QATs

Quality Assessment Tasks

UNIT 4 ■ OUTCOMES 1, 2 & 3

VCE Mathematics Methods (CAS)

SCHOOL-ASSESSED COURSEWORK

Introduction

Outcome 1
Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Outcome 2
Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

Outcome 3
Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modeling or investigative techniques or approaches.

Task

Item Response

The task has been designed to allow achievement up to and including the highest level in the Performance Descriptors. It covers a broad range of **key knowledge** and **key skills** over the three outcomes for Unit 4.

It will contribute 20 out of the total (40) marks allocated for SAC in Unit 4.

This task will be marked out of 60 and then will be converted to a proportion of the contribution of this task to SAC in this unit.

The marks for each part are indicated in brackets. Answer in space provided or as directed.

You have 120 minutes over no more than two days. Work in progress will be collected by your teacher.

Each part is worth 30 marks as indicated.

You can access your logbook and an approved CAS calculator.

T Indicates where use of the technology is specifically required in order to answer the question.

Your teacher will advise you of any variation to these conditions.

*This task is an Item Response type. Each question is in the form of a multiple choice question. As well as determining the **correct** answer you must also provide a reason for why each of the **incorrect** answers is incorrect. Each question is thus worth five marks, one for each of the five possible responses.*

NAME:

Task

PART 1 **Name:**_____

You have 60 minutes for this section.

*Indicate if each option is correct OR incorrect **and** provide a reason for each judgement you make.*

Question 1

The function $f:[a,\infty)\rightarrow R$ where $f(x)=e^{(x-1)(x+3)}-1$, will have an inverse function if

A	a = -3	B	-3 <a < -1	C	a = -1	D	a < -1	E	a ∈R

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 2

If $f'(x) = 3f(x)$ and $f(0) = e$ then $f(x)$ could be:

A	e^{3x}
B	$e^{3x} + 1$
C	$e^{x/3}$
D	e^{3x+1}
E	e^{3x+3}

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 3

The equation of the tangent to the curve of $f(x) = (x-3)^{3/5} + 2$ at $x = 3$ is:

A	$y = 3$
B	$x = 3$
C	$\frac{3}{5}(x-3)^{-2/5}$
D	$\frac{3}{5}(x-3)^{2/5}$
E	undefined

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 4

Logarithms and exponentials are inverses of each other. Using this information determine which of the following is the same as $\int_0^a \ln(x+1)\ dx$:

A	$\int_0^a (e^x - 1)\ dx$
B	$\int_a^0 (e^x - 1)\ dx$
C	$\int_0^{\ln(a+1)} (e^x - 1)\ dx$
D	$a\ln(a+1) - \int_0^{\ln(a+1)} (e^x - 1)\ dx$
E	$a\ (\mathrm{e}^{\mathrm{a}} - 1) - \int_0^{\ln(a+1)} (e^x - 1)\ dx$

Working Space

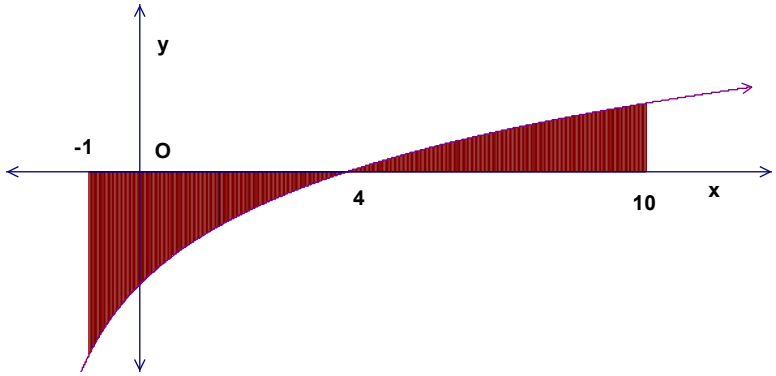
Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 5

