



Trial Examination 2014

VCE Mathematical Methods (CAS) Units 3&4

Written Examination 1

Suggested Solutions

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Question 1

$$\begin{aligned} \text{a. } \frac{dy}{dx} &= 5\left(\frac{x^2}{3} - 2x\right)^4 \times \left(\frac{2x}{3} - 2\right) \\ &= 10\left(\frac{x}{3} - 1\right)\left(\frac{x^2}{3} - 2x\right)^4 \end{aligned} \quad \text{A1}$$

$$\begin{aligned} \text{b. } f'(x) &= \frac{2x \cos(x) - x^2(\sin(x))}{\cos^2(x)} \quad \text{M1} \\ &= \frac{2x \cos(x) + x^2(-\sin(x))}{\cos^2(x)} \end{aligned}$$

$$f'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2\sqrt{3}}{9} \quad \text{A1}$$

Question 2

$$\begin{aligned} \text{a. } \int \frac{2}{(3x-2)^4} dx &= 2 \int (3x-2)^{-4} dx \\ &= 2\left[-\frac{1}{3}(3x-2)^{-3} \times \frac{1}{3}\right] \quad \text{M1} \\ &= \frac{-2}{9(3x-2)^3} \quad \text{A1} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{1}{3} \int_3^a \frac{1}{(x-2)} dx &= -1 \\ \int_3^a \frac{1}{(x-2)} dx &= -3 \end{aligned}$$

$$[\log_e(x-2)]_3^a = -3 \quad \text{M1}$$

$$\log_e(a-2) - \log_e(3-2) = -3$$

$$\log_e(a-2) = -3$$

$$a-2 = e^{-3}$$

$$a = e^{-3} + 2 \quad \text{A1}$$

Question 3

Finding stationary points first:

$$\frac{df(x)}{dx} = \frac{d(x^2 e^{-x})}{dx} = 2xe^{-x} - x^2 e^{-x} = e^{-x}(2x - x^2), \quad \text{M1}$$

$$2x - x^2 = 0 (e^{-x} \neq 0)$$

$$x = 0 \text{ or } x = 2$$

Using sign diagram, there is a minimum at 0 and maximum at 2.

$$f(0) = 0, f(2) = \frac{4}{e^2} \quad \text{A1}$$

Find values of the function at the endpoints.

$$f(-1) = e, f(3) = \frac{9}{e^3}$$

Comparing values, we can make a conclusion that absolute minimum is (0, 0) and absolute maximum is (-1, e).

A1

Question 4

a. $\cos\left(2x + \frac{\pi}{2}\right) = -\frac{\sqrt{3}}{2}$

$$2x + \frac{\pi}{2} = \frac{5\pi}{6} + 2\pi k \text{ or } 2x + \frac{\pi}{2} = \frac{7\pi}{6} + 2\pi k \quad \text{M1}$$

$$2x = \frac{\pi}{3} + 2\pi k \text{ or } 2x = \frac{2\pi}{3} + 2\pi k$$

$$x = \frac{\pi}{6} + \pi k \text{ or } x = \frac{\pi}{3} + \pi k$$

Checking which solutions are in the required interval gives

$$x = -\frac{11\pi}{6}, -\frac{5\pi}{3}, -\frac{5\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{3} \quad \text{A1}$$

