TEST 1

Technology active end-of-year examination
Functions and graphs
Section A: 15 marks
Section B: 25 marks
Suggested writing time: 60 minutes

Section A: Multiple-choice questions

Specific instructions to students

- A correct answer scores 1 mark, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one letter is shaded on the answer sheet.
- Choose the alternative that most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 A B C D — USE PENCIL ONLY

- Use pencil only.
- Working space is provided under those questions that require working out.

QUESTION 1

The maximal domain for the function $y = 2\sqrt{2 - x}$ is

A R

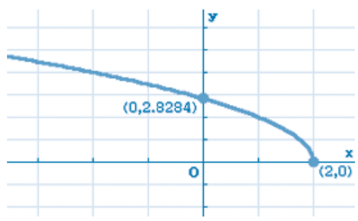
B $[2, \infty)$

C $(2, \infty)$

D $(-\infty, 2)$

E $(-\infty, 2]$

$y = 2\sqrt{2 - x}$ has the graph below with domain $(-\infty, 2]$.



QUESTION 2

The derivative of the function $f(x) = 3 - 7x$ is $f'(x) =$

A 3

B –3

C –7

D 7

E $3x - \frac{7x^2}{2}$

$f(x) = 3 - 7x$ so $f'(x) =$ gradient $= -7$

QUESTION 3

The rule of the inverse of the function $f(x) = 3 - 7x$ is $f^{-1}(x) =$

A $\frac{3 - x}{7}$

B $\frac{x - 3}{7}$

C $3 - 7y$

D $3 - 7x$

E $7 - 3x$

Let $y = 3 - 7x$
Swap x and y and rearrange to determine the inverse.
$x = 3 - 7y$
So $f^{-1}(x) = \frac{3 - x}{7}$

QUESTION 4

For $f(x) = \begin{cases} x, & x \geq 0 \\ 1, & x < 0 \end{cases}$

$f(2) =$

A 1

B 2

C 3

D 4

E 0

$f(x) = \begin{cases} x, & x \geq 0 \\ 1, & x < 0 \end{cases}$

$x = 2$ is in domain $x \geq 0$, so $f(2) = 2$

QUESTION 5

For $f(x) = \begin{cases} x^2, & x \geq 0 \\ 1, & x < 0 \end{cases}$

$f'(2) =$

A 1

B 2

C 3

D 4

E 0

$f(x) = \begin{cases} x^2, & x \geq 0 \\ 1, & x < 0 \end{cases}$

$x = 2$ is in domain $x \geq 0$, so $f'(2) =$ gradient at $x = 2$
Answer = 4.

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QUESTION 6

The domain and range, respectively, of the function

$y = -\frac{2}{(x-1)^2} + 1$ are

A R and R

B $R\backslash\{0\}$ and $R\backslash\{0\}$

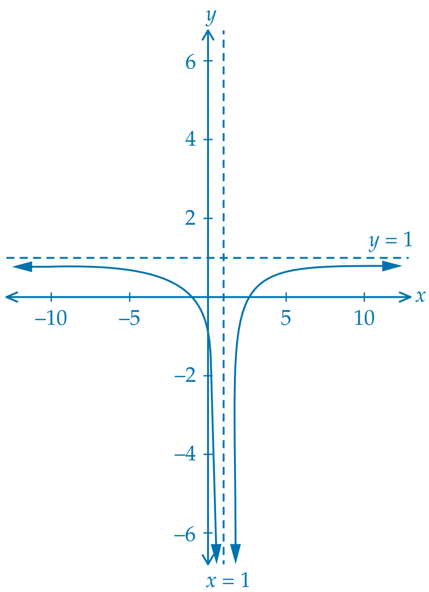
C $R\backslash\{0\}$ and $R\backslash\{1\}$

