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Mathematical Methods(CAS)

2014

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1 The rule of the inverse of $f:[-1,1)\to R, f(x)=\frac{x}{x-1}$ is

A. $f^{-1}(x)=\frac{x}{x-1}$

B. $f^{-1}(x)=\frac{x-1}{x}$

C. $f^{-1}(x)=\frac{x}{x+1}$

D. $f^{-1}(x)=\frac{x-1}{x+1}$

E. $f^{-1}(x)=\frac{x+1}{x-1}$

Question 2 The equation $\log_a(1-x)=1-\log_a x$ has two distinct solutions in x if

A. $a=0.5$

B. $0<a\le 0.25$

C. $a\le 0.2$

D. $0<a\le 0.2$

E. $0<a<0.5$

Question 3 Given $a,b\in R^+$, the maximal domain of $f(x)=\sqrt{\frac{x+a}{x-b}}$ is

A. $[-a,b)$

B. $(-a,b]$

C. $[b,\infty)$

D. $R\setminus[-a,b)$

E. $(-\infty,-a]\cup(b,\infty)$

Question 4 The range of $f(x)=\dfrac{x^2-a}{x+\sqrt{a}}$ is

A. $R\setminus\{a\}$

B. $R\setminus\{-\sqrt{a}\}$

C. $R\setminus\{2a\}$

D. $R\setminus\{-2\sqrt{a}\}$

E. R

Question 5 $y=mx-2m$ intersects $y=x^3-6x^2+8x$ once only when

A. $m<-3$

B. $m>-5$

C. $m<-2$

D. $m>-6$

E. $m<-4$

Question 6 Given $f(1-x)=-f(x-1)$, a possible $f(x)$ is

A. $(x-1)^2$

B. $(x+1)^2$

C. $x(x^2-1)$

D. $(x+1)^3$

E. $x^2(x^2-1)$

Question 7 If $\cos A+\sin B=0$, $\pi<A<\dfrac{3\pi}{2}$ and $\dfrac{\pi}{2}<B<\pi$, then

A. $B=A-\dfrac{\pi}{2}$

B. $B=A-\dfrac{3\pi}{4}$

C. $A=B+\dfrac{\pi}{4}$

D. $A=B+\dfrac{\pi}{3}$

E. $A+B=2\pi$

Question 8 The equation $c-9x+6x^2-x^3=0$ has exactly three positive solutions in x if

A. $0<c<5$

B. $0\le c<4$

C. $0\le c\le 4$

D. $0<c<4$

E. $c<4$

Question 9 The graph of $y=(1-x)^3+x-1$ is translated in the positive x direction by 1 unit and then reflected in the y-axis. The equation of the resulting graph is

A. $y=(x-1)(x-2)(x-3)$

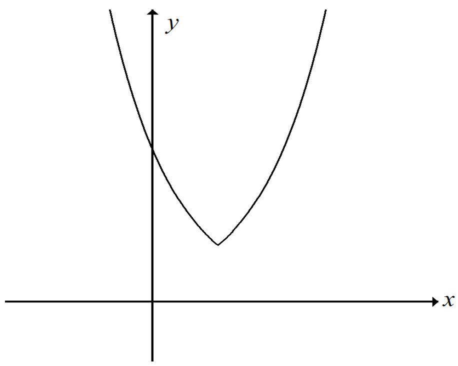
B. $y=(x+1)(x+2)(x+3)$

C. $y=(3-x)(2-x)(1-x)$

D. $y=-(x+1)(x+2)(x+3)$

E. $y=(x+1)(x-2)(x+3)$

Question 10 The following graph is a composite function of $f(x)=|x|$ and $g(x)$. $g(x)$ is differentiable everywhere and it is strictly increasing.



Given $a\in R$, a possible equation of the graph is

A. $y=|g(x)|+a$

B. $y=g(|x|)+a$

C. $y=|g(x-a)|$

D. $y=g(|x+a|)$

E. $y=|g(x)+a|$

Question 11 Consider hybrid function $f(x)=\begin{cases} 2x-1, & x<a \\ ax^2+bx, & x \ge a \end{cases}$, where $a, b \in R$.

If $f(x)$ is differentiable at $x=a$, then

A. $a=b$

B. $a>b$

C. $a<b$

D. $a=1$

E. $0 \le a \le 1$

Question 12 Consider the graph of a polynomial function of degree 4.
Which one of the following statements is ***false***?

A. The graph has at least 1 inflection point.

B. The graph may have 1, 2 or 3 stationary points.

C. The graph may have 0, 1, 2, 3 or 4 *x*-intercepts.

D. The graph does not display any asymptotic behaviour.

E. The graph has a maximal domain of *R*, the set of real numbers.

Question 13 Consider $f:[-1, 3) \to R, f(x)=2\sin\left(\frac{\pi x}{2}\right)$ and $g:[1, 2) \to R, g(x)=\sqrt{x-1}$.

