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2014 Trial Examination

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STUDENT NUMBER

Figures									Letter:
Words									

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 17 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 –Multiple-choice questions

Instructions for Section 1

Answer all questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The function with the rule $f(x) = -\frac{1}{2}\tan\left(\frac{\pi}{3}x\right)$ has period:

A. $\frac{1}{3}$

B. 3

C. 3π

D. $\frac{\pi}{3}$

E. 6

Question 2

If the function $f: (-\infty, 3) \to R,\ f(x) = log_e(3-x)\ and\ g: [-3, \infty) \to R,\ g(x) = -\sqrt{x+3}$ then the maximal domain of the function $f(x) + g(x)$ is:

A. $[-3, 3)$

B. $(-3, 3)$

C. $(-\infty, 3)$

D. $[-3, \infty)$

E. $(-\infty, \infty)$

Question 3

The square of the distance between the two points $(a, -b)\ and\ (2a, 2b)$ is given by:

A. $\sqrt{9a^2 + b^2}$

B. $\sqrt{a^2 + 9b^2}$

C. $a^2 + 9b^2$

D. $9a^2 + b^2$

E. $9a^2 - b^2$

SECTION 1 – continued

Question 4

If the equation $x^3 + ax^2 + 41x + 56 = 0$ has a solution $x = -1$, then the value of a is:

A. 14
B. ± 14
C. 16
D. -16
E. -14

Question 5

The inverse of the function $f: R \to R,\ f(x) = 3 - e^x$ is given by:

A. $f^{-1}: R \to R, f^{-1}(x) = log_e(x-3)$
B. $f^{-1}: R \to R, f^{-1}(x) = log_e(3-x)$
C. $f^{-1}: (3, \infty) \to R, f^{-1}(x) = log_e(x-3)$
D. $f^{-1}: (-\infty, 3) \to R, f^{-1}(x) = log_e(3-x)$
E. $f^{-1}: R \to R, f^{-1}(x) = -log_e(x-3)$

Question 6

The average value of the function $f(x) = 2sin\left(x - \frac{3\pi}{4}\right)$ over the interval $\left[\frac{1}{2},\ 3\right]$ is closest to:

A. -2.16

B. 2.16

C. -0.87

D. 0.87

E. -0.22

SECTION 1 – continued
TURN OVER

Question 7

For independent events A and B, $\Pr(A \cap B) = p, \Pr(A' \cap B') = \Pr(A \cap B') = p + \frac{1}{4}$ then the value of p is:

A. 0

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{2}$

E. $\frac{3}{8}$

Question 8

If the tangent to the graph of $f(x) = 2e^{x+2}$ at $x = a$ passes through the origin, then a equals:

A. 1

B. 0

C. -2

D. 2

E. 3

Question 9

The graph of $y = kx^2 - 5x$ intersects the graph of $x - y - 3 = 0$ at one point. The value of k is:

A. $\frac{1}{3}$

B. -3

C. 3

D. $\frac{4}{3}$

E. $\frac{3}{4}$

SECTION 1 – continued

Question 10

Let $f: R \to R$ be a differentiable function. The derivative of $f\left(\cos\left(g(x)\right)\right)$ is:

A. $f'\left(\cos\left(g(x)\right)\right) \times -\sin\left(g(x)\right)$

B. $f'\left(\cos\left(g(x)\right)\right) \times -\sin\left(g'(x)\right)$

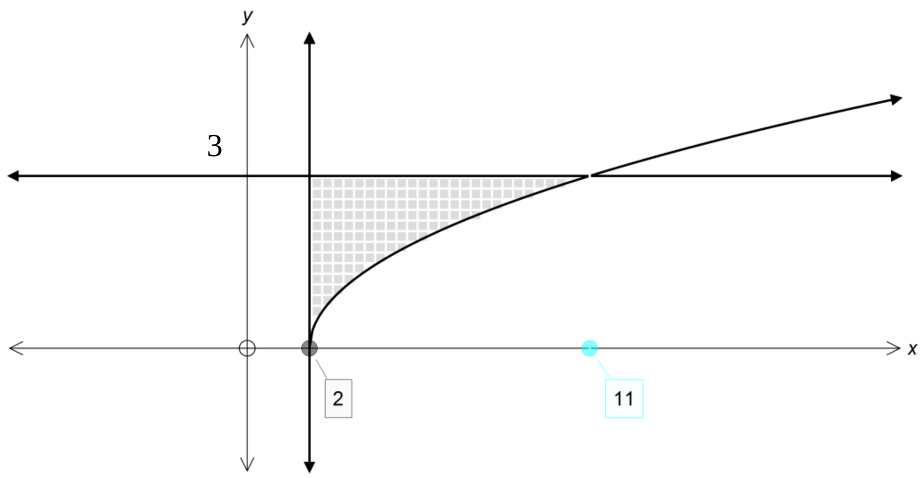
C. $f'\left(-\sin\left(g'(x)\right)\right)$

