MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

TSSM
Creating VCE Success

*(*TSSM's *2014 trial exam updated for the current study design)*

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: B

Explanation:

Period is: $\frac{\pi}{\pi/3} = 3$

Question 2

Answer: A

Explanation:
The domain of $f(x) + g(x)$ is given by $dom\ of\ f \cap dom\ of\ g$

Question 3

Answer: C

Explanation:

$d^2 = \left(\sqrt{(2a-a)^2 + (2b+b)^2}\right)^2 = a^2 + 9b^2$

Question 4

Answer: E

Explanation:

$-1 + a - 41 + 56 = 0$
$a = -14$

Question 5

Answer: D

Explanation:

The domain of the inverse is the range of the function = $(-\infty, 3)$
$x = 3 - e^y$
$e^y = 3 - x \ \ whcih\ implies \ \ y = log_e(3 - x)$

Question 6

Answer: C

Explanation:

$\frac{1}{3-1/2}\int_{1/2}^{3} 2sin\left(x - \frac{3\pi}{4}\right)\ dx$

Question 7

Answer: C

Explanation:

Draw the probability table and use $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ to find the value of p.

Question 8

Answer: A

Explanation:

$y - 2e^{a+2} = 2e^{a+2}(x - a)$
$Solve: \ -2e^{a+2} = 2e^{a+2}(-a)\ for\ a$

Question 9

Answer: C

Explanation:
$kx^2 - 5x = x - 3$
$\Delta = 0 \ \ implies\ k = 3$

Question 10

Answer: D

Explanation:

Chain rule. First differentiate *f*, then cos, then *g*

Question 11

Answer: B

Explanation:

$Shaded\ Area = Area\ of\ rectangle - Area\ under\ the\ curve$
$Shaded\ Area = 3 \times 9 - \int_2^{11} \sqrt{x-2}\ dx$

Question 12

Answer: A

Explanation:
For $f(g(x))$ to be defined $(Range\ of\ g)\ should\ be\ a\ subset\ of\ (domain\ of\ f\ )$

Question 13

Answer: C

Explanation:

Use matrix transformations to get $y - 1 = (-x + 4 - 1)^2$

Question 14

Answer: E

Explanation:

$\Pr(X < 90) = \Pr\left(Z < \frac{-30}{\sigma}\right)$
$\frac{-30}{\sigma} = invnorm\ \left(\frac{3}{40}, 0,1\right)$

Question 15

Answer: A

Explanation:

$M = 1.96 \times \sqrt{\frac{0.2 \times 0.8}{50}}$
$= 0.11$

Question 16

Answer: B

Explanation:
$solve\left(\frac{d}{dx}(5e^x \sin(x)) = 0\right)\ for\ x\ and\ choose\ the\ second\ value\ of\ x.$

Question 17

Answer: E

Explanation:

$0 = (1 + a)^2\ \ implies\ a = -1$
$f(x) = \frac{(x-1)^3}{3} + c$

Question 18

Answer: B

Explanation:

$\hat{p}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\Pr(\hat{P}=\hat{p})$	0.4096	0.4096	0.1536	0.0256	0.0016

$$\frac{Pr(0<\hat{p}<0.8)}{Pr(\hat{p}<0.8)} = \frac{0.5888}{0.9984} = 0.590$$

Question 19

Answer: A

Explanation:

As the graph of the function passes through 3, its gradient changes from negative to positive hence a point of minimum.

Question 20

Answer: C

Explanation:

X~Bi(5,0.2)
$\Pr(X=3) = \binom{5}{3}(0.2)^3(0.8)^2$

Question 21

Answer: E

Explanation:

$1(f(3)+f(4)+f(5))$

Question 22

Answer: A

Explanation:

$\frac{f(3)-f(1)}{3-1} = 7$

Solve for a.