The total area of the shaded regions in the diagram is given by

A	$\int_{-1}^{10} f(x)\,dx$
B	$\int_{-1}^{4} f(x)\,dx + \int_{4}^{10} f(x)\,dx$
C	$-\left(\int_{4}^{10} f(x)\,dx + \int_{-1}^{4} f(x)\,dx \right)$
D	$\int_{4}^{10} f(x)\,dx - \int_{1}^{4} f(x)\,dx$
E	$\int_{4}^{10} f(x)\,dx - \int_{-1}^{4} f(x)\,dx$



Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 6 **T**

The function $f(x) = |e^{x-2} - 3| - 2$ has two x-intercepts. *g(x)* and *h(x)* each have one x-intercept. Which combination of *g(x)* and *h(x)* has ***both*** of the intercepts as *f(x)*?

A	$g(x) = e^{x-2} + 1$ and $h(x) = -e^{x-2} + 1$
B	$g(x) = e^{x-2} - 5$ and $h(x) = -e^{x-2} + 1$
C	$g(x) = e^{x-2} - 5$ and $h(x) = e^{x-2} - 1$
D	$g(x) = -e^{x-2} - 5$ and $h(x) = -e^{x-2} + 1$
E	$g(x) = -e^{x+2} - 5$ and $h(x) = -e^{x+2} + 1$

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

PART 2 **Name:**_____

You have 60 minutes for this section.

*Indicate if each option is correct OR incorrect **and** provide a reason for each judgement you make.*

Question 7 **T**

The functions, $f(x) = \frac{x^2}{4}$ and $g(x) = 2x + 3$ intersect at two points. The x-values of these points of intersection can be found by solving:

A	$0 = x^2 - 2x - 3$
B	$0 = x^2 + 8x + 3$
C	$0 = x^2 - 8x + 12$
D	$0 = x^2 - 8x - 12$
E	$0 = x^2 - 6x - 12$

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 8

The functions $f(x) = x \ln(x^2)$ and $g(x) = 2x\, ln(x)$

A	Have the same derivative and the same stationary points
B	Have the same derivative but share only one stationary point
C	Have different derivatives but share the same stationary points
D	Have different derivatives but share one stationary point
E	Are identical to one another

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 9

A matrix equation that would produce a reflection in the y axis and a translation of 2 units in the positive x direction is:

A $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

B $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

C $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

D $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

E $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 10 **T**

The function $f(x) = \sin(\ln(x))$ has two stationary points between x = 0 and x = 2π. The sum of the y-values of these stationary points is closest to:

A	0
B	2
C	5.018
D	0.9645
E	undefined

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 11

Let $f(x)=x^3$ and $g(x)=\ln(x)$

Then *f(g(x))* is:

A	$[ln(x)]^3$
B	$ln(x^3)$
C	$x^3\ ln(x)$
D	$3\ ln(x)$
E	$\frac{3\ln(x^2)}{x}$

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Task

Question 12 **T**

The average value of the function $f(x) = e^{x-x^3} - 1$ between x = -1 and x = 1 is closest to:

A	0.07690
B	0.50838
C	0.03845
D	0.25419
E	undefined

Working Space

Option	Correct/Incorrect	Reason
A		
B		
C		
D		
E		

Teacher Advice

Assessment planning

This task covers assessment in:

- Matrix Transformations
- Inverses of Functions
- Exponent and Logarithm Laws
- Methods of differentiation and integration.

This task contributes 20 of the 40 SAC marks in Unit 4.
The coursework scores for this task are:

Outcome 1	8 marks	40%
Outcome 2	7 marks	35%
Outcome 3	5 marks	25%
TOTAL	20 marks	

This weighting can be used in the conversion of their mark out of 50.
For example, a score of 40 results in:

OUTCOME 1	OUTCOME 2	OUTCOME 3
40/60*20*0.4	40/60*20*0.35	40/60*20*0.25
= 5.333	= 4.6666	= 3.3333
= 5 (round down)	= 5 (rounded up)	= 3 (rounded down)

The above can be established in an Excel file.

Alternatively each mark can be assigned as in the following table.

