SACRED HEART GIRLS' COLLEGE
OAKLEIGH

Mathematical Methods CAS 2013

Unit 3 SAC 2: TEST
Part A

Name: SOLUTIONS

Teacher (please circle): Ms Gates Mr Smith Mrs Mak

No CAS and no summary notes permitted

Part A: 5 short answer questions

Writing Time: 20 minutes

Marks: 14

SHORT ANSWER QUESTIONS

Instructions:
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this test are **not** drawn to scale.

Question 1

Solve the equation $9^x - 6\left(3^x\right) = 27$ for all x.

$(3^2)^x - 6(3^x) - 27 = 0$

$(3^x)^2 - 6(3^x) - 27 = 0$

Let $a = 3^x$

$a^2 - 6a - 27 = 0$

$(a-9)(a+3) = 0$

$3^x = 9$ or $3^x = -3$

$x = 2$ no solution

$\therefore x = 2$

3 marks

Question 2

Solve the equation $3\log_2(2x) - 2\log_2(x) = 1$

$\log_2 \frac{(2x)^3}{x^2} = 1$

$2^1 = \frac{8x^3}{x^2}$

$2x^2 = 8x^3$

$0 = 8x^3 - 2x^2$

$0 = 2x^2(4x - 1)$

$x = 0$ or $x = \frac{1}{4}$

but $x > 0$

$\therefore x = \frac{1}{4}$

2 marks

Question 3

The unhealed area, A cm, of a particular wound, t days after it was sustained, is given by the function $A(t) = 8e^{\frac{-t}{10}}, t \geq 0$

According to this model, what is the time required for the area of unhealed wound to be halved?

$4 = 8e^{-\frac{t}{10}}$

$\frac{1}{2} = e^{-\frac{t}{10}}$

$\log_e\left(\frac{1}{2}\right) = -\frac{t}{10}$

$t = -10\log_e\left(\frac{1}{2}\right)$

3 marks

Question 4

Show that $2\log_a 2x + \log_a 4x - 5\log_a x = -2\log_a\left(\frac{x}{4}\right)$