The maximal domain of $g \circ f$ is

A. $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

B. $\left[\frac{1}{3}, \frac{5}{3}\right]$

C. $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

D. $\left[\frac{1}{3}, 1\right) \cup \left(1, \frac{5}{3}\right]$

E. $\left[\frac{\pi}{3}, \pi\right) \cup \left(\pi, \frac{5\pi}{3}\right]$

Question 14 Function $f(x)$ has the property $f(-x)=-f(x)$ for $x\in R$.

The value of definite integral $\int_0^2 f(1-x)\,dx$ is

A. 2

B. 1

C. 0

D. -1

E. -2

Question 15 Using $f(x+h)\approx f(x)+h f'(x)$ appropriately, the approximation of $\sqrt{50}$ is

A. $\frac{99}{14}$

B. 7.0711

C. $\frac{140+\sqrt{2}}{20}$

D. $5\sqrt{2}$

E. $\frac{1+70\sqrt{2}}{10\sqrt{2}}$

Question 16 A system of simultaneous linear equations expressed as a matrix equation is shown below.

$$\begin{bmatrix} 2 & 0 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ -1 & -1 & 2 & 1 \\ -2 & 2 & -2 & -2 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -2 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Which one of the following statements is ***true***?

A. The square matrix has a non-zero determinant.

B. The system of simultaneous linear equations has no solution.

C. The discriminant of the system of simultaneous linear equations equals zero.

D. The square matrix has an inverse.

E. The 4×4 matrix is a regular matrix.

Question 17 $\int_{-a}^{a} \cos^2 \theta \, d\theta =$

A. $a + \sin a \cos a$

B. $a - \sin a \cos a$

C. $\cos(2a)$

D. $a - \cos a$

E. a

Question 18 X is a discrete random variable and its probability has a binomial distribution. The mean of X is 9 and the variance is 6.
$\Pr(X > 12)$ is closest to

A. 0.079

B. 0.036

C. 0.014

D. 0.005

E. 0.995

Question 19 Suppose the probability that you are late to school if you are late the school day before is 0.15, and the probability that you are on time if you are on time the school day before is 0.90.
If you have forgotten whether you were late on your first school day, in the long run the probability that you are on time is

A. $\frac{6}{7}$

B. $\frac{7}{8}$

C. $\frac{9}{10}$

D. $\frac{17}{19}$

E. $\frac{18}{19}$

Question 20 The probability density function of continuous random variable X is given by

$$f(x)=\begin{cases} p, & -4\le x\le 0 \\ 4p, & 1\le x\le 9p \\ 0, & \text{elsewhere} \end{cases}\text{, where } p \text{ is a real constant.}$$

The value of p is

A. $\frac{1}{4}$

B. $\frac{1}{5}$

C. $\frac{1}{6}$

D. $\frac{1}{7}$

E. $\frac{1}{8}$

Question 21 If $\Pr(A\cap B)=\frac{1}{2}$, then $\Pr(B'\mid A)=$

A. $\frac{1}{\Pr(B')\Pr(A)}$

B. $1-\frac{1}{2\Pr(B)}$

C. $1-\frac{1}{2\Pr(A)}$

D. $1-\frac{1}{2\Pr(B')}$

E. $1-\frac{1}{2\Pr(A')}$

Question 22 If $2^x=5^y=100^z$, then

A. $z=\frac{xy}{2(x+y)}$

B. $z=\frac{x+y}{2xy}$

C. $z=\frac{2xy}{x+y}$

D. $z=\frac{x-y}{x+y}$

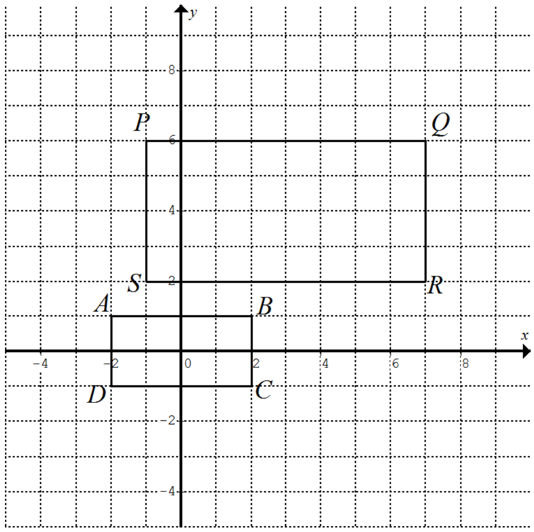
E. $z=\frac{x+y}{x-y}$

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

The following rectangles are similar.



