2014 Mathematical Methods (CAS) Examination 1

GENERAL COMMENTS

In general, students were able to comprehend what was required by a question and demonstrated the formulation skills that would lead to a viable solution. However, often poor arithmetic skills, mainly when operating with fractions, or poor algebraic manipulations, especially with transpositions and dealing with negatives, led to errors. This was particularly evident in Questions 1b., 3, 4, 5c., 6, 7, 8a., 9b. and 10bi.

Students are reminded that when dealing with fractions, final answers should be simplified. Answers such as $\frac{15}{32}+\frac{3}{24}$, $\frac{1+0.5}{2}$ and $\frac{9}{6}$ are considered incomplete, and are not awarded an answer mark.

Most students were able to apply the product rule successfully in Question 1a. However, the application of the chain rule in Question 1b., was not as well handled. Incorrect application of rules, particularly those for anti-differentiation and logarithmic laws was evident in Questions 2, 6 and 8.

Some students highlighted key components of a question. This strategy can help to reduce transcription errors and to ensure that the specific question is answered in its entirety. Checking the final answer against highlighted question components can also be beneficial.

As is often the case, students who produced responses that were detailed, clearly communicated and precise generally scored well. Questions worth more than one mark require methodology and an obvious conclusion. A diagram such as a unit circle in Question 3 or tree diagram in Question 9 can be an invaluable aid in formulating and then presenting reasoning.

Students' notation was good, especially with respect to *dx* and use of the given variables in Questions 1–8 and 10. It should be noted that coordinates are represented by curved brackets as square brackets define an interval of values, not a point.

While handwriting is not assessed, students should ensure that their writing is legible so that their working and intended answers are clear. In particular, π was written like x in Question 7 and students confused the two in their working.

Questions 8b., 9bii. and 10 were the most challenging questions for students. In Question 8b. many students did not extend the concepts of part a. to follow through to part b. Question 9bii. involved conditional probability. Question 10 could have been formulated in a variety of ways, but in all parts required students to set up an equation that was manipulated to produce a final answer. More practice with this style of question is advised.

SPECIFIC INFORMATION

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Question 1a.

Marks	0	1	2	Average
%	8	4	88	1.8

$$\frac{dy}{dx} = x^2 \cos(x) + 2x\sin(x)$$

Although this question was generally very well handled, some students made errors in an attempt to factorise, which was not necessary.

Question 1b.

Marks	0	1	2	3	Average
%	10	16	18	55	2.2

$$f'(x) = \frac{1}{2}\left(x^2+3\right)^{\frac{-1}{2}} \times 2x = \frac{x}{\sqrt{x^2+3}}$$

$$f'(1) = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

Many students only gave only gave the expression for $f'(x)$, not the specific value of $f'(1)$. Students should also note that $\frac{1}{\sqrt{4}} \neq \pm\frac{1}{2}$, $\frac{1}{\sqrt{4}} = \frac{1}{2}$.

Question 2

Marks	0	1	2	Average
%	18	28	54	1.4

$$\frac{1}{2} \times 2\left[\log_e\left(|2x-1|\right)\right]_4^5 = \log_e(9) - \log_e(7)$$

$$b = \frac{9}{7}$$

This was a routine application of $\int\left(\frac{a}{ax+b}\right)dx = \log_e\left(|ax+b|\right) + c$.

Question 3

Marks	0	1	2	Average
%	19	27	55	1.4

$2x = \frac{5\pi}{6}, \frac{7\pi}{6}$

$x = \frac{5\pi}{12}, \frac{7\pi}{12}$

Many students were unsure of exact values and did not ascertain the correct basic angle of $\frac{\pi}{6}$ or produced solutions beyond the specified domain.

Question 4

Marks	0	1	2	Average
%	7	18	76	1.7

$2^{3x-3} = \left(2^3\right)^{2-x}$

$3x - 3 = 6 - 3x$

$x = \frac{3}{2}$

Some students chose to work with a common base of 8. Students are reminded to simplify their final answer, especially for fraction answers.

Question 5a.

Marks	0	1	2	Average
%	14	20	66	1.5

$f'(x) = 0$

$6x - 3x^2 = 0$

$3x(2 - x) = 0$

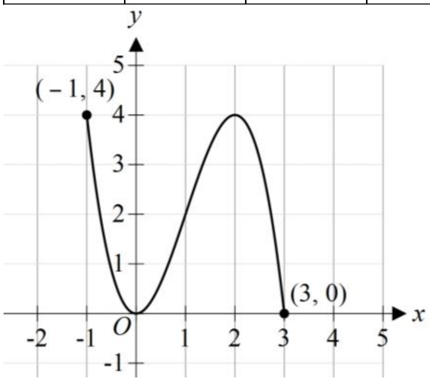
$\therefore x = 0, 2$

Coordinates $(0,0)$ and $(2,4)$

Students must be vigilant in ensuring they answer the specific question. In this question, coordinates were required and not simply *x* values. Some students omitted the turning point (0,0) and others incorrectly found *x* intercepts.