L.H.S. $= \log_a \frac{(2x)^2 \times 4x}{x^5}$

$= \log_a \left(\frac{16x^3}{x^5}\right)$

$= \log_a \left(\frac{16}{x^2}\right)$

$= \log_a \left(\frac{x}{4}\right)^{-2}$

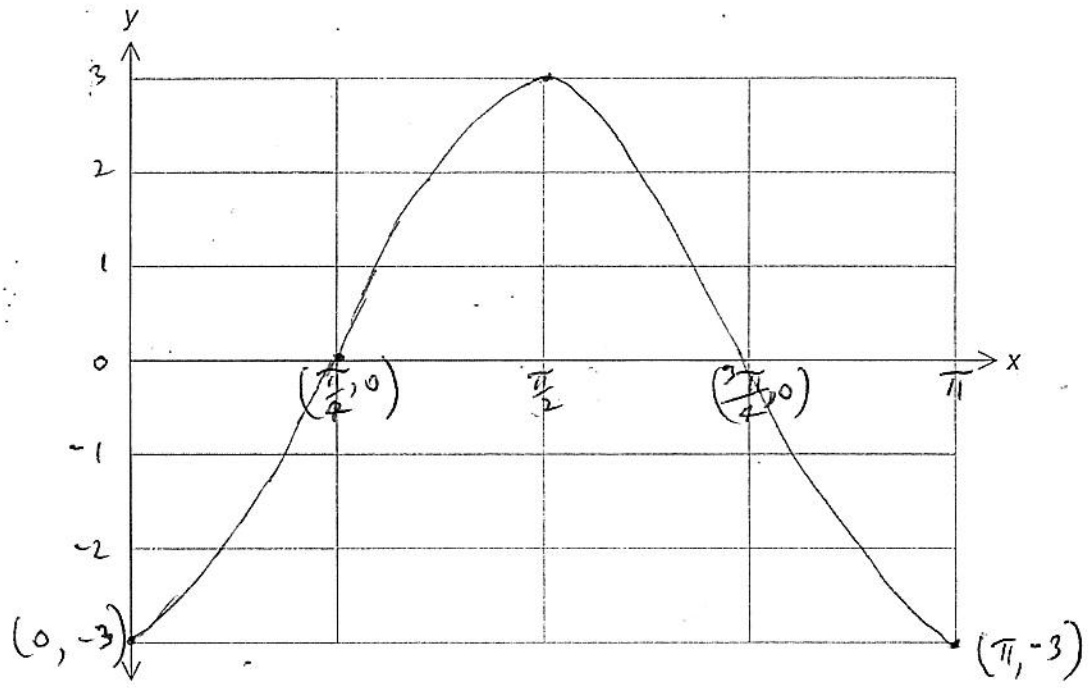
2 marks

$= -2\log_a\left(\frac{x}{4}\right)$

Question 5

For the function $f:[0,\pi] \to R, f(x) = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$

Sketch the graph of the function f on the set of axes below. Find the amplitude and period and label axes intercepts as well as endpoints with their coordinates



AMPLITUDE = 3

PERIOD = $\frac{2\pi}{2}$

= π

4 marks

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Mathematical Methods CAS 2013

Unit 3 SAC 2: TEST
Part B

Name: SOLUTIONS

Teacher (please circle): Ms Gates Mr Smith Mrs Mak

Part B: 5 multiple choice questions and 2 extended response questions.

CAS and a bound reference of summary notes permitted

Writing Time: 30 minutes
Marks: 20

MULTIPLE CHOICE

Instructions:
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for that question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Only the answers on the Answer Sheet will be marked.

Question 1

The expression $9^{-\frac{3}{2}}+(8x)^{-\frac{1}{3}}$ equals

A. $27-\dfrac{1}{2\sqrt[3]{x}}$

B. $\dfrac{1}{27}+\dfrac{2}{x^3}$

C. $-27+\dfrac{1}{2\sqrt[3]{x}}$

D. $-27+\dfrac{2}{2\sqrt[3]{x}}$

E. $\dfrac{1}{27}+\dfrac{1}{2\sqrt[3]{x}}$

$\frac{1}{27} + \frac{1}{8^{\frac{1}{3}} \times x^{\frac{1}{3}}}$

$= \frac{1}{27} + \frac{1}{2\sqrt[3]{x}}$

Question 2

The range of the function $f(x)=\log_e(x^2-2x+4)$ is

A. $[1,\infty)$

B. $(\log_e 5,\infty)$

C. $[\log_e 3,\infty)$

D. $[\log_e 5,\infty)$

E. $(\log_e 3,\infty)$

MINIMUM VALUE OF x^2-2x+4

MIN. AT $x = \frac{-b}{2a}$

$= \frac{-(-2)}{2}$

$= 1$

where $(1)^2-2(1)+4 = 3$

so range of $f(x)$ is $[\log_e 3, \infty)$

Question 3

A certain radioactive isotope is decaying exponentially so that its mass is given by $M = M_0 e^{-kt}$ and has a half life of two seconds (i.e. after 2 seconds only half of the initial mass remains).
The approximate time, in seconds, it will take to lose 80% of its original mass is closest to

A. 0.6
B. 4.0
C. 4.3
D. 4.6
E. 8.1

$\frac{1}{2}M_0 = M_0 e^{-2k}$

$\frac{1}{2} = e^{-2k}$

$k = \frac{1}{2}\log_e 2$

loses 80% of original mass

$0.2M_0 = M_0 e^{\left(-\frac{1}{2}\log_e 2\right)t}$

$t = 4.64$

Question 4

The period and amplitude of the function $f(x) = -3\sin\left(\frac{2(x-\pi)}{3}\right)$ respectively are