a. Describe two transformations which will centre rectangle $PQRS$ at the origin. 1 mark

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Let the new rectangle centred at the origin in part a be $WXYZ$. Line segments WX and ZY are parallel to the x-axis and equal in length. Vertices W, X, Y and Z correspond to vertices P, Q, R and S respectively.

b i. Write down the equation for segment WX including its domain. 1 mark

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b ii. Write down the equation for segment WZ including its range. 1 mark

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c. Use dilations to explain why the rectangle $WXYZ$ and rectangle $ABCD$ are similar. 2 marks

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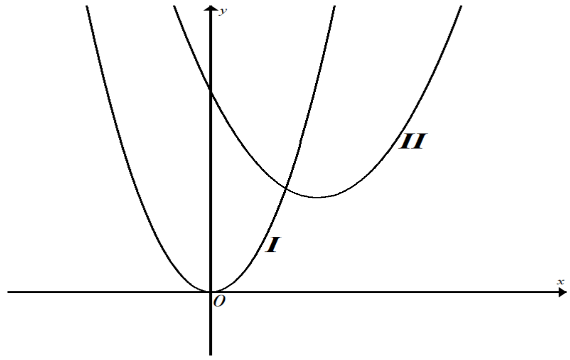
The following parabolas, ***I*** and ***II***, are ***similar***.

Equation of parabola ***I***: $y = x^2$

Equation of parabola ***II***: $2y = x^2 - 6x + 17$



d. Describe fully two transformations of parabolas ***II*** which will shift its vertex to the origin. 2 marks

e. Write down the equation of the new parabola after the transformations described in part d. 1 mark

f. Explain with calculations that parabolas ***I*** and ***II*** are similar. 2 marks

g. *All parabolas are similar*. Explain whether this statement is true or false. 3 marks

Question 2 The concentration of a drug in the blood t units of time after injection is given by

$$y=\frac{1}{a-b}\left(e^{-bt}-e^{-at}\right)$$

where a and b are positive integers, and $a>b$.

a. Show that $y=\frac{2^a-2^b}{(a-b)2^{a+b}}$ when $t=\log_e 2$. 2 marks

The graph of $y=\frac{1}{a-b}\left(e^{-bt}-e^{-at}\right)$ passes through point $P\left(1,\frac{e-1}{e^2}\right)$ and point $Q\left(2,\frac{e^2-1}{e^4}\right)$.

b. Write down two simultaneous equations that can be used to find the parameters a and b. 2 marks

c. Solve the two simultaneous equations in part b for the parameters a and b without using CAS.
2 marks

d i. Find the derivative of $\frac{1}{a-b}\left(e^{-bt}-e^{-at}\right)$ with respect to t in terms of a and b. 1 mark

d ii. Hence find t in terms of a and b when the concentration of the drug in the blood is maximum.
2 marks

d iii. Find the maximum concentration of the drug in the blood if $a=2$ and $b=1$. 2 marks

Use $a=2$ and $b=1$ in the following parts.

e. Sketch the graph of $y=\frac{1}{a-b}\left(e^{-bt}-e^{-at}\right)$, showing and labeling the important features of the graph. Make full use of the grid below and show graph up to $t=5$.
2 marks

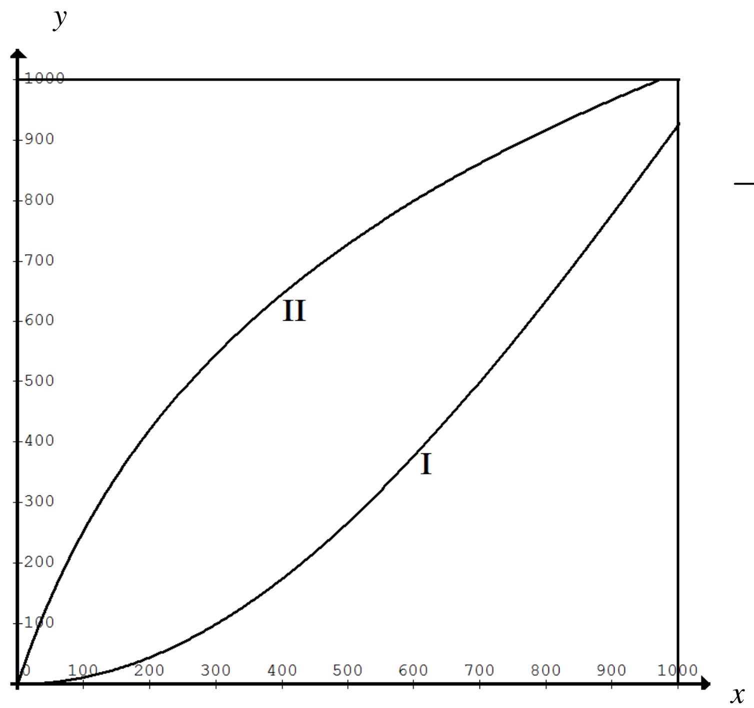
A second identical injection is given to the same person at $t = 2$ after the first injection.
The concentration of the drug in the blood at $t \geq 2$ is the sum of the concentrations of the first and the second injections.