SECTION 2: Analysis Questions

Question 1

a. $f(x) = x^3e^{-2x}$

$f'(x) = -2x^3e^{-2x} + 3x^2e^{-2x}$

$f'(x) = x^2e^{-2x}(3 - 2x)$

$a = 2 \ and \ b = -2$

M1+A2

3 marks

b. $f(0) = 0$

$(0,0)$

A1

1 mark

c.

$f'(x) = 0 \ \rightarrow \ x^2e^{-2x}(3 - 2x) = 0$

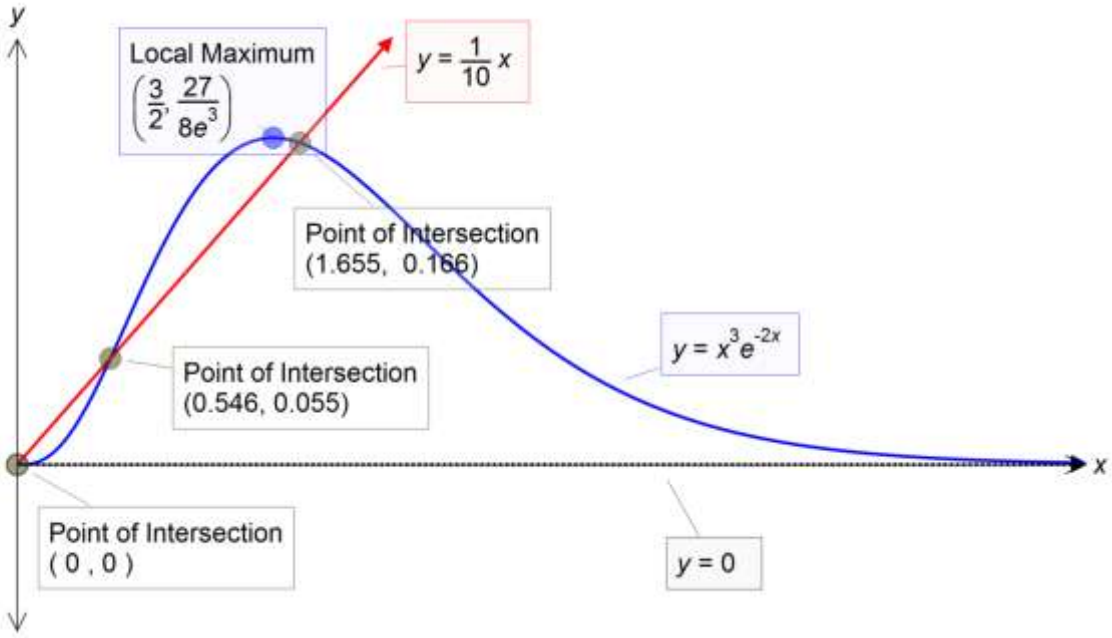
$x = 0, \ x = \frac{3}{2}$

Stationary points are $(0, 0) \ and \ \left(\frac{3}{2}, \ \frac{27}{8e^3}\right)$

M1+A2

3 marks

d.



1 mark for turning point, 1 mark for asymptote, 1 mark for shape, 1 mark for axes intercepts

4 marks

e. See the graph above.

1 mark for sketching the line, 1 mark for the intersection points correct to 3 dp.

2 marks

f.

$Area = \int_0^{0.546} \left(\frac{1}{10}x - x^3e^{-2x}\right) dx + \int_{0.546}^{1.655} \left(x^3e^{-2x} - \frac{1}{10}x\right) dx$