Teacher Advice

Question	Outcome 1	Outcome 2	Outcome 3
1A			1
1B		1	
1C	1		
1D		1	
1E		1	
2A	1		
2B	1		
2C	1		
2D	1		
2E	1		
3A		1	
3B	1		
3C		1	
3D		1	
3E		1	
4A		1	
4B		1	
4C		1	
4D		1	
4E		1	
5A		1	
5B		1	
5C		1	
5D		1	
5E		1	
6A	1		
6B			1
6C	1		
6D			1
6E		1	
7A			1
7B	1		
7C		1	
7D	1		
7E			1

Teacher Advice

8A	1		
8B	1		
8C	1		
8D	1		
8E	1		
9A	1		
9B	1		
9C	1		
9D	1		
9E	1		
10A			1
10B			1
10C			1
10D			1
10E			1
11A		1	
11B	1		
11C	1		
11D	1		
11E		1	
12A			1
12B			1
12C			1
12D			1
12E			1

This task has been designed to meet the highest level in the performance descriptors provided by VCAA for each outcome in unit 3 in the Assessment Handbook.

Solution Pathway

Below are suggested responses. Teachers should consider the merits of alternative responses.

5 marks per question.

PART 1

Question 1

The function $f:[a,\infty)\rightarrow R$ where $f(x)=e^{(x-1)(x+3)}-1$, will have an inverse function if

A	a = -3	B	a <-3	C	a = -1	D	a < -1	E	a ∈R

Working Space

For *f(x)* to have an inverse function it must itself be a one to one function. Looking at the graph shows that *f(x)* has a single turning point.

Differentiating gives $f'(x) = (2x + 2)e^{(x-1)(x+3)}$

Solving for *f'(x)= 0* gives x = -1

Hence, for *f(x)* to have an inverse function, $a \geq -1$

The only option given that satisfies this restriction is C, where a = - 1.

Option	Correct/Incorrect	Reason
A	Incorrect	a = -3, is an x-intercept, not a stationary point. The function is not one to one over this domain.
B	Incorrect	-3 < a < -1, denotes values between the left x-intercept and the turning point. Although moving closer to the correct value of a, all values will still give a many to one function.
C	Correct	See working above.
D	Incorrect	Here the correct a value is quoted but the inequality sign denotes that the domain begins to the left of the turning point, not exactly on it. This in turn means that the function is not one to one.
E	Incorrect	a ∈R means that a can take any value. While this means that the function will sometimes be one to one, it is not always the case. The question requires values of a that will all satisfy the condition.

Solution Pathway

Question 2

If $f'(x) = 3f(x)$ and $f(0) = e$ then $f(x)$ could be:

A	e^{3x}
B	$e^{3x} + 1$
C	$e^{x/3}$
D	e^{3x+1}
E	e^{3x+3}

Working Space

Differentiate and substitute x = 0 into each option.

Option	Correct/Incorrect	Reason
A	Incorrect	$f'(x) = 3e^{3x} = 3f(x)$ $f(0) = 1$. This condition is not correct.
B	Incorrect	$f'(x) = 3e^{3x} = 3f(x)$. This condition is not correct. $3f(x) = 3(e^{3x} + 1)$ $f(0) = 2$. This condition is not correct.
C	Incorrect	$f'(x) = \frac{e^{x/3}}{3} \neq 3f(x)$. This condition is not correct.
D	Correct	$f'(x) = 3e^{3x+1} = 3f(x)$ $f(0) = e$. Both conditions correct.
E	Incorrect	$f'(x) = 3e^{3x+3} = 3f(x)$ $f(0) = e^3$. This condition is not correct.

Solution Pathway

Question 3

The equation of the tangent to the curve of $f(x) = (x-3)^{3/5} + 2$ at x = 3 is:

A	$y = 3$
B	$x = 3$
C	$\frac{3}{5}(x-3)^{-2/5}$
D	$\frac{3}{5}(x-3)^{2/5}$
E	undefined

Working Space

Differentiating gives $f'(x) = \frac{3(x-3)^{-\frac{2}{5}}}{5} = \frac{3}{5\,(x-3)^{2/5}}$

The gradient at $x = 3$ then is undefined. This means that the tangent is a vertical line.