D. $f'\left(\cos\left(g(x)\right)\right) \times -\sin\left(g(x)\right) \times g'(x)$

E. $f'\left(\cos\left(g(x)\right)\right)$

Question 11

The graph of the function $f: [2, 11] \to R,\ f(x) = \sqrt{x-2}$ is shown below:



Which one of the following definite integrals could be used to find the area of the shaded region?

A. $\int_2^{11} \sqrt{x-2}\ dx$

B. $27 - \int_2^{11} \sqrt{x-2}\ dx$

C. $33 - \int_2^{11} \sqrt{x-2}\ dx$

D. $27 - \int_0^{3} (x^2+2)\ dx$

E. $33 - \int_0^{3} (x^2+2)\ dx$

SECTION 1 – continued

TURN OVER

Question 12

If $g(x) = x^3 + 1$ *and* $f(x) = \sqrt{2-x}$ then the largest possible range of g for which $f(g(x))$ is defined is:

A. $(-\infty, 2]$

B. $[-2, \infty)$

C. $(-1, \infty)$

D. R^+

E. $[0, 2]$

Question 13

A transformation $T: R^2 \to R^2$, $T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ maps the graph of a function *f* to the graph of $y = (x-1)^2$, $x \in R$. The rule of *f* is:

A. $y = (x-3)^2 - 1$

B. $y = -(x-3)^2 - 1$

C. $y = (x-3)^2 + 1$

D. $y = -(x-3)^2 + 1$

E. $y = x^2 - 4$

Question 14

A random variable *X* is normally distributed with mean 120 and standard deviation of σ . If $\Pr(X < 90) = \frac{3}{40}$, the value of σ is:

A. 7

B. 13

C. 17

D. 18

E. 21

SECTION 1 – continued

Question 15

Which of the following functions satisfies $f(2x) = 2f(x)$ for all values of x?

A. $f(x) = 2x^2$

B. $f(x) = \sqrt{2x}$

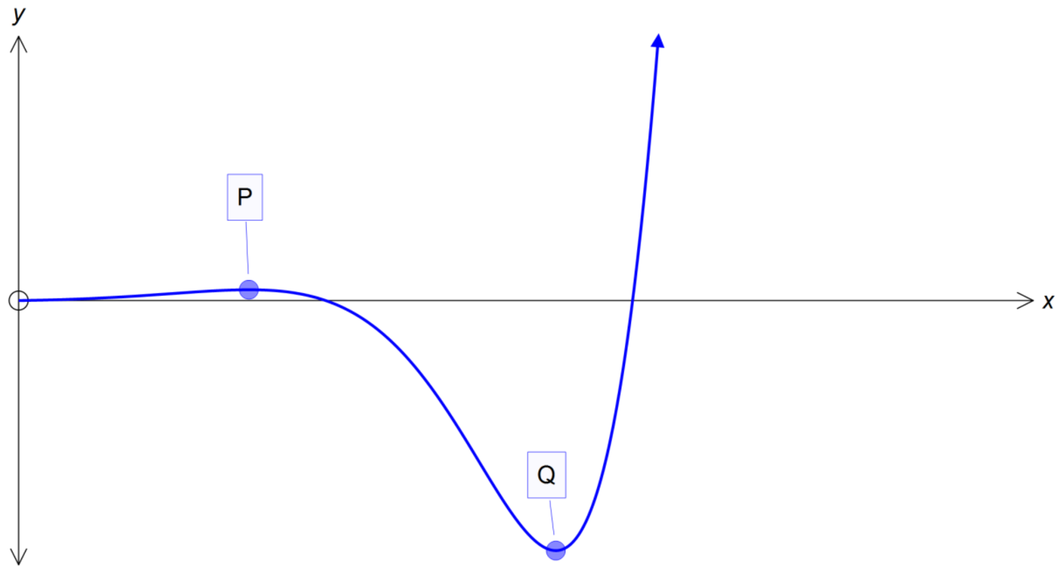
C. $f(x) = 2x$

D. $f(x) = x + 2$

E. $f(x) = \frac{1}{2}log_e x$

Question 16

Part of the graph of the function $f: (0, \infty) \to R, f(x) = 5e^x \sin(x)$ is shown in the diagram below.