$Area = 0.0322\ sq\ units$

M2+A1

3 marks

Question 2

a. $Max\ d(t) = 6m$

$6 = 4 + 2sin\left(\frac{\pi(t+2)}{6}\right)$

$t = 1, 13$

$At\ 10.00am\ \ and\ \ 10.00pm$

M1+A2

3 marks

b. $\text{Period} = \frac{2\pi}{\pi/6} = 12hours$

A1

1 mark

c. $3.6 = 4 + 2sin\left(\frac{\pi(t+2)}{6}\right)$

$t = 4.38457, 9.61543, 16.3846,\ \ 21.6154$

$At\ 6{:}37pm$

M2+A1

3 marks

d. $2 = 4 + 2sin\left(\frac{\pi(t+2)}{6}\right)$

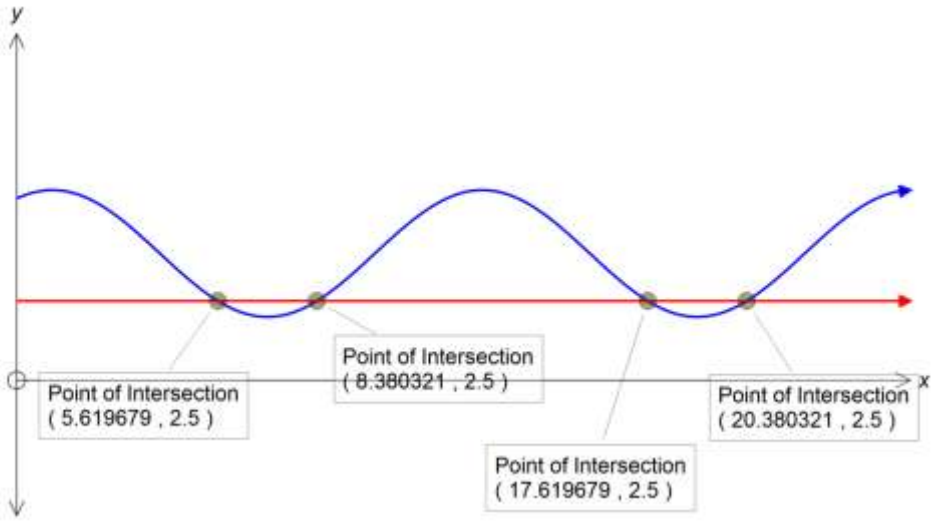
$t = 7,\ \ 19$

$At\ 4pm\ \ and\ \ 4am\ (next\ day)$

A2

2 marks

e. $4 + 2sin\left(\frac{\pi(t+2)}{6}\right) \leq 2.5$



$Between\ 2{:}37pm\ \ and\ \ 5{:}23pm$

M2+A2
4 marks

Question 3

a. $(-1, \infty)$

1 mark

b. $Translate\ by -1\ unit\ along\ the\ x-axis$
$Reflect\ the\ graph\ across\ the\ x-axis$
$Translate\ by +2\ units\ along\ the\ y-axis$

A3
3 marks

c. $let\ y = 2 - log_e(x+1)$
$x = 2 - log_e(y+1)$
$log_e(y+1) = 2 - x$
$f^{-1}(x) = e^{2-x} - 1$
Domain of f^{-1} is R

M2+A2
4 marks

d. $2 - log_e(x+1) = x$
$(1.2079, 1.2079)$

M1+A1

2 marks

e. $\frac{\mathrm{d}}{dx}(2 - log_e(x+1)) = \frac{-1}{x+1}$
$m = \frac{-1}{5}$

M1+A1
2 marks

f. $(4, 2 - log_e 5)$
$y - (2 - log_e 5) = 5(x-4)$
$y = 5x - 18 - log_e 5$

M2+A1
3 marks

Question 4

a. $0.85^5 = 0.4437$

M1+A1
2 marks

b. $5 \times 0.15 \times 0.85^4 = 0.3915$

M1+A1
2 marks

c. $1 - 0.15^0 \times 0.85^5 = 0.5563$

M1+A1
2 marks

d. Let $X \sim Bi(115, 0.15)$
$\Pr(15 \leq X \leq 20) = 0.5520$
(on CAS use $binomcdf(110, 0.15, 15, 20)$)

M2+A1
3 marks

e. Let $Y \sim N(20700, 2915^2)$
$\Pr(Y > 21000) = 0.4590$
(on CAS use $normcdf(21000, \infty, 20700, 2915)$)

M2+A1
3 marks

f. $\Pr(18000 < Y < 25000) = 0.7528$
(on CAS use $normcdf(18000, 25000, 20700, 2915)$)

M1+A1
2 marks