ln\left(\sqrt{x}\right) - \frac{x}{2}\right]_1^9$$
$$= \left(9\ln\left(\sqrt{9}\right) - \frac{9}{2}\right) - \left(ln\sqrt{1} - \frac{1}{2}\right)$$
$$= 9\ln(3) - 4 \text{ u}^2$$

(1 mark)

i) To use the inverse it is first necessary to find the value of *g(1)* and then find the area of the rectangle with width 1 and height *g(1)*. Remember that *g(x)* is the inverse function.

Only after this can the area be found by subtracting the area under the curve of the inverse.

(1 mark)

Question 4

a) 10 cm. Let t = 0 and r(0) = 10

(1 mark)

b) $r'(t) = -\frac{t}{10}$ cm/sec (1 mark for negative sign, 1 mark for expression)

(2 marks)

c)

Let $r(t) = 0 = 10 - \frac{t^2}{20}$

$10 = \frac{t^2}{20}$

$t^2 = 200$ (1 mark method)

$t = \pm 14.142$ seconds

However, time is positive only so +14.142 seconds (1 mark answer)

(2 marks)

d) By substituting into the formula $A = 4\pi r^2$

$$A(t) = 4\pi\left(10 - \frac{t^2}{20}\right)^2$$

$$A(t) = 4\pi\left(100 - t^2 + \frac{t^4}{400}\right) \quad (1)$$

$$A(t) = \pi\left(400 - 4t^2 + \frac{t^4}{100}\right) \quad (1)$$

$$A(t) = \pi\left(\frac{t^4}{100} - 4t^2 + 400\right)$$

(2 marks)

e) Let t = 0, A(0) = 400π cm^2

(1 mark)

f)

$$V(t) = \int \pi\left(\frac{t^4}{100} - 4t^2 + 400\right) dt$$

$$V(t) = \pi\left(\frac{t^5}{500} - \frac{4t^3}{3} + 400t\right) + C$$ (1 mark)

g)

$V = \frac{4}{3}\pi r^2$

When t = 0, r(0) =10 and V(0) = C

So, V(0) =$V(0) = \frac{4}{3}\pi \times 10^2 = \frac{400\pi}{3}$ cm^3.

(1 mark)

h)

$$average\ rate = \frac{-400\pi}{3} \div \sqrt{200}$$

$average\ rate = \frac{-400\pi}{3\sqrt{200}} \approx -29.62$ cm^3/sec

Therefore the average rate of decrease is 29.6 cm^3/sec

(1 mark)

i)

Rate of change of volume is the same as the equation for Surface area. (1 mark)

$$\frac{-400\pi}{3\sqrt{200}} = -\pi\left(\frac{t^4}{100} - 4t^2 + 400\right)$$

$$\frac{-400}{3\sqrt{200}} = \frac{-t^4}{100} + 4t^2 - 400$$ (sign - 1 mark, expression - 1)

Using solve on the calculator gives four values for t: -15.19, -13.01, 13.01 and 15.19.

Since the domain is 0 $\leq$ t $\leq$ 14.142, then at t = 13.01 seconds the instantaneous rate is the same as the average rate. (1 mark)

(4 marks)