The first two turning points are labelled as P and Q. The x-coordinate of the point Q is:

A. $\frac{\pi}{4}$

B. $\frac{7\pi}{4}$

C. $\frac{9\pi}{4}$

D. $\frac{5\pi}{6}$

E. π

SECTION 1 – continued
TURN OVER

Question 17

If $f'(x) = (x + a)^2$, where a is a real constant, and $f'(1) = 0$ then $f(x)$ is equal to:

A. $(x - 1)^3$

B. $\frac{(x)^3}{3} + 1$

C. $\frac{(x+1)^3}{3}$

D. $\frac{(x+1)^3}{3} - 1$

E. $\frac{(x-1)^3}{3}$

Question 18

Using Euler's rule for approximation, the best approximation for $\sqrt{109}$ is given by:

A. $f(10) + 9f'(10)$

B. $f(100) + 9f'(100)$

C. $f(121) - 12f'(121)$

D. $f(11) - 12f'(11)$

E. $f(100 - 9f'(100)$

Question 19

Let $f: R \to R$ be a function such that $f'(3) = 0$ and $f'(x) > 0$ when $x > 3$, and $f'(x) < 0$, when $x < 3$. The graph of f at $x = 3$ has a:

A. $local\ minimum$

B. $local\ maximum$

C. $staionary\ point\ of\ inflection$

D. $point\ of\ discontinuity$

E. $gradient\ 3$

Question 20

The probability of hitting a target with any particular arrow is 0.8. If 5 arrows are shot, the probability that 3 targets are missed can be best represented by:

A. $(0.2)^3$

B. $C(5,3)(0.8)^3(0.2)^2$

C. $C(5,3)(0.2)^3(0.8)^2$

D. $(0.2)^3(0.8)^2$

E. $(0.8)^3(0.2)$

SECTION 1 – continued

Question 21

Using the right rectangle approximation with rectangles of width 1, the area bounded by the curve $y = \frac{1}{x}$, the x-axis and the lines $x = 2$ and $x = 5$ is approximated by:

A. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

B. $2\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)$

C. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

D. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

E. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

Question 22

If the average rate of change of the function $f(x) = 2x^2 + ax - 3$ over the interval $[1, 3]$ is 7, the value of a is:

A. -1

B. 1

C. 15

D. 3

E. 0

END OF SECTION 1

TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (16 marks)

Consider the function $f(x) = x^3 e^{-2x}$.

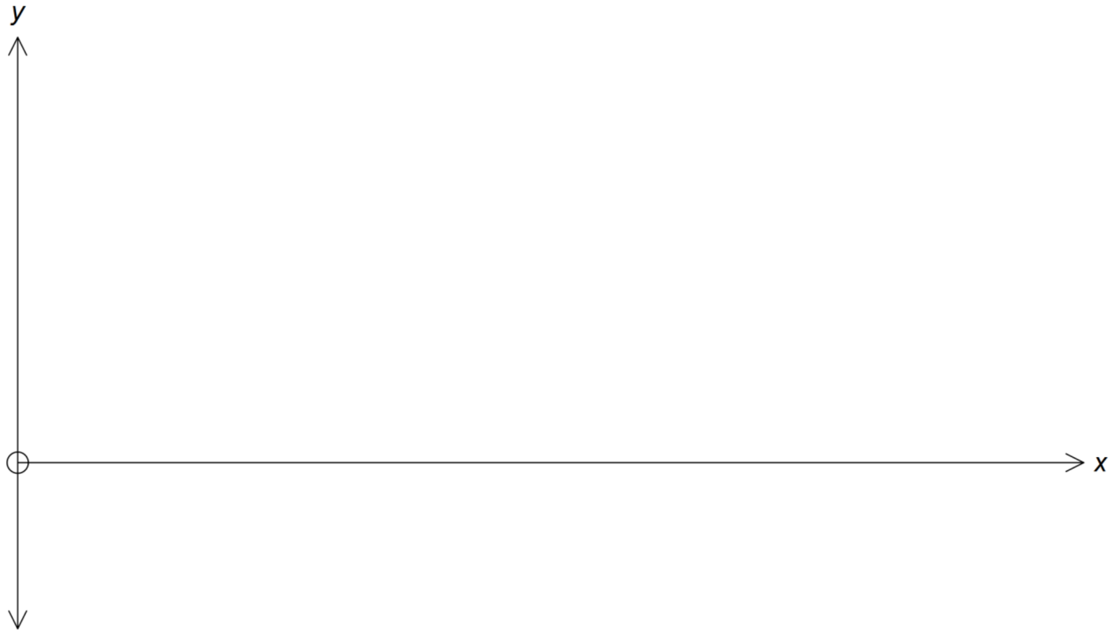
a. Find the values of a and b so that $f'(x) = x^a e^{-2x}(3 + bx)$. 3 marks