A. -3 and $\frac{2}{3}$
B. 3π and 3
C. -3 and 3π
D. -3 and $\frac{2\pi}{3}$
E. π and 3

$\text{PERIOD} = \frac{2\pi}{\frac{2}{3}}$

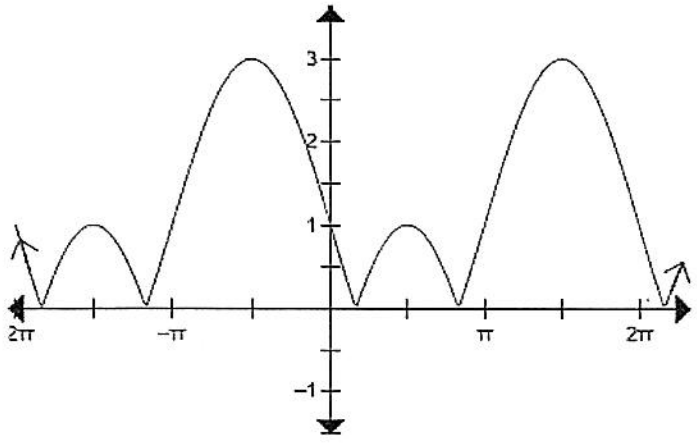
$= 2\pi \times \frac{3}{2}$

$= 3\pi$

AMPLITUDE $= 3$

Question 5

The equation of the graph shown could be



A. $f(x) = |2\sin(x)+1|$
B. $f(x) = 2\sin(x)-1$
C. $f(x) = 2\sin(x)+1$
D. $f(x) = |2\sin(x)-1|$
E. $f(x) = 2|\sin(x)|-1$

obviously a modulus function
so not B or C.

graph A then D in calc.

EXTENDED RESPONSE

Instructions:
Answer **all** questions in the spaces provided.
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Question 1

A study is being conducted on the population of a particular species of possum.
The population, P, at time t months after the study started is assumed to follow the model

$$P(t) = \frac{3000ke^{0.25t}}{1+ke^{0.25t}}$$

a. Given that there are 750 possums when the study started, show that $k = \frac{1}{3}$.

when $t=0$, $P=750$

$750 = \frac{3000k}{1+k}$ 1

$750 + 750k = 3000k$

$750 = 2250k$

$k = \frac{750}{2250}$ 1

$k = \frac{1}{3}$

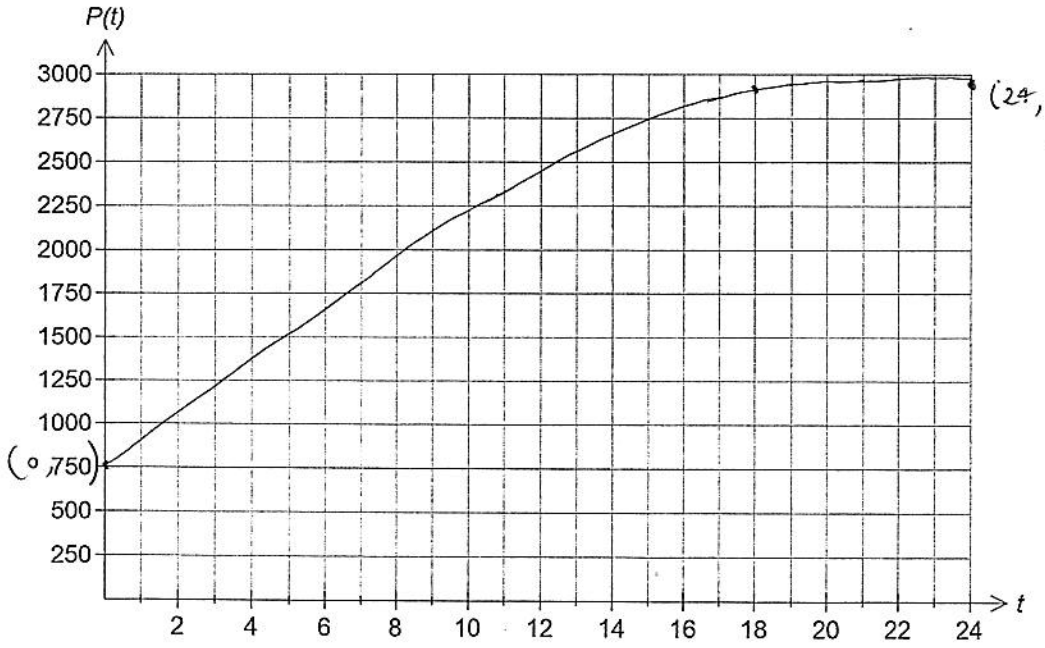
2 marks

b. Determine the possum population after 18 months to the nearest whole number.

$P(18) = 2903$

1 mark

c. Sketch the graph of the possum population over the first two years. Label the endpoints with coordinates.



↑ or accept (24, 2977.8558...) or (24, 2978)

1 for endpoints
1 for curve

2 marks

d. In which month will the possum population have increased by 150%?