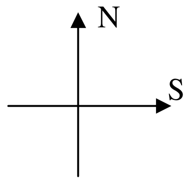
f. Find the drug concentration y in the blood as a function of t for $t \geq 2$. 2 marks

g. Use addition of ordinates to sketch the concentration-time graph for $2 \leq t \leq 5$. 2 marks

h. Find $t \in [2, 5]$ when the concentration is dropped to $e^{-2} + e^{-4}$. Write your answer correct to 2 decimal places. 1 mark

Question 3 A mathematician turned farmer divides his parcel of flat farmland **equally** in area among his three children using curves **I** and **II** as boundaries. Both curves pass through the origin of the x, y axes. The square parcel of land has side length measuring 1000 m. See the diagram below.





The equations for the two curves inside the square are:

$y = 1000(1 - \cos nx)$ for curve **I** ; $y = A\log_e\left(1 + \frac{x}{150}\right)$ for curve **II**

a i. Use calculus to show that $3\sin(1000n) = 2000n$ for curve **I**. 2 marks

a ii. Hence show the equation of curve **I** is $y = 1000(1 - \cos(0.001496x))$ approximately. 1 mark b i. Show that the equation of the reflection of curve **II** in the line $y = x$ is $y = 150\left(e^{\frac{x}{A}} - 1\right)$. 2 marks

b ii. Hence use calculus to show that $A\log_e\left(1 + \frac{29000}{9A}\right) = 1000$ for curve **II**. 3 marks

b iii. Hence show the equation of curve **II** is $y = 496.7\log_e\left(1 + \frac{x}{150}\right)$ approximately. 1 mark

c. Use calculus to find the longest distance in the northerly direction between curve **I** and curve **II**. Write your answer to the nearest metre. 3 marks

Question 4 The National Assessment Program—Literacy and Numeracy (NAPLAN) tests are conducted in May for all students across Australia in Years 3, 5, 7 and 9. Each year, over one million students nationally sit the NAPLAN tests. All students in the same year level are assessed on the same test items in the assessment domains of reading, writing and numeracy.

In 2013 numeracy test for Year 9 students across Australia has a mean score of 583.6 (out of 1000) and a standard deviation of 82.2. Across Victoria the mean is 588.4, and the standard deviation is 77.9.

Assume that the Year 9 numeracy test scores in Victoria and nationally are normally distributed in 2013.

a. What percentage of Victorian Year 9 students is above the national mean in the 2013 numeracy test?
1 mark

b. Determine the lowest and the highest 2013 numeracy test scores (correct to the nearest unit) of the middle 68% of Year 9 students in Victoria. 1 mark

c. What percentage of Year 9 students nationally scored within 588.4 ± 77.9 in the 2013 numeracy test?
1 mark

d. Find the probability (correct to 2 decimal places) that there are more than 6 among 10 randomly chosen Victorian Year 9 students scoring above the national mean in the 2013 numeracy test. 1 mark

e. What percentage (correct to 1 decimal place) scored below 788 among the Victorian Year 9 students scoring above the national mean in 2013? 1 mark

f. In the 2013 numeracy test, 80% of NSW Year 9 students scored below 668 and 70% scored above 544. Calculate the mean and standard deviation of the 2013 NSW numeracy test scores assuming a normal distribution of the scores. 2 marks

g. Fadel did the Year 9 numeracy test in Victoria in 2013 and he was in the top 5% in Victoria. If he did the same test in NSW he would be in the top *x*% in NSW. Find *x* correct to the nearest whole number. 2 marks

All students across Australia do the NAPLAN tests in Years 3, 5, 7 and 9. Suppose the probability that a student scores above the national mean in a test is 0.90 if the student scores above the national mean in the previous test; and the probability that the student scores below the national mean in a test is 0.60 if the student scores below the national mean in the previous test.

h. A student scored below the national mean in the Year 3 test in 2013. What is the probability (correct to 2 decimal places) that the student will score above the national mean in the next three tests? 1 mark

i. Another student scored above the national mean in the Year 3 and Year 5 tests. What is the probability (correct to 2 decimal places) that the student will score above the national mean in at least one of the next two tests? 2 marks

j. If a student scores above the national mean in the Year 3 and Year 7 tests, what is the expected number of times scoring above the national mean in the four tests? 3 marks

End of exam 2

