The Mathematical Association of Victoria
Trial Exam 2014

MATHEMATICAL METHODS (CAS)

Written Examination 2

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 21 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page and on the answer sheet for multiple- choice questions.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The solution to the equation $2\cos\left(\frac{x}{2}\right)+\sqrt{3}=0$ is given by

A. $2\pi n \pm \frac{\pi}{3}, n \in Z$

B. $4\pi n \pm \frac{\pi}{3}, n \in Z$

C. $4\pi n \pm \frac{5\pi}{3}, n \in Z$

D. $\pm\frac{5\pi}{3}$ only

E. $\frac{2\pi}{3}, \frac{4\pi}{3}$

Question 2

The range of the graph of $f(x)=-4\sin(a(x+b))-b$, where a and b are both non-zero real constants, is

A. $\left[-\frac{b}{a}, \frac{b}{a}\right]$

B. $[a-b, a+b]$

C. $[-4-b, -4+b]$

D. $[-4-b, 4-b]$

E. $[-4a-5b, -4a+3b]$

Question 3

Let $f:[-\pi,2\pi]\to R$, where $f(x)=2\pi\tan(x-\pi)+3$. The equations of the asymptotes of the graph of *f* are

A. $x=-\dfrac{3\pi}{2}$ only

B. $x=\dfrac{\pi}{2}, x=\pm\dfrac{3\pi}{2}$

C. $x=\dfrac{3\pi}{2}, x=\pm\dfrac{\pi}{2}$

D. $x=\dfrac{\pi}{2}$ only

E. $y=2\pi+3$ only

Question 4

The equation of the image of the graph of $y=\sin(x)$ under a translation of $\dfrac{\pi}{2}$ in the negative direction of the *x* axis and a translation of $\dfrac{1}{2}$ in the positive direction of the *y* axis, followed by a dilation from the *y* axis of a factor of 2 is

A. $y=\sin\left(\dfrac{x}{2}+\pi\right)+\dfrac{1}{2}$

B. $y=\sin\left(\dfrac{x}{2}-\dfrac{\pi}{2}\right)+\dfrac{1}{2}$

C. $y=\dfrac{1}{2}\sin\left(x+\dfrac{\pi}{2}\right)+1$

D. $y=\cos\left(\dfrac{x}{2}\right)+\dfrac{1}{2}$

E. $y=\cos(2x)+\dfrac{1}{2}$

Question 5

The maximal domain and range of $f(x)=\dfrac{2x+2}{x-1}$ are respectively

A. $R\backslash\{2\}$ and $R\backslash\{1\}$

B. $R\backslash\{1\}$ and $R\backslash\{2\}$

C. $R\backslash\{1\}$ and R

D. $R\backslash\{0\}$ and $R\backslash\{0\}$

E. $R\backslash\{1\}$ and $R\backslash\{4\}$

Question 6

The graph of the function with rule $f(x) = \log_e(2|x|+1)$ defined on its maximal domain has

A. no asymptotes
B. one asymptote
C. two asymptotes
D. three asymptotes
E. four asymptotes

Question 7

$(x-1)(x^2+ax+b)=0$, where *a* and *b* are real constants, will always have two unique solutions only if

A. $a^2-4b>0$
B. $a^2=4b, a \neq -2$
C. $a^2-4b<0$
D. $a^2=4b$
E. $a^2-4b \geq 0$

Question 8

If $f:[-2,\infty) \to R, f(x)=\sqrt{2+x}$ and $g:(-5,\infty) \to R, g(x)=x^2+5$ and $h(x)=g(f(x))$ then

A. $h:[-2,\infty) \to R, h(x)=x+7$
B. $h:[-2,\infty) \to R, h(x)=\sqrt{x^2+7}$
C. $h:(-5,\infty] \to R, h(x)=x+7$
D. $h:(-5,\infty] \to R, h(x)=\sqrt{x^2+7}$
E. $h:(-5,-2] \to R, h(x)=x+7$

