

Trial Examination 2014

## VCE Mathematical Methods (CAS) Units 3&4

Written Examination 1

### Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

#### Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

#### Materials supplied

Question and answer booklet of 11 pages and a sheet of miscellaneous formulas.

Working space is provided throughout the booklet.

#### Instructions

Write your **name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2014 VCE Mathematical Methods (CAS) Units 3&4 Written Examination 1.

**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

**Question 1 (3 marks)**

- a. If  $y = \left(\frac{x^2}{3} - 2x\right)^5$ , find  $\frac{dy}{dx}$ . 1 mark

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- b. If  $f(x) = \frac{x^2}{\cos(x)}$ , find  $f'\left(\frac{\pi}{3}\right)$ . 2 marks

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**Question 2 (4 marks)**

- a. Find an anti-derivative of  $\frac{2}{(3x-2)^4}$ , with respect to  $x$ . 2 marks

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- b. Given that  $\int_3^a \frac{1}{3(x-2)} dx = -1$ , find the exact value of  $a$ , where  $a > 3$ . 2 marks

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**Question 3 (3 marks)**

For the function  $f: [-1, 3] \rightarrow R, f(x) = \frac{x^2}{e^x}$  find the absolute maximum and minimum.

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**Question 4 (3 marks)**

a. Solve the equation  $2 \cos\left(2x + \frac{\pi}{2}\right) + \sqrt{3} = 0$  for  $x \in [-2\pi, \pi]$ . 2 marks

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b. State the range of the function  $f: R \rightarrow R, f(x) = 2 \cos\left(2x + \frac{\pi}{2}\right) + \sqrt{3}$ . 1 mark

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**Question 5 (4 marks)**

Solve the equations.

**a.**  $5^{4x} - 6 \times 25^x = -5$

2 marks

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**b.**  $2\log_3 x + \log_{\frac{1}{3}} x = 3$

2 marks

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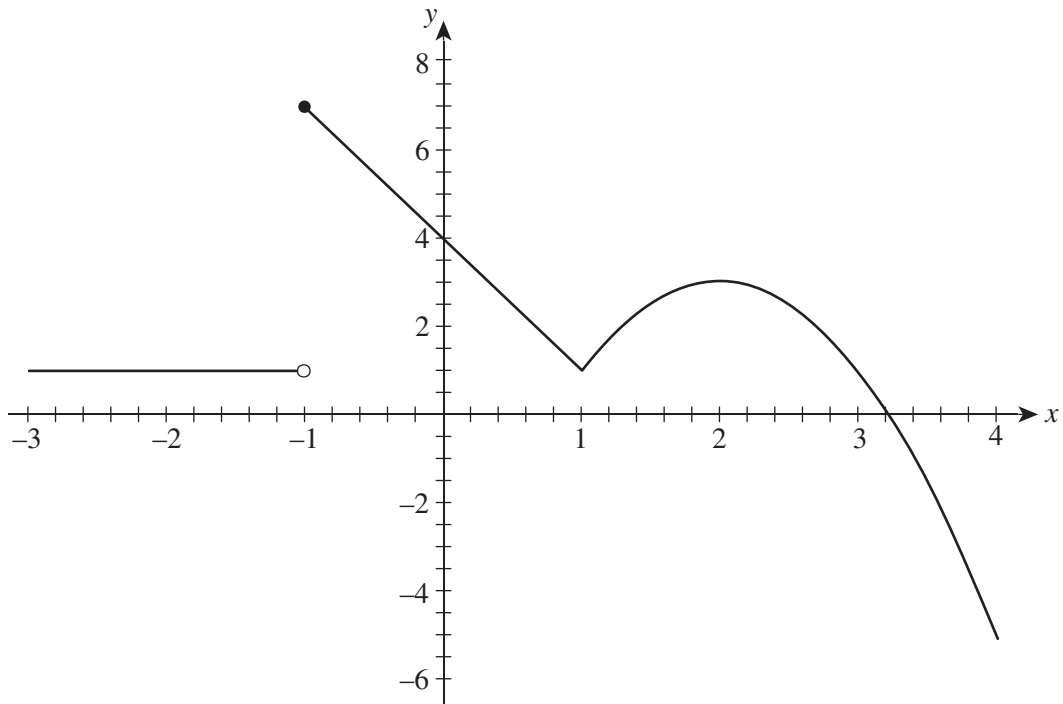
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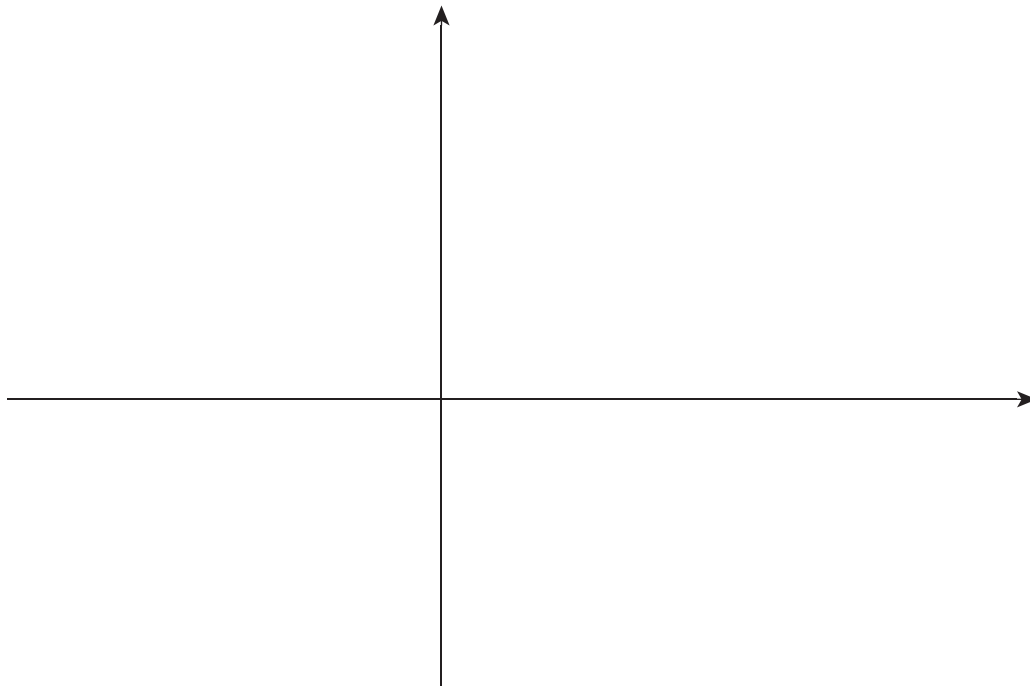
**Question 6 (3 marks)**