Hence the equation of the tangent is x = 3

Correct option is B

Option	Correct/Incorrect	Reason
A	Incorrect	This is a horizontal line, not a vertical line.
B	Correct	See above.
C	Incorrect	This is the equation of the derivative, not of the tangent.
D	Incorrect	This is an attempt to find the derivative, but the sign of the power is incorrect.
E	Incorrect	The gradient of the tangent is undefined but the equation of the tangent can be defined since it is a vertical line.

Solution Pathway

Question 4

$\int_0^a \ln(x+1)\ dx$ is the same as:

A	$\int_0^a (e^x - 1)\ dx$
B	$\int_a^0 (e^x - 1)\ dx$
C	$\int_0^{\ln(a+1)} (e^x - 1)\ dx$
D	$a\ln(a+1) - \int_0^{\ln(a+1)} (e^x - 1)\ dx$
E	$a\ (e^a - 1) - \int_0^{\ln(a+1)} (e^x - 1)\ dx$

Working Space

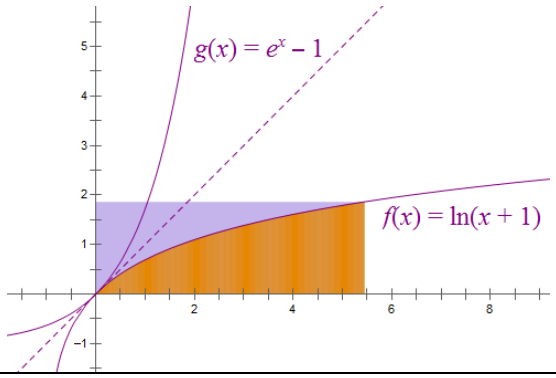
Determining the integral of a logarithmic function directly is beyond the scope of this course, but by using the properties of inverses it can be found indirectly.

$\int_0^a \ln(x+1)\ dx = Area\ of\ rectangle - area\ of\ inverse\ funtion$

The rectangle in question is $a\ ln(a+1)$

The inverse of f(x) is $e^x - 1$

Hence the correct option is D.



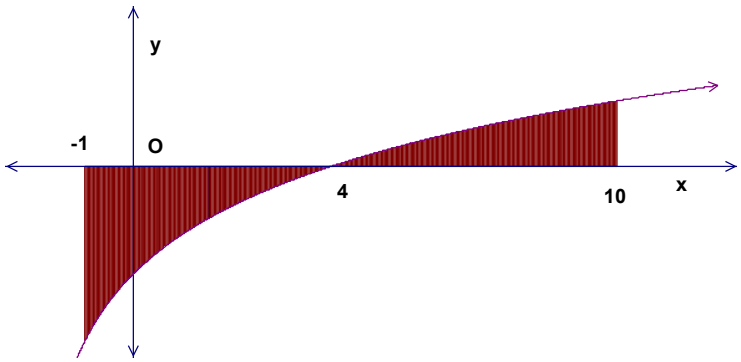
Option	Correct/Incorrect	Reason
A	Incorrect	This is the inverse function but over the wrong domain. It also does not account for the area of the rectangle.
B	Incorrect	In addition to not accounting for the rectangle, this option also swaps the terminals around unnecessarily.
C	Incorrect	This is just the area of the inverse. It does not account for the area of the rectangle.
D	Correct	See above.
E	Incorrect	The area of the rectangle is incorrectly calculated.

Solution Pathway

Question 5

The total area of the shaded regions in the diagram is given by

A	$\int_{-1}^{10} f(x)\,dx$
B	$\int_{-1}^{4} f(x)\,dx + \int_{4}^{10} f(x)\,dx$
C	$-\left(\int_{4}^{10} f(x)\,dx + \int_{-1}^{4} f(x)\,dx \right)$
D	$\int_{4}^{10} f(x)\,dx - \int_{1}^{4} f(x)\,dx$
E	$\int_{4}^{10} f(x)\,dx - \int_{-1}^{4} f(x)\,dx$



Working Space

The given area has two regions one below and the other above the x-axis. The region below must be reversed in sign before the two are added together to give the complete area.