Question 9

The function $g:\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right] \to R$, where $g(x)=\sin|x|$, is **not** differentiable when

A. $x=\pm\frac{5\pi}{2},\ x=0$
B. $x=-2\pi, x=-\pi, x=0, x=\pi, x=2\pi$
C. $x=-\frac{5\pi}{2}, x=\frac{5\pi}{2}$ only
D. $x=\pm 2\pi, x=\pm\pi$
E. $x=0$ only

Question 10

If the linear approximation formula $f(x+h) \approx f(x) + hf'(x)$ is used to find an approximate value of $f(x + h)$ and an overestimate of $f(x + h)$ is found then the rule for f could be

A. $f(x) = e^{x+1} - 2$
B. $f(x) = -\log_e(2x+1)$
C. $f(x) = -\sqrt{x-1}$
D. $f(x) = \dfrac{2}{(x-1)^2}$
E. $f(x) = \dfrac{3}{x-2} + 1$

Question 11

Consider the function $g: R \setminus \{-1\} \to R$, $g(x) = \dfrac{1}{x+1}$. An expression for the gradient function could be given as

A. $\lim\limits_{h \to 0}\left(\dfrac{1}{x+h+1} - \dfrac{1}{x+1}\right)$
B. $\dfrac{1}{h}\left(\dfrac{1}{x+h+1} - \dfrac{1}{x+1}\right)$
C. $\lim\limits_{h \to 0}\left(\dfrac{1}{x+h+1} + \dfrac{1}{x+1}\right)$
D. $\dfrac{1}{(x+1)^2}$
E. $\lim\limits_{h \to 0}\left(\dfrac{-1}{x^2 + xh + 2x + h + 1}\right)$

Question 12

A quartic function, f has a stationary point of inflection at $(-1,3)$ and passes through (1, 3).
The equation of this function could be

A. $f(x) = (x-1)^2(x+1)^2 + 3$
B. $f(x) = x^4 + 2x^3 - x + 2$
C. $f(x) = \dfrac{1}{16}(x+1)^4 + 2$
D. $f(x) = (x+1)^3(x-1) + 3$
E. $f(x) = (x-1)^3(x+1) + 3$

Question 13

If $f(x) = x^2 + 2x + 3$ and $g(x) = -x^4 + x^2 + 7$ then the maximum value of $g - f$ is

A. 0

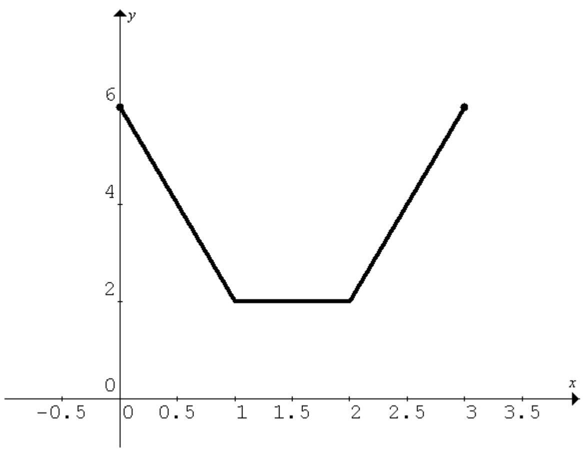
B. $\dfrac{-1}{\sqrt[3]{2}}$

C. $\dfrac{3}{2^{\frac{4}{3}}} + 4$

D. $\dfrac{1}{\sqrt[3]{2}}$

E. 7

Question 14



The average value of the function shown in the above diagram is closest to

A. 6

B. 5

C. $\dfrac{10}{3}$

D. 2

E. 1.5

Question 15

Left endpoint rectangles with widths 0.5 are used to find the approximate area bounded the graph with equation $y = \log_e(x-1)$, the x-axis and the ordinates $x = 2$ and $x = 4$. This area is given by