The graph of the function  $f$  is shown below.



a. Sketch the graph of the derived function  $f'$  on the set of axes below.

2 marks



b. State the domain of the derivative function.

1 mark

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**Question 7 (5 marks)**

- a. The graph of the function  $f: [-3, 3] \rightarrow R, f(x) = |2(x-a)^2 - b - 3|$  has  $x$ -intercepts at  $-\frac{3}{2}$  and 2.

Find the values of  $a$  and  $b$ .

3 marks

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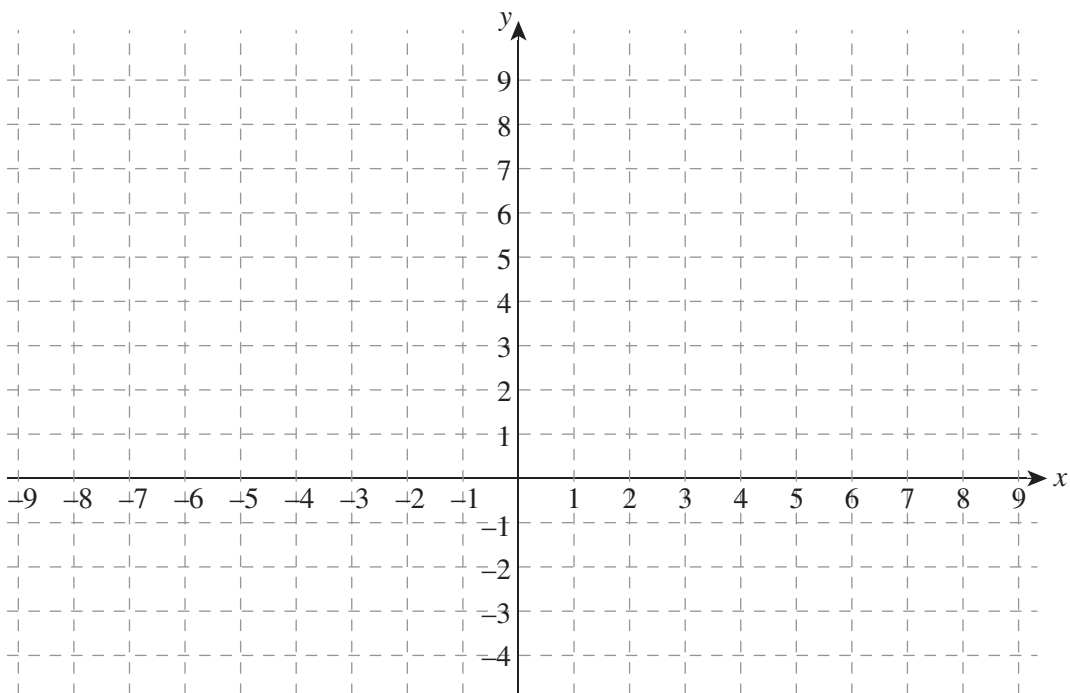
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- b. Sketch the graph of the image of  $f(x)$  on the interval  $[-3, 3]$  under the transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Label the axes intercepts and endpoints with their coordinates.

2 marks



**Question 8 (4 marks)**

- a. If  $f(x) = x^2 \log_e(x)$ , find  $f'(x)$ . 1 mark

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- b. Hence, find the value of  $\int_{\frac{1}{e}}^e x \log_e(x) dx$ . 3 marks

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**Question 9 (5 marks)**

Jacob is playing table tennis with 4 friends, playing once with each. The probability of winning a certain number of games is a random variable with a probability distribution given in the table below:

$x$	0	1	2	3	4
$\Pr(X = x)$	$p^2$	$\frac{4p^2}{3}$	$\frac{3p}{2}$	$\frac{p}{2}$	$\frac{2p^2}{3}$

- a. Find value of  $p$ . 2 marks

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- b. Find the probability that Jacob will win at least 1 game. 1 mark

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- c. Jacob won the first 2 games.  
Find the probability that he will win at least 3 games in total. 2 marks

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**Question 10 (6 marks)**

- a.** A continuous random variable has a probability density function defined by  $f(x)$  where

$$f(x) = \begin{cases} k(x+2)(2x^2 - 5x + 3) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of  $k$ .

2 marks

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- b.** A national study was commissioned to learn how far elite soccer players could kick the ball. Each player was given several attempts to produce their longest kick. It was found that the length of their best kicks were normally distributed with a mean of 50 m and a standard deviation of 5 m. It is known that  $\text{Pr}(Z < 1.4) = 0.75$ .

- i.** Find the probability that a randomly selected player will have a longest kick of at least 43 m.

2 marks

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- ii.** If a player was randomly chosen from those who were able to kick a ball further than 50 m, what is the probability that the player can kick further than 57 m? 2 marks

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**END OF QUESTION AND ANSWER BOOKLET**