b. $[\sqrt{3} - 2, \sqrt{3} + 2]$ A1

Question 5

a. $5^{4x} - 6 \times 5^{2x} + 5 = 0$

Let $u = 5^{2x}$

$u^2 - 6u + 5 = 0$

M1

$u = 1$ or $u = 5$

$5^{2x} = 1$ or $5^{2x} = 5$

$x = 0$ or $x = 0.5$

A1

b. $\log_{\frac{1}{3}}(x) = \frac{\log_3(x)}{\log_3\left(\frac{1}{3}\right)}$

M1

$= -\log_3(x)$

A1

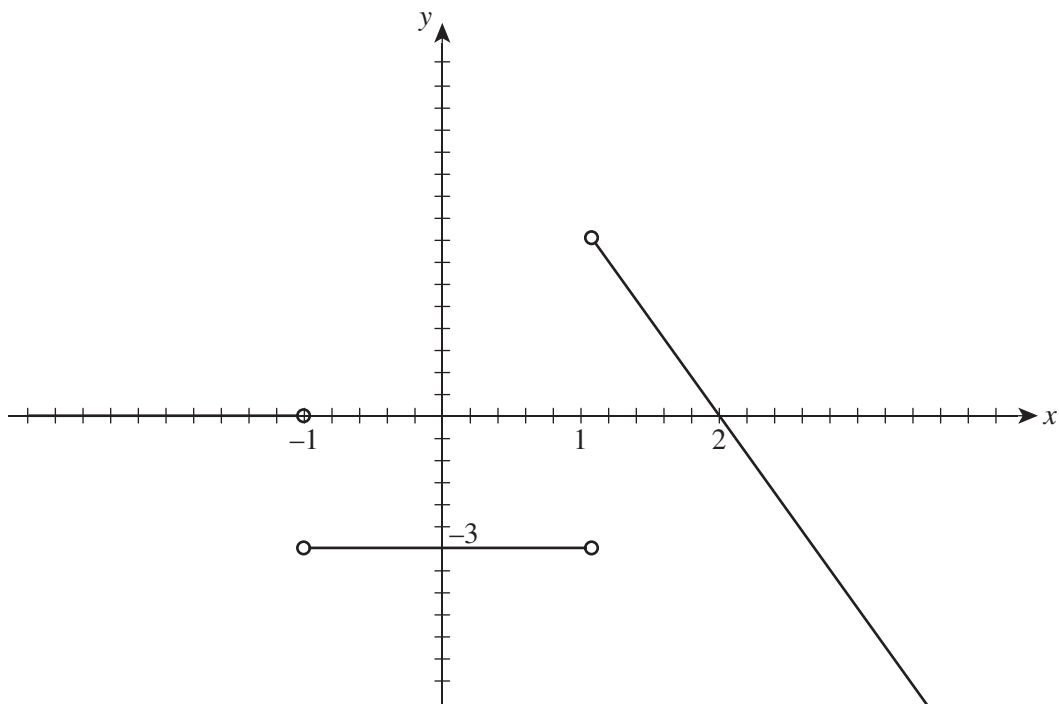
$2\log_3 x - \log_3(x) = 3$

$\log_3(x) = 3$

$x = 27$

Question 6

a.



correct shape A1
correct endpoints A1

b. Domain: $x \in (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

A1

Question 7

- a. As the graph of $f(x) = 2(x - a)^2 - b - 3$ is parabola, modulus will not change the x -intercepts, it will just reflect part of the parabola below the x -axis in the x -axis. So to solve the problem we can remove the modulus sign.

$2(x - a)^2 - b - 3 = 0$, substitute $-\frac{2}{3}$ and 2 instead of x and we will obtain 2 simultaneous equations

$$2(2 - a)^2 - b - 3 = 0$$

$$2\left(-\frac{3}{2} - a\right)^2 - b - 3 = 0$$

M1

solve them by eliminating b (subtract first equation from second)

$$2\left(-\frac{3}{2} - a\right)^2 - 2(2 - a)^2 = 0$$

$$a = \frac{1}{4}$$

A1

Substitute this value back into any of the equations and solve for b

$$b = \frac{25}{8}$$

A1

- b. This transformation is a dilation from y -axis by factor 3 and a translation 1 unit in the positive direction of y -axis. This transformation will move x -intercepts (cusp points) 3 times further away from y -axis to -4.5 and 6 and then move the graph 1 up.

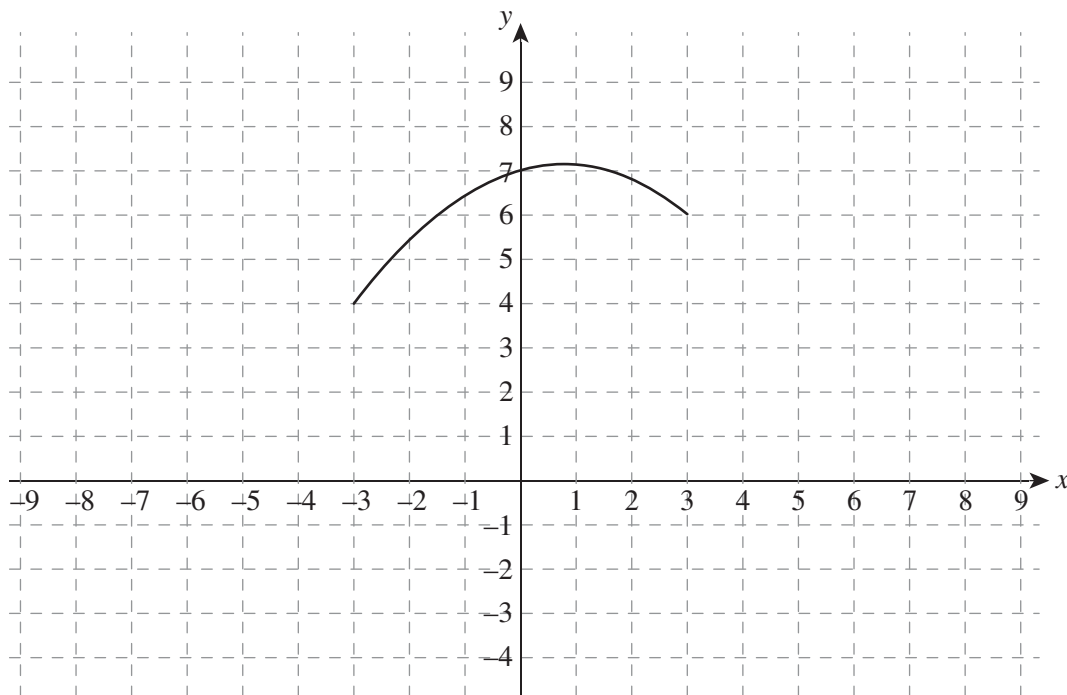
So the rule for the function will become $y = \left|2\left(\frac{x}{3} - \frac{1}{4}\right)^2 - \frac{49}{8}\right| + 1$

M1

Substituting $x = 0$, $x = -3$ and $x = 3$ will give y -intercept and end points.