D $R\backslash\{1\}$ and $(-\infty, 2)$

E $R\backslash\{1\}$ and $(-\infty, 1)$

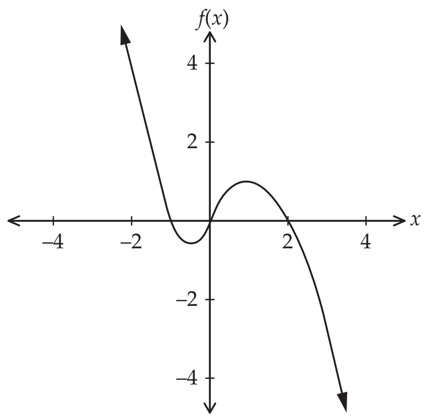
$y = -\frac{2}{(x-1)^2} + 1.$

Domain = $R\backslash\{1\}$ and range = $(-\infty, 1)$.



QUESTION 7

A cubic graph is shown.



The equation of the graph could be

A $y = (x + 1)(x - 1)(x - 2)$

B $y = x(x + 1)(x - 2)$

C $y = 0.5x(x + 1)(2 - x)$

D $y = 0.5x(x + 1)(x - 2)$

E $y = -0.5(x + 1)(x - 1)(x - 2)$

–ve cubic graph with x-intercepts of –1, 0, 2.

The equation of the graph could be

$y = 0.5x(x + 1)(2 - x)$

QUESTION 8

The graph of $y = e^{x+1} - 2$ has asymptotes at

A $y = -2$ only

B $y = -2$ and $x = -1$

C $x = -1$ only

D $x = -2$ and $y = -1$

E $y = 1$ only

$y = e^{x+1} - 2$ has an asymptote at $y = -2$ only.

QUESTION 9

The graph of $y = 2\log_e(2x + 2)$ has asymptotes at

A $y = -2$ only

B $y = 2$ and $x = -1$

C $x = -1$ only

D $x = -2$ and $y = -1$

E $x = -2$ only

$2x + 2 = 0 \Rightarrow x = -1$

$y = 2\log_e(2x + 2)$ has an asymptote at $x = -1$ only.

QUESTION 10

The graph of $y = -\cos(2x)$ has a period and amplitude respectively of

A 1 and 2π

B 2π and 1

C π and 1

D π and –1

E 1 and π

$y = -\cos(2x)$ has a period $= \frac{2\pi}{2} = \pi$ and amplitude = 1.

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QUESTION 11

The coordinates of A and B are (–1, 1) and (0, 1) respectively. The equation of the line that passes through the points A and B is

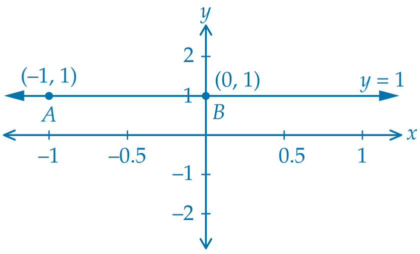
A $y = 1$

B $x = 0$

C $y = x$

D $x = -1$

E $y = x + 1$



The y-coordinate in the two points (–1, 1) and (0, 1) is the same. The equation of the line is $y = 1$.

QUESTION 12

The coordinates of A and B are (–1, 1) and (0, 1) respectively. The coordinates of the midpoint between A and B are

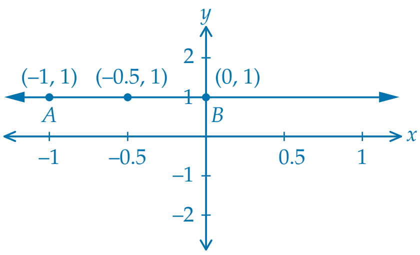
A (0, 0)

B (0, 1)

C (–0.5, 0)

D (–0.5, 1)

E (0.5, 2)



The y-coordinate in the two points (–1, 1) and (0, 1) is the same, so find the middle x value.