Question 5b.

Marks	0	1	2	Average
%	21	15	63	1.4



The correct response to this question required a smooth curve (not V shapes at turning points) and endpoints clearly labelled with their coordinates.

Question 5c.

Marks	0	1	2	3	Average
%	25	27	27	21	1.5

Method1

$$A = 12 - \int_{-1}^{2} \left(3x^2 - x^3\right) dx$$

$$= 12 - \left[x^3 - \frac{x^4}{4}\right]_{-1}^{2}$$

$$= 12 - \left[(8-4) - \left(-1 - \frac{1}{4}\right)\right]$$

$$= 6\frac{3}{4} \left(\text{or } \frac{27}{4} \; or \; 6.75\right)$$

OR

Method 2

$$\int_{-1}^{2} \left(4 - (3x^2 - x^3)\right) dx$$

$$= \left[4x - x^3 + \frac{x^4}{4}\right]_{-1}^{2}$$

Most students knew to seek a difference of two areas and were adept with basic integration; however, quite often arithmetic errors in evaluations or the incorrect use of negative signs marred their progress towards acquiring full marks. Some students unnecessarily ‘overworked’ the problem by creating three or four integrations, increasing the likelihood of an error. A few students took a more direct route that involved symmetry of the curve.

Question 6

Marks	0	1	2	Average
%	23	33	44	1.2

$$\log_e\left(\frac{x}{\sqrt{x}}\right) = 3$$

$$\frac{x}{\sqrt{x}} = e^3$$

$$\sqrt{x} = e^3$$

$$x = e^6$$

Poor performance on this question was mainly attributed to the incorrect application of logarithm or index laws. Students should have good facility with logarithm and exponent laws.

Question 7

Marks	0	1	2	3	Average
%	19	12	30	38	1.9

$f(x) = 2\sin(x) + \frac{1}{2}\cos(2x) + c$

since $f\left(\frac{\pi}{2}\right) = \frac{1}{2}$ $\quad \frac{1}{2} = 2\sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2\times\frac{\pi}{2}\right) + c$

$\frac{1}{2} = 2 - \frac{1}{2} + c$

$c = -1$

$\therefore\ f(x) = 2\sin(x) + \frac{1}{2}\cos(2x) - 1$

The students who omitted a constant of integration (+*c*) when completing the anti-differentiation were then unable to find the specific equation required.

Question 8a.

Marks	0	1	2	Average
%	32	25	43	1.1

$\int_0^m \left(\frac{1}{5}e^{-\frac{x}{5}}\right)dx = \frac{1}{2}$

$\left[-e^{-\frac{x}{5}}\right]_0^m = \frac{1}{2}$

$-e^{-\frac{m}{5}} + 1 = \frac{1}{2}$

$m = -5\log_e\left(\frac{1}{2}\right)$ or $m = 5\log_e(2)$ *or* $\log_e(32)$

While most students knew to set up a definite integral involving *m* (median) and equating to ½, errors with mishandling negatives in the anti-differentiation or problems with algebraic skills in solving an indicial equation worked against progress in obtaining the correct answer. Some students confused median with the mean or the law of total probability.

Question 8b.

Marks	0	1	2	Average
%	48	28	24	0.8

$$\Pr(X < 1 \mid X \le m) = \frac{\Pr(X < 1)}{\Pr(X \le m)}$$

$$= \frac{\int_0^1 \left(\frac{1}{5}e^{-\frac{x}{5}}\right)dx}{\left(\frac{1}{2}\right)}$$

$$= 2\left(1 - e^{-\frac{1}{5}}\right)$$

This question was not well handled. The most common error was assuming that the value of m (obtained in part a.) was equivalent to $\Pr(X \le m)$.

Question 9a.

Marks	0	1	2	Average
%	42	8	50	1.1

Pr(at least 1 morning walk) = 1 – Pr (no walk)

$$=1-\frac{1}{4}\times\frac{2}{3}=\frac{5}{6} \qquad \text{OR}$$

$w = walk$

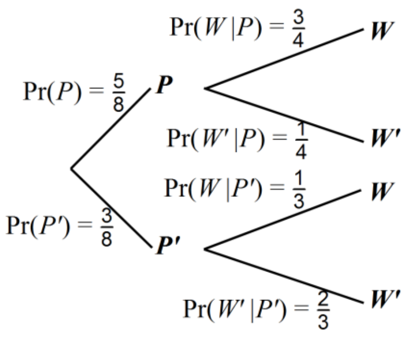
$\Pr(w'w + ww' + ww)$

$$=\frac{3}{4}\times\frac{1}{3}+\frac{3}{4}\times\frac{2}{3}+\frac{1}{4}\times\frac{1}{3}=\frac{5}{6}$$

Many students attempted to use matrices but did not recognise the basic nature of the problem. A tree diagram or listing the sample space were the best options.

