

## 2014 Trial Examination

STUDENT NUMBER

Figures									Letter
Words									

# MATHEMATICAL METHODS (CAS)

## Units 3 & 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

### QUESTION & ANSWER BOOK

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

#### Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.

#### Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1(6 marks)**

- a. If  $f(x) = 2x^2 \log_e(3x)$ , find  $f'(x)$ . 2 marks

---

---

---

---

- b. Find  $f'\left(\frac{1}{3}\right)$ . 1 mark

---

---

- c. Find the value(s) of  $x$  for which  $f'(x) = 0$ . 3 marks

---

---

---

---

---



**Question 3 (2 marks)**

Solve the equation  $2 \cos\left(\frac{x}{3}\right) + 1 = 0$ ,  $x \in [3\pi, 6\pi]$

---

---

---

---

---

---

---

**Question 4 (9 marks)**

**a.** Find the value of  $k$  for which  $4e^{2x} - ke^x + 9 = 0$  has a unique solution. 2 marks

---

---

---

**b.** Find the unique solution for the equation  $4e^{2x} - ke^x + 9 = 0$  for the positive value of  $k$  found in part **a.** 3 marks

---

---

---

---

---

**Question 4 - continued**

Let  $g(x) = 2e^x - 3$

- c. Find the inverse,  $g^{-1}(x)$ , of the function  $g(x)$ . 2 marks

---

---

---

---

---

---

---

- d. State the domain and range of  $g^{-1}(x)$ . 2 marks

---

---

---

---

**TURN OVER**



**Question 6 (6 marks)**

The probability distribution of a discrete random variable,  $X$ , is given by the table below.

$X$	0	1	2	3	4
$Pr(X)$	$\frac{1}{2k}$	$\frac{k}{10}$	$\frac{k}{10}$	$\frac{3}{10k}$	$\frac{2}{5k}$

- a.** Show that  $k = 3$  or  $k = 2$ . 3 marks

---



---



---



---



---

Let  $k = 3$ .

- b.** Calculate  $E(X)$ . 2 marks

---



---



---

- c.** Find  $Pr(X \leq E(X))$ . 1 mark

---



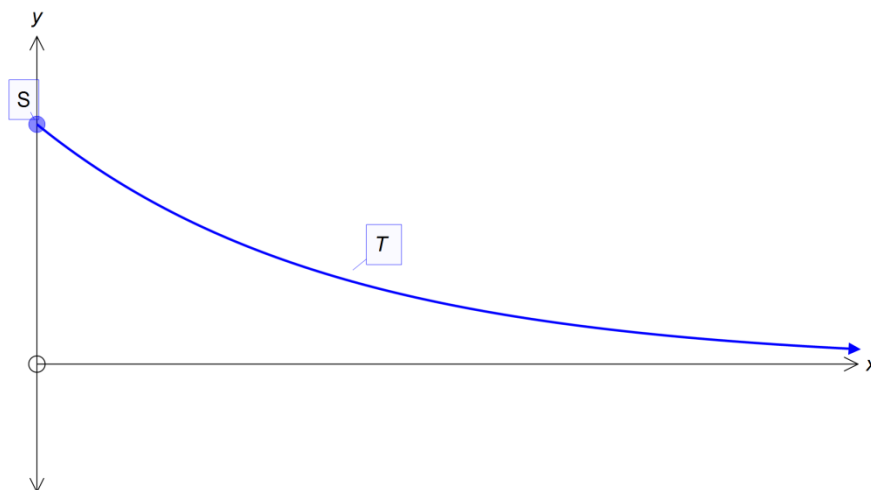
---

**TURN OVER**

**Question 7 (4 marks)**

Let  $f: [0, \infty) \rightarrow R$ ,  $f(x) = 2e^{-\frac{x}{3}}$

Let  $S$  be the point on the graph of  $f$  on the  $y$ -axis and let  $T$  be the point on the graph of  $f$  with the  $y$ -coordinate  $\frac{1}{2}$ .



Find the area of the region bounded by the graph of  $f$  and the line segment  $ST$ .

---



---



---



---



---



---



---



---



---



---



**Question 8 (3 marks)**

A continuous random variable,  $X$ , has a probability density function

$$f(x) = \begin{cases} \frac{1}{2} \sin(x) & 0 \leq x \leq \pi \\ 0 & \text{elsewhere} \end{cases}$$

**a.** Find the median of  $X$ .

2 marks

---

---

---

---

---

**b.** Find the mode of  $X$ .

1 mark

---

---

---

---

---

**END OF QUESTION AND ANSWER BOOK**