Midpoint = (–0.5, 1)

QUESTION 13

The coordinates of A and B are (1, 1) and (2, 4), respectively. The distance between A and B is

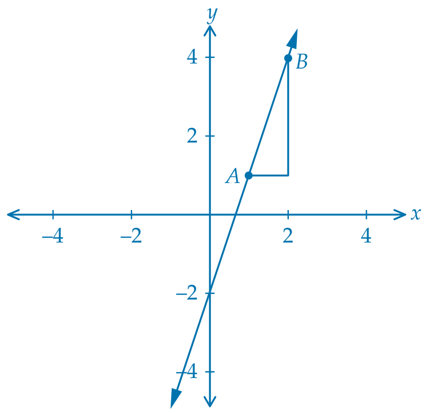
A 10

B $\sqrt{10}$

C $\pm\sqrt{10}$

D 2

E 4



(1, 1) and (2, 4).

Distance = $\sqrt{(2-1)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10}$

QUESTION 14

The maximal domain of the inverse of the graph of $y = -(x-2)^2 - 1$ is

A R

B $[2, \infty)$

C $(-1, \infty)$

D $(-\infty, 1]$

E $(-\infty, -1]$

Upside-down parabola with turning point (2, –1).
Domain of inverse = range of $y = -(x-2)^2 - 1$, and is $(-\infty, -1]$.

QUESTION 15

An equivalent expression to $y = a^{2x}$ is

A $x = \log_a(y)$

B $y = \log_a(x)$

C $x = \frac{1}{2}\log_a(y)$

D $y = \frac{1}{2}\log_a(2x)$

E $x = a^{2y}$

$y = a^{2x} \Leftrightarrow 2x = \log_a(y)$

$\Rightarrow x = \frac{1}{2}\log_a(y)$

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ONE ANSWER PER LINE — **USE PENCIL ONLY**

	A	B	C	D	E
1	A	B	C	D	■
2	A	B	■	D	E
3	■	B	C	D	E
4	A	■	C	D	E
5	A	B	C	■	E
6	A	B	C	D	■
7	A	B	■	D	E
8	■	B	C	D	E
9	A	B	■	D	E
10	A	B	■	D	E
11	■	B	C	D	E
12	A	B	C	■	E
13	A	■	C	D	E
14	A	B	C	D	■
15	A	B	■	D	E

Section B: Extended-response questions

Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 16

A polynomial is defined as $P(x) = ax^3 + bx^2 + 2x + 10$

a It is known that $x - 2$ is a factor of $P(x)$ and that if $P(x)$ is divided by $x + 1$, the remainder is –3. Show that $a = \frac{5}{2}$ and $b = -\frac{17}{2}$

Let $f(x) = ax^3 + bx^2 + 2x + 10$

And $f(2) = 0$

And $f(-1) = -3$

define $f(x)=ax^3+bx^2+2x+10$ done

$\begin{cases} f(2)=0 \\ f(-1)=-3 \end{cases} \Big| a, b$

$\left\{a=\frac{5}{2}, b=-\frac{17}{2}\right\}$

Solve simultaneously to get $a = \frac{5}{2}$ and $b = -\frac{17}{2}$

(For a 'show that' question it would be necessary to solve these equations simultaneously by hand.)

3 marks

b Hence find the solutions for the equation $y = P(x) = 0$.

$y = P(x) = \frac{5}{2}x^3 - \frac{17}{2}x^2 + 2x + 10$

We know that $f(2) = 0$, so $x - 2$ is a factor.

By division, the quadratic factor is $\frac{5}{2}x^2 - \frac{7}{2}x - 5$

propFrac$\left(\left(\frac{5}{2}x^3-\frac{17}{2}x^2+2x+10\right)/(x-2)\right)$

$\frac{5\cdot x^2}{2}-\frac{7\cdot x}{2}-5$

$y = (x-2)\left(\frac{5}{2}x^2 - \frac{7}{2}x - 5\right)$

$= \frac{1}{2}(x-2)(5x^2 - 7x - 10)$

rFactor$\left(\frac{5\cdot x^2}{2}-\frac{7\cdot x}{2}-5\right)$

$\frac{5\cdot\left(x+\frac{\sqrt{249}}{10}-\frac{7}{10}\right)\cdot\left(x-\frac{\sqrt{249}}{10}-\frac{7}{10}\right)}{2}$