A. $\log_e\left(\sqrt{\frac{15}{2}}\right)$

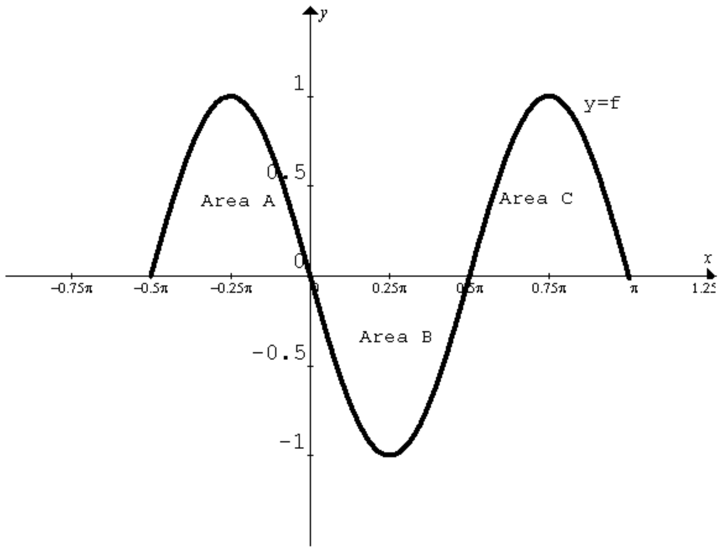
B. $\log_e(3) + \log_e(5) - \log_e(2)$

C. $\int_2^4 \left(\log_e(x-1)\right) dx$

D. $0.5\left(\log_e(1.5) + \log_e(2) + \log_e(2.5) + \log_e(3)\right)$

E. 1.007

Question 16



If Area A = Area B = Area C and Area A + Area B + Area C is the area bounded by the curve of f and the x-axis between $x = -\frac{\pi}{2}$ and $x = \pi$ then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(-f(-x)\right) dx$ is equal to

A. $-$Area A

B. 0

C. Area A

D. $2 \times$ Area A

E. $3 \times$ Area A

Question 17

Consider the discrete random variable described by the table shown.

X	0	1	2	3	4
$\Pr(X = x)$	$2p^2$	0.1	0.02	p	p^2

The standard deviation of X is closest to

A. 0.4
B. 1.98
C. 6.34
D. 2.4196
E. 1.5555

Question 18

Within a certain group of actors in a country town the probability a person is cast in one of the leading roles in the forthcoming musical 0.05. What is the probability correct to four decimal places that out of 50 randomly selected people from this population, less than 2 are cast in a leading role?

A. 0.5405
B. 0.2794
C. 0.7206
D. 0.9231
E. 0.2611

Question 19

The probability that Canada wins a medal in the first downhill skiing event at a particular Winter Olympics is judged to be 0.7. The probability that Canada wins a medal in a particular downhill skiing event given that it won a medal in the previous downhill skiing event is 0.55. The probability that Canada does not win a medal in a downhill skiing event given that it does not win a medal in the previous downhill skiing event is 0.35. The probability that Canada wins a medal in the fourth downhill skiing event is

A. 0.4092
B. 0.7
C. 0.5908
D. 0.59092
E. 0.5912

Question 20

For two events, P and Q, $\Pr(P \cap Q) + \frac{1}{2}\Pr(P' \cap Q) = \frac{3}{10}$.

P and Q will necessarily be mutually exclusive if

A. $\Pr(P' \cap Q) = 0$

B. $\Pr(P \cap Q') = \Pr(P' \cap Q)$

C. $\Pr(P \cap Q') = \frac{3}{5}$

D. $\Pr(P \cap Q') + \Pr(P' \cap Q') = \frac{2}{5}$

E. $\Pr(P') = 1$

Question 21

A flower show displays 1000 begonia bushes, from which 85 have a height greater than 80 cm. It is found by the curators of the flower show that the heights of the begonia bushes are distributed normally with a mean μ cm.

