Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Technology-free Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 10 pages, including a formula sheet on the last page.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

a. If $y = x \, ln(x)$, find $\frac{dy}{dx}$.

1 mark

b. If $y = (x^3 - 2x)^2$, find $\frac{dy}{dx}$ at $x = 2$.

2 marks

Total: 3 marks

Question 2
Find an antiderivative of $\cos(-2x + 3)$.

2 marks

Question 3
Given $g(x) = 2e^{3x} + 1$:

a.   Find the inverse of g.

2 marks

b.   Find $g(g^{-1}(x))$.

2 marks

Total: 4 marks

Question 4

The number of phone calls, X, that Bob receives at home in a day is a random variable with probability distribution given by:

x	0	1	2	3
$\Pr(X = x)$	0.25	0.1	0.25	0.4

a.   Find the mean of X.

_____

_____

_____

_____

2 marks

b.   What is the probability that Bob receives two calls three days in a row?

_____

_____

_____

_____

1 mark

c.   What is the probability that Bob receives three calls over a two day span?

_____

_____

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_____

3 marks

Total: 6 marks

Question 5
$f(x) = |-x^2 - 4x - 3|, -3 \leq x \leq 3$

a.   Sketch the graph of $f(x)$ (all important points except for turning points should be labelled).

3 marks

b.   Find the coordinates of the image of the point $(0, -3)$ under a reflection in the y-axis and a translation of two units in the positive y-direction.

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1 mark

c.   Find the equation of the image of $f(x)$ after this series of transformations (a reflection in the y-axis and a translation of two units in the positive y-direction) has been applied to it.

2 marks

Total: 6 marks

Question 6

Find the solutions of $2\sin(2x - \frac{\pi}{2}) + \sqrt{2} = 0$ for $x \in [0, 2\pi]$.

3 marks

Question 7

Solve $\ln(x - 3) - 2\ln(x + 1) + \ln(2x - 1) = 0$, where $x > 1$.

3 marks

Question 8

a.   The random variable X is normally distributed with mean 150 and standard deviation 11. If $\Pr(X < 139) = q$, find $\Pr(X > 161 \,|\, X > 150)$ in terms of q.

2 marks

b.   The probability density function f of the random variable X is given by $f(x) = \frac{x-2}{8}, 2 \leq x \leq 6$. Find k such that $\Pr(X \leq k) = \frac{1}{2}$.

3 marks

Total: 5 marks

Question 9

a. $f(x) = x^2 \ln x$. Find $f'(x)$.

1 mark

b. Use your answer from part a to find $\int_1^e x \ln(x) \ dx$.

3 marks

Total: 4 marks

Question 10
Given $f(x) = x^3 - kx$, $g(x) = kx$, show that the area bounded by f and g is four times the size of the area bounded by g and the x-axis.

4 marks

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule $\quad \dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule $\quad \dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\quad \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

approximation $\quad f(x+h) = f(x) + hf'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

transition matrices $\quad S_n = T^n \times S_0$

mean $\quad \mu = E(X)$

variance $\quad var(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

	probability distribution	mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$