Only option E accounts for this correctly.

Option	Correct/Incorrect	Reason
A	Incorrect	This evaluates the integral rather than finding the area.
B	Incorrect	This correctly splits the integral into two regions but fails to correct for the negative answer in the first region.
C	Incorrect	This changes the sign of the entire evaluation, rather than just the first region.
D	Incorrect	The lower bound terminal on the second integral has the incorrect sign.
E	Correct	See above.

Solution Pathway

Question 6

The function $f(x) = |e^{x-2} - 3| - 2$ has two x-intercepts. *f(x)* shares one intercept each with the functions:

A	$g(x) = e^{x-2} + 1$ and $h(x) = -e^{x-2} + 1$
B	$g(x) = e^{x-2} - 5$ and $h(x) = -e^{x-2} + 1$
C	$g(x) = e^{x-2} - 5$ and $h(x) = e^{x-2} - 1$
D	$g(x) = -e^{x-2} - 5$ and $h(x) = -e^{x-2} + 1$
E	$g(x) = -e^{x+2} - 5$ and $h(x) = -e^{x+2} + 1$

Working Space

$f(x) = |e^{x-2} - 3| - 2$ is the same as

$$f(x) = \begin{cases} e^{x-2} - 3 - 2 & x > 3.0986 \\ -(e^{x-2} - 3) - 2 & x < 3.0986 \end{cases}$$

This simplifies to $f(x) = \begin{cases} e^{x-2} - 5 & x > 3.0986 \\ -e^{x-2} + 1 & x < 3.0986 \end{cases}$

Hence the answer is B.

Option	Correct/Incorrect	Reason
A	Incorrect	Both functions have the same constant, +1. *g(x)* is incorrect.
B	Correct	See above.
C	Incorrect	*h(x)* is incorrect. The negative sign was not placed in front of the expression when the modulus was removed.
D	Incorrect	*g(x)* is incorrect. A negative sign has been placed in front of both expressions without reason.
E	Incorrect	*h(x)* is incorrect. The power has a +2 not a -2. Signs in powers are not affected in modulii.

Solution Pathway

PART 2

Question 7

The functions, $f(x) = \frac{x^2}{4}$ and $g(x) = 2x + 3$ intersect at two points. The x-values of these points of intersection can be found by solving:

A	$0 = x^2 - 2x - 3$
B	$0 = x^2 + 8x + 3$
C	$0 = x^2 - 8x + 12$
D	$0 = x^2 - 8x - 12$
E	$0 = x^2 - 6x - 12$

Working Space

Let $2x + 3 = \frac{x^2}{4}$

Then $8x + 12 = x^2$

Rearranging gives $0 = x^2 - 8x - 12$. This is D.

Option	Correct/Incorrect	Reason
A	Incorrect	The 4 has not been multiplied through correctly.
B	Incorrect	The signs are wrong for the second and third terms and the 3 has not been multiplied by 4.
C	Incorrect	The sign of the constant term is incorrect.
D	Correct	See above.
E	Incorrect	The coefficient of the middle term is incorrect. Perhaps 2 and 4 were added rather than multiplied together.

Solution Pathway

Question 8

The functions $f(x) = x \ln(x^2)$ and $g(x) = 2x\, ln(x)$

A	Have the same derivative and the same stationary points
B	Have the same derivative but share only one stationary point
C	Have different derivatives but share the same stationary points
D	Have different derivatives but share one stationary point
E	Are identical to one another

Working Space

Differentiating both functions shows that they do have the same derivative.

The two functions are algebraically equivalent but the use of x^2 in f(x) means that it is defined over all $x \in$ R\{0} whereas g(x) is only defined for R+.