If it is also found that 1% of the bushes have a height less that 10 cm, then the value of μ is closest to

A. 54

B. -2.3263

C. 19

D. 1.3722

E. 0.085

Question 22

The variance, correct to four decimal places, of the random variable X which has a probability density function of $f(x) = \begin{cases} |\sin(x)| & \frac{3\pi}{2} \le x \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$ is

A. 1.0000

B. 1.1416

C. 28.0536

D. 27.9120

E. 0.1416

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (10 marks)
Tasmania Jones is painting the outside of his house. He is at the top of a 5 m electronic ladder which leans against a vertical wall. He sets the ladder so that its base moves at a rate of 2 m/s away from the wall. Let b m be the horizontal distance the base is from the wall and h m be the vertical height of the top of the ladder from the ground.

a. **i.** How fast is the top of the ladder sliding down the wall when the base is 3 m from the wall?

3 marks

ii. How long does it take for the ladder to be lying flat on the ground if the initial height of the ladder was $2\sqrt{6}$ m? 1 mark

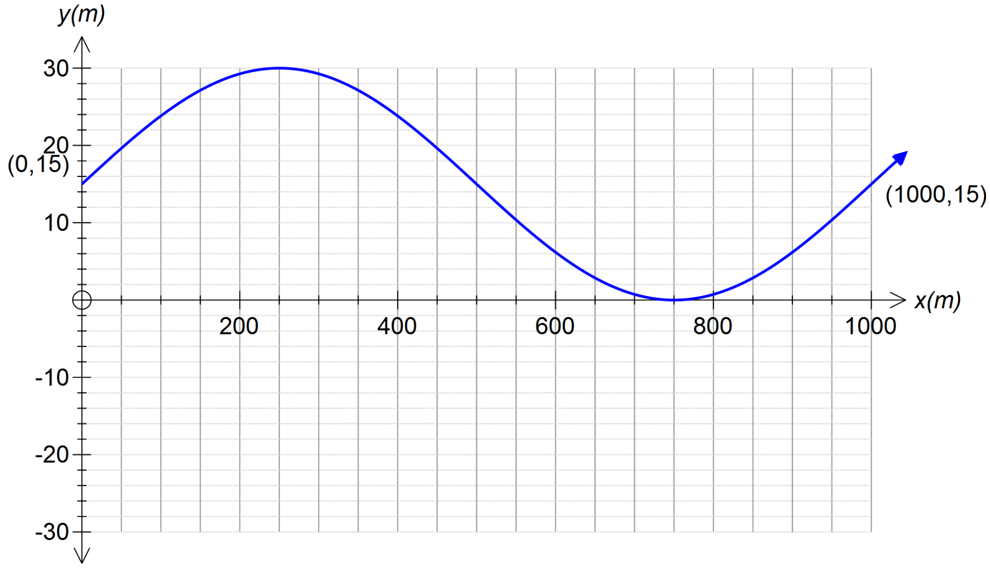
Tasmania is painting at a rate of $\frac{dA}{dt} = 2\log_e(t+1)$ m^2/min, where A is the area of wall in m^2 and t is the time, in minutes, he spends painting.

b. **i.** Find A in terms of t. 2 marks

ii. Hence or otherwise, what area of wall does he paint in the first half hour? Give your answer correct to one decimal place in m^2. 2 marks

iii. How much longer does it take him to paint a further 50 m^2? Give your answer correct to the nearest minute. 2 marks

Question 2 (17 marks)
The graph below represents the depth of a lake, in metres, at certain distances, in metres, from a café on the edge of the lake.