Question 9bi.

Marks	0	1	2	Average
%	40	17	44	1.1



$$\Pr(W)=\frac{5}{8}\times\frac{3}{4}+\frac{3}{8}\times\frac{1}{3}=\frac{19}{32}$$

Many students made a good attempt at this question. Most students correctly identified the required sum of two products; however, made errors in the evaluation of the final fraction.

Question 9bii.

Marks	0	1	2	Average
%	52	21	27	0.8

$$\Pr(P\,|\,W)=\frac{\Pr(W\cap P)}{\Pr(W)}=\frac{\left(\frac{5}{8}\times\frac{3}{4}\right)}{\frac{19}{32}}=\frac{15}{19}$$

Many students were able to identify the conditional probability and use their answer to part bi. in the denominator; however, used an incorrect numerator.

Question 10a.

Marks	0	1	2	3	Average
%	28	30	12	30	**1.5**

Substitute $x = 2$ and $y = 4$

$$4 = 4a + 2b \quad \text{...equation 1}$$

$$\frac{dy}{dx} = 2ax + b = \frac{0-4}{6-2} \text{ at } x = 2 \therefore -1 = 4a + b \quad \text{...equation 2}$$

Solve simultaneously, $a = -\frac{3}{2}$ and $b = 5$

Most students knew to set up two equations to solve simultaneously. The most common error was to substitute $x = 6$ and $y = 0$ into $y = ax^2 + bx$, stating incorrectly that the point (6, 0) lay on the parabola.

Question 10bi.

Marks	0	1	Average
%	68	32	**0.3**

ΔVOU similar to ΔQPU

or $m_{VU} = m_{UQ}$

$$\frac{v}{u} = \frac{4}{u-2}$$

$$v = \frac{4u}{u-2}$$

or alternatively

$m_{VQ} = m_{UQ}$

$$\frac{v-4}{0-2} = \frac{0-4}{u-2}$$

$$v = \frac{8}{u-2} + 4$$

$$= \frac{8 + (4u - 8)}{u-2} = \frac{4u}{u-2}$$

Some students were able to set up a suitable equation; however, errors in algebraic manipulation (or poor setting out of working) often resulted in incorrect final answers.

Question 10bii.

Marks	0	1	2	Average
%	74	17	9	0.4

$$\begin{aligned} A &= \frac{1}{2}\times u\times v - 8 \\ &= \frac{1}{2}\times u\times \frac{4u}{u-2} - 8 \\ &= \frac{2u^2}{u-2} - 8 \end{aligned}$$

or

$$\begin{aligned} A &= \frac{1}{2}\times u\times v - 8 \\ &= \frac{1}{2}\times u\times \left(\frac{8}{u-2} + 4\right) - 8 \\ &= \frac{4u}{u-2} + 2u - 8 \end{aligned}$$

For local max/min

$$\frac{dA}{du} = \frac{(u-2)\times 4u - 2u^2}{(u-2)^2} = 0$$

$\therefore 2u^2 - 8u = 0$

$u = \not{0}$ or $u = 4$

$u = 4$ and $A = 8$

or

$$\frac{dA}{du} = -\frac{8}{(u-2)^2} + 2 = 0$$

$\therefore (u-2)^2 - 4 = 0$

$(u-2+2)(u-2-2) = 0$

$u = 4,\ A = 8$

Test area at endpoint

When $u = \frac{5}{2}, A = 17$ and $u = 6, A = 10$

$\therefore$ min when $u = 4$ (not at an endpoint).

Thus, the final answer is u = 4 and minimum area = 8 (sq units).

This question was answered poorly. A complete solution to this question required setting up an equation for the area in one variable and then testing turning points as well as endpoints to determine the minimum value. Many students set up overly complex equations for the area or had difficulty in differentiating their equation correctly. Other students did the reverse by simply assuming u = 6.

Question 10biii.

Marks	0	1	Average
%	92	8	0.1

Maximum at the lower endpoint

When $u = \frac{5}{2}, A = 17$ (sq units)

This question was also answered poorly. Many students who attempted this question incorrectly assumed that the value of the maximum area occurred at a local stationary point. Some students only gave a partial answer to the question and not the values for both u and area.