$2.5 \times 750 = 1875$

$1875 = \dfrac{3000 \times \frac{1}{3} e^{0.25t}}{1 + \frac{e^{0.25t}}{3}}$ 1

$t = 6.44$

DURING 7TH. MONTH 1

2 marks

Question 2

One of the largest earthquakes of the twentieth century was in Japan on March 2, 1933. It measured 8.9 on the Richter scale. This scale is a logarithmic scale and the relationship between the magnitude, R, of an earthquake and the amplitude, a, of the seismic wave is given by the formula:

$$R = log_s\left(\frac{a}{T}\right) + 4.24 \text{ where}$$

a = the amplitude of the seismic wave (vertical motion relative to normal ground level) in microns
T = the period of the seisic wave in seconds

The San Francisco earthquake in 1906 had magnitude of 8.3 on the Richter scale.

a. If the period of this wave was 2.5 seconds find the amplitude to the nearest micron.

145 microns

1 mark

Another earthquake has a seismic wave which is given by the formula

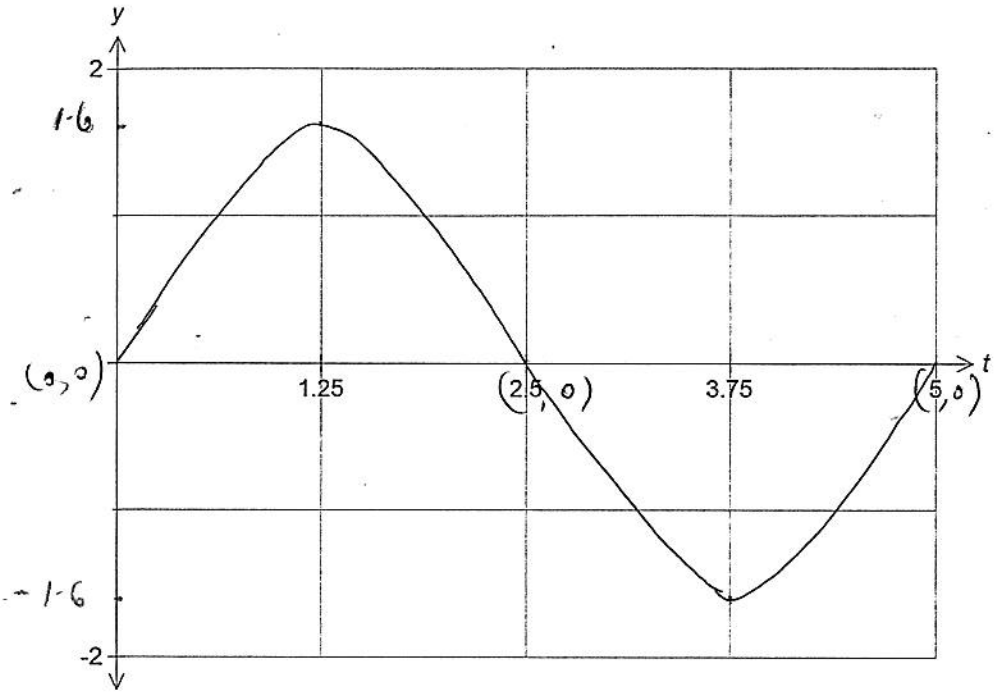
$$y = 1.6sin\left(\frac{2\pi t}{5}\right) \text{ where}$$

y = the vertical motion relative to normal ground level of the wave measured in cm

t = the time, in seconds, since the earthquake started.

b. Sketch the graph of $y = 1.6sin\left(\frac{2\pi t}{5}\right)$ for one period. Label endpoints and intercepts as coordinates.

PERIOD $= \frac{2\pi}{\frac{2\pi}{5}} = 5$ sec



1 for endpoints
1 for x-intercept
1 for correct curve

3 marks

A "*critical time*" is when the ground rises or falls by more than 1 cm.

c. i Find, correct to two decimal places, between what times the ground first rises and first falls by more than1 cm.

$1 = 1.6 \sin\left(\frac{2\pi t}{5}\right)$ given $0 < t < 2.5$

between $t = 0.54$ s. or $t = 1.96$ s.

2 marks

ii Find the proportion of time during the first period that is "*critical time*".

$\frac{1.42}{5} = 0.284$ or $\frac{71}{250}$ or 28.4%.

1 mark

iii If the wave continues for 12.5 seconds, find the proportion of time that is "*critical time*".

1 mark

END OF SAC