The equation for the graph above is in the form of

$$y = c + a\sin\left(\frac{\pi x}{b}\right), 0 \le x \le 1000$$

where a, b, c are integer values. y is the depth of water in the lake, in metres. x is the distance, in metres, from a café on the edge of the lake.

a. **i.** State the value of a. 1 mark

ii. State the value of c. 1 mark

iii. Show that $b = 500$. 1 mark

b. Using your values of *a, b, c* write down the equation for y. 1 mark

c. **i.** What is the depth of the lake 600 metres from the café. Write your answer to the nearest **centimetre.** 1 mark

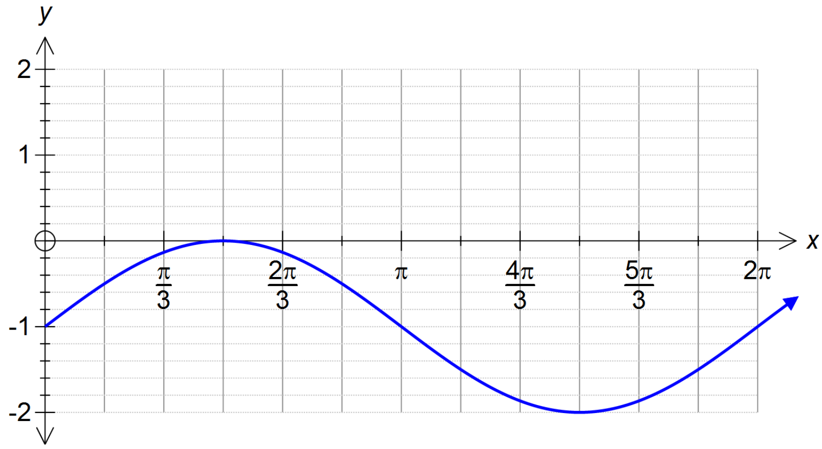
ii. For the domain $0 \le x \le 1000$, at what distance(s), expressed to the nearest **metre,** from the café is the same depth of water found, as in part **c. i.**? 1 mark

World rowing officials want to hold rowing titles on this lake. It is found that boats of this calibre can only compete when there is at least 5 metres of water beneath them.

d. **i.** At what distances from the café will the depth of water be suitable? Write your answer in interval notation, rounded to the nearest **metre.** 2 marks

ii. Hence, find the length of the race, rounded to the nearest **metre,** before it is interrupted by shallow water, if the boats start their race from a jetty that is placed at the café. 1 mark

Mr Row begins to design a model for a rowing course. Initially he draws the graph of $f(x) = \sin(x) - 1$ for $0 \le x \le 2\pi$, where x is the distance, in metres, from a café on the edge of the lake and $f(x) = 0$ represents zero depth underground, $f(x) = -2$ represents a depth of 2 metres underground. The origin (0, 0) represents the location of the café.



He decides that tourists might want to walk on a path across the lake. During their walk, tourists are happy to stand in water that is less than 50 cm deep.

e. State the coordinates of the point where the tourists will stand when there is zero depth of water. 1 mark

f. In the model of $f(x) = \sin(x) - 1$ for $0 \le x \le 2\pi$, at what distance(s), as represented by the x-axis, from the café can tourists walk? Presume that a boat can take the tourists to their walk. Write your answer to the nearest **centimetre.** 2 marks

A more realistic model that Mr Row comes up with is

$$g(x) = -10\sin\left(\frac{\pi x}{500}\right) - 10 \text{ for } 0 \le x \le 1000$$

where $g(x) = 0$ represents zero depth underground and $g(x) = -10$ represents a depth of 10 metres underground. The origin (0, 0) represents the location of the café.

g Using this more realistic model**,** Mr Row tempts tourists by installing an underground water slide, in the shape of a straight line, between the points $x = 750$ and $x = 875$. What is the equation of the line that represents the slide? 3 marks

h. State the transformation(s) that will create the image graph $g(x) = -10\sin\left(\frac{\pi x}{500}\right) - 10$ from the graph of $y = c + a\sin\left(\frac{\pi x}{b}\right)$ as defined in **part b**. 2 marks

Question 3 (16 marks)