$(-3, 4)$, $(0, 7)$ and $(3, 6)$.

A1



Question 8

a. $f'(x) = 2x \log_e(x) + x$ A1

b. $\int_{\frac{1}{e}}^e 2x \log_e(x) + x \, dx = x^2 \log_e(x)$

$$\int_{\frac{1}{e}}^e 2x \log_e(x) \, dx + \int_{\frac{1}{e}}^e x \, dx = \left[x^2 \log_e x \right]_{\frac{1}{e}}^e$$

$$\int_{\frac{1}{e}}^e 2x \log_e(x) \, dx = \left[x^2 \log_e x \right]_{\frac{1}{e}}^e - \int_{\frac{1}{e}}^e x \, dx$$
 M1

$$2 \int_{\frac{1}{e}}^e x \log_e(x) \, dx = \left[x^2 \log_e x \right]_{\frac{1}{e}}^e - \int_{\frac{1}{e}}^e x \, dx$$

$$\int_{\frac{1}{e}}^e x \log_e(x) \, dx = \frac{1}{2} \left(\left[x^2 \log_e x \right]_{\frac{1}{e}}^e - \int_{\frac{1}{e}}^e x \, dx \right)$$

$$= \frac{1}{2} \left[x^2 \log_e(x) - \frac{x^2}{2} \right]_{\frac{1}{e}}^e$$
 M1

$$= \frac{1}{2} \left[\left(e^2 \log_e(e) - \frac{e^2}{2} \right) - \left(\frac{1}{e^2} \log_e\left(\frac{1}{e}\right) - \frac{1}{2e^2} \right) \right]$$

$$= \frac{1}{2} \left[\left(e^2 - \frac{e^2}{2} \right) - \left(-\frac{1}{e^2} - \frac{1}{2e^2} \right) \right]$$

$$= \frac{1}{2} \left(\frac{e^2}{2} + \frac{3}{2e^2} \right)$$

$$= \frac{1}{4} \left(e^2 + \frac{3}{e^2} \right)$$
 A1

Question 9

a. As all probabilities should add up to 1

$$3p^2 + 2p = 1$$

$$3p^2 + 2p - 1 = 0$$
 M1

$$(3p - 1)(p + 1) = 0$$

Solve for p : $p = \frac{1}{3}$ or $p = -1$. Reject -1 as probability cannot be negative. A1

- b. Subtract probability of Jacob not winning any games from 1.

$$\begin{aligned}\Pr(X > 0) &= 1 - \Pr(X = 0) \\ &= 1 - p^2\end{aligned}$$

$$\Pr(X > 0) = 1 - p^2 = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} \quad \text{A1}$$

- c. Recognise that this is conditional probability – it is given that Jacob won at least 2 games.

$$\text{So } \Pr(X \geq 3 | X \geq 2) = \frac{\Pr(X \geq 3)}{\Pr(X \geq 2)} \quad \text{M1}$$

$$\Pr(X \geq 3 | X \geq 2) = \frac{\frac{p}{2} + \frac{2p^2}{3}}{2p + \frac{2p^2}{3}} = \frac{\frac{13}{54}}{\frac{40}{54}}$$

$$\Pr(X \geq 3 | X \geq 2) = \frac{13}{40} \quad \text{A1}$$

Question 10

a. $f(x) = k(2x^3 - x^2 - 7x + 6)$

$$\int_0^1 k(2x^3 - x^2 - 7x + 6) dx = 1 \quad \text{M1}$$

$$k \left[\frac{x^4}{2} - \frac{x^3}{3} - \frac{7x^2}{2} + 6x \right]_0^1 = 1$$

$$k \left(\frac{1}{2} - \frac{1}{3} - \frac{7}{2} + 6 \right) = 1$$

$$k = \frac{3}{8} \quad \text{A1}$$

- b. i. $\Pr(X > 43)$ is required

$$Z = \frac{43 - 50}{5} = -1.4$$

$$\Pr(X > 43) = \Pr(Z > -1.4) \quad \text{M1}$$

$$\Pr(Z > -1.4) = \Pr(Z < 1.4) \text{ by symmetry}$$

$$\text{Therefore } \Pr(X > 43) = 0.75 \quad \text{A1}$$

- ii. $\Pr(X > 57 | X > 50)$ is required (conditional probability)

$$\Pr(X > 57 | X > 50) = \frac{\Pr(X > 57)}{\Pr(X > 50)} \quad \text{M1}$$

$$= \frac{\Pr(Z > 1.4)}{\Pr(Z > 0)}$$

$$= \frac{1 - 0.75}{0.5}$$

$$= 0.5 \quad \text{A1}$$