This in turn means that f(x) has two stationary points while g(x) has only one.

Answer is B.

Option	Correct/Incorrect	Reason
A	Incorrect	Since g(x) is restricted the functions cannot share both stationary points
B	Correct	See above.
C	Incorrect	The derivatives are the same and there is only one common stationary point
D	Incorrect	The derivatives are the same.
E	Incorrect	The use of x^2 and x in the logarithmic term mean the functions are not the same.

Solution Pathway

Question 9

Matrix Transformation

A matrix equation that would produce a reflection in the y axis and a translation of 2 units in the positive x direction is:

A $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

B $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

C $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

D $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

E $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

Working Space

A reflection in the y axis means the sign of x changes. This mean -1 in the top left position of the 2 x 2 matrix.

A translation of 2 in the positive x-direction means 2 in the top position of the 2 x 1 matrix.

This gives E as the correct option.

Option	Correct/Incorrect	Reason
A	Incorrect	The translation is to the left not to the right.
B	Incorrect	The reflection is in the x-axis, not the y-axis.
C	Incorrect	The translation is in the positive y-direction.
D	Incorrect	The translation is in the negative y-direction.
E	Correct	See above.

Solution Pathway

Question 10

The function $f(x) = \sin(\ln(x))$ has two stationary points between x = 0 and x = 2π. The sum of the y-values of these stationary points is closest to:

A	0
B	2
C	5.018
D	0.9645
E	undefined

Working Space

The y-values of the function are dependent only on the sin( ) part of the function. From the graph, or otherwise, one of these is +1 and the other is -1. It is not necessary to determine the x-values of the turning points.

The sum of the y-values is then equal to zero.

The correct option is A.

Option	Correct/Incorrect	Reason
A	Correct	See above.
B	Incorrect	The sign of the left turning point was ignored, or the values were subtracted from one another.
C	Incorrect	This is the sum of the x-values of the turning points.
D	Incorrect	This is the value of $f(2\pi)$.
E	Incorrect	This is the value of $f(0)$.

Solution Pathway

Question 11

Let $f(x)=x^3$ and $g(x)=\ln(x)$

Then *f(g(x))* is:

A	$[ln(x)]^3$
B	$ln(x^3)$
C	$x^3\ ln(x)$
D	$3\ ln(x)$
E	$\frac{3\ln(x^2)}{x}$

Working Space

Insert *ln(x)* in place of *x* in x^3 to give $(ln(x))^3$. This is the same as A.

Option	Correct/Incorrect	Reason
A	Correct	See above.
B	Incorrect	The power is inside the logarithm not outside.
C	Incorrect	This is the product of the functions not the nested function.
D	Incorrect	The power was placed inside the logarithm not outside and then the power rule was applied.
E	incorrect	This is the derivative of the nested function.

Solution Pathway

Question 12

The average value of the function $f(x) = e^{x-x^3} - 1$ between x = -1 and x = 1 is closest to:

A	0.07690
B	0.50838
C	0.03845
D	0.25419
E	undefined

Working Space

Average value is $\frac{1}{b-a}\int_a^b e^{x-x^3} - 1\,dx = \frac{1}{1--1}\int_{-1}^1 e^{x-x^3} - 1\,dx$

Using the calculator gives 0.03845. This is C.

Option	Correct/Incorrect	Reason
A	Incorrect	This is the integral, not the average value
B	Incorrect	This is the area between x = -1 and x = 1. $\int_{-1}^1 e^{x-x^3} - 1\,dx = 0.50838$
C	Correct	$\frac{1}{1--1}\int_{-1}^1 e^{x-x^3} - 1\,dx = \frac{1}{2}\int_{-1}^1 e^{x-x^3} - 1\,dx = 0.03845$
D	Incorrect	This is the area divided by b – a. Area = 0.50838. $\frac{0.50838}{2} = 0.2519$
E	Incorrect	This is arrived at only if the divisor is zero. This in turn occurs if the sign of a is entered incorrectly.