A school decides to operate daily fitness classes for staff to use. It is decided that the fitter the staff, the less sick days they will take, and student outcomes will improve.
The school offers Rowing Ergo Classes (R), Cycle Classes (C) and Step Classes (S). There are 60 staff members and all take part in at least one class.
The probability that a staff member takes a Rowing Ergo Class is 0.25.
The probability that a staff member takes a Cycle Class is 0.55.
The probability that a staff member takes a Step Class is 0.75.
The events R, C and S are independent.

a. **i.** What is the probability that a randomly selected staff member will take both a Rowing Class and a Step Class? 1 mark

ii. What is the probability that a randomly selected staff member will take a Rowing Class given that we know they always take a Step Class? 1 mark

b. **i.** What is the probability that a randomly selected staff member will take *R* and *S* but not *C*? 2 marks

ii. If 2 staff members take all 3 classes, 5 take *R* and *C* and 10 take *R* and *S*, how many staff members take *C* only? 2 marks

We still assume there are 60 staff members and all take part in at least one class daily.

A pattern emerges and it is found that if a staff member takes a Step Class one day there is a 20% chance of them taking a Step Class the next day. If a staff member takes a Cycle Class one day there is a 35% chance of them taking a Step Class the next day.

On a particular week the Rowing instructor is unwell and no *R* classes are offered and so all staff participate in either Step Classes or Cycle Classes (but not both).

On Monday 15 take Cycle Class.

c. **i.** How many staff, to the nearest staff member, take Cycle class on Tuesday of that week?

2 marks

ii. How many staff, to the nearest staff member, take Step class on Friday of that week? 1 mark

iii. What is the long term prediction, to the nearest staff member, of each of Cycle and Step class participation?

2 marks

The PE faculty observe a small group of 10 Mathematics staff members.
They find that for this group of Mathematics staff members the event of selecting each class is independent, and results suggest that 2 out of 10 take Cycle on a regular basis and 6 out of 10 take Step.

d. What is the probability that a randomly selected staff member out of this group of 10 Mathematics staff members will take a Step class on Monday and Tuesday of a particular week and Cycle on the next three days?

1 mark

e. What is the probability, correct to 4 decimal places, that on any particular day, more than six out of this small group of 10 will take Step? 2 marks

Consider a new group of Science staff members.

f. The probability of taking a Cycle class for this group of Science staff members is 0.3. What is the minimum number of Science staff members needed so that the probability of taking at least two Cycle classes is more than 0.65? 2 marks

Question 4 (15 marks)

Rolle's theorem states that if a function f is continuous on a closed interval $\left[a,b\right]$ and differentiable on the open interval $\left(a,b\right)$ and $f(a)=f(b)$ then there exists a value of c in the open interval $\left(a,b\right)$ such that $f'(c)=0$.

Consider the function $f:\left[1,b\right]\to R,$ where $f(x)=-x^4+8x^3-24x^2+32x-15$.

a. Express $f(x)$ in the form $A(x-B)^4+C$, where A,B and C are integers. 2 marks

b. Find the value of b such that $f(1)=f(b)$. 2 marks

c. Using Rolle's theorem, what is the maximum interval over which f is

i. continuous? 1 mark

ii. differentiable? 1 mark

d. Find the value of c such that $f'(c)=0$. 1 mark

e. If $g:\left[\frac{5}{2},\frac{7}{2}\right]\to R$, where $g(x)=\left|\log_e(x-2)\right|$, explain why the conclusion does not hold in Rolle's theorem for this function. 1 mark

f. **i.** Find k, the average value of f. 2 marks

ii. **Hence**, find the values of m and n such that $\int_m^n \left(f(x)-k\right)dx = 2\int_1^m \left(k-f(x)\right)dx$.

Give your answers correct to four decimal places. 2 marks

Working space

Consider the function $f_1 : [1,3] \to R, f_1(x) = -x^4 + 8x^3 + ax^2 + bx + d$ where a, b and d are real constants.

g. Find the values of d for which f_1 has three stationary points between $x = 1$ and $x = 3$, and $f_1(1) = f_1(3) = 0$. 3 marks

END OF QUESTION AND ANSWER BOOK