b. Find the coordinates of the axes intercepts, if any. 1 mark

SECTION 2 – Question 1 - continued

c. Find the stationary points of $f(x) = x^3 e^{-2x}$. 3 marks

d. Sketch the graph of $f(x)$ clearly labelling all axes intercepts and turning point(s). 4 marks



e. On the axes above, sketch the line $y = \frac{1}{10}x$, labelling the point(s) of intersection of the two graphs correct to three decimal places. 2 marks

SECTION 2 – Question 1 - continued

TURN OVER

f. Find the area, correct to 4 decimal places, between the curve $y = f(x)$ and the line $y = \frac{1}{10}x$. 3 marks

Question 2 (13 marks)

The distance, *d* metres, of water from a fixed point *O* on the sand at any time *t* hours after 9.00am is modelled by the equation

$$d(t) = 4 + 2sin\left(\frac{\pi(t+2)}{6}\right)$$

a. State the maximum distance of water from the fixed point and the time when it occurs in the first 24 hours. 3 marks

b. State the period of the function $d(t)$. 1 mark

SECTION 2 – Question 2 - continued

c. When will the distance of the water be 3.6m from *O* the second time in the first 24 hours?
3 marks

d. At what time(s) is the water closest to the fixed point on the sand, in the first 24 hours?
2 marks

Children are allowed to play on the sand when the water is 2.5 m or less from the fixed point *O*.

e. At what time(s) between 9am and 6pm are the children likely to play? 4 marks

SECTION 2 – continued
TURN OVER

Question 3 (15 marks)

Let $f: D \to R,\ f(x) = 2 - log_e(x+1)$ and D is the largest possible domain for which f is defined.

a. Find D. 1 mark

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b. Describe the sequence of transformations which when applied to the graph of $y = log_e x$, produces the graph of $f(x)$. 3 marks

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c. Find the rule for f^{-1}, the inverse of f . State the domain of f^{-1}. 4 marks

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d. Find the point of intersection of $f\ \ and\ f^{-1}$. 2 marks

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e. Use calculus to find the exact value of the gradient of the graph of f when x = 4. 2 marks

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SECTION 2 – Question 3 - continued

f. Find the equation of the normal to the graph of $y = f(x)$ when $x = 4$. 3 marks

Question 4 (14 marks)

The probability of a student getting into a TAFE institution is 0.15. Five students of the same school submit an application to gain entry into TAFE, correct to 4 decimal places.

a. Find the probability that none of the five gain entry into TAFE. 2 marks

b. Find the probability that only one of the five gain entry into TAFE. 2 marks

c. Find the probability that at least one of the five gain entry into TAFE. 2 marks

SECTION 2 – Question 4 - continued

TURN OVER

d. If 110 candidates were applying for TAFE, find the probability of between 15 to 20 (inclusive) students gaining an entry into TAFE. 3 marks

Over the past five years, the number of students gaining entry into TAFE is normally distributed with mean 20 700 and standard deviation 2915.

e. Find the probability of the number of TAFE students exceeding 21 000. 3 marks

f. Find the probability that a particular year will have between 18 000 and 25 000 TAFE students. 2 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name:___________________________________

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E