$y = \frac{5}{2}(x-2)\left(x - \frac{7 \pm \sqrt{249}}{10}\right)$

$y = 0$ at $x = 2$, $x = \frac{7 \pm \sqrt{249}}{10}$

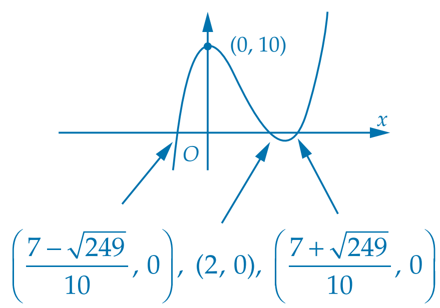
Or **solve** can be used.

solve$\left(\frac{5}{2}\cdot x^3-\frac{17}{2}x^2+2\cdot x+10=0, x\right)$

$\left\{x=2, x=\frac{-\sqrt{249}}{10}+\frac{7}{10}, x=\frac{\sqrt{249}}{10}+\frac{7}{10}\right\}$

3 marks

c Sketch the graph of $y = P(x)$, labelling the coordinates of any axial intercepts.



4 marks

A section of the graph drawn in part **c** represents a proposed ski slope, where y metres is the height above ground level and x metres is the horizontal distance from the end of the ski slope. The ski slope ends at the point (0, 10).

Consider $f: [0, 5] \to \boldsymbol{R}, f(x) = \frac{5}{2}x^3 - \frac{17}{2}x^2 + 2x + 10$

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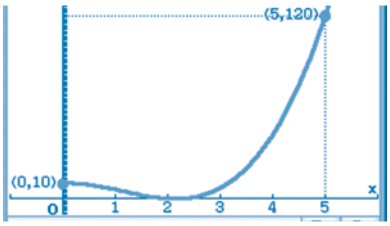
d Find the maximim height of the ski slope.



Max at the beginning = 120 m

2 marks

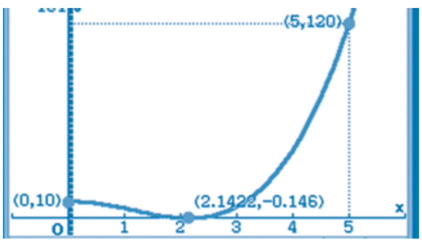
e How high will the engineers have to build the platform for the end of the slope?

The ski slope ends at the point (0, 10).

Height at the end = 10 m

1 mark

f The engineers do not permit the ski slope to be opened if the path goes underground. Will the ski slope go ahead?



The minimum point (2.14, –0.15) goes underground to a depth of approximately 0.15 m.

The ski slope will not go ahead.

2 marks

(Total: 15 marks)

QUESTION 17

A jet of water from a hose is pointed over the fence by children into their neighbour's garden. The water stream from the hose starts at a distance from the ground of 2.88 metres. Amy, who is holding the hose, is standing at a horizontal distance of 14 metres from the fence. The water goes over the fence and lands on the other side in the neighbour's garden at a horizontal distance of 4 metres from the fence.

a Use the function $h(x) = a + b(x + 6)^2$ to model the path of the water in the air, where a and b are constants and the variables, h and x, are in metres. Find the values of a and b.

define h(x)=a+b(x+6)^2

done

{h(−14)=2.88, h(4)=0 | a, b

$\left\{a=8,\ b=-\frac{2}{25}\right\}$

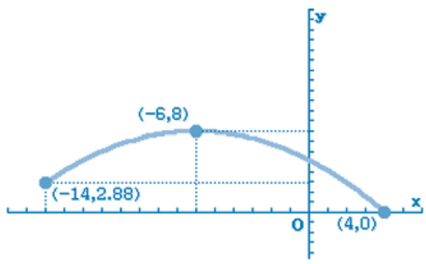
Define the function $h(x) = a + b(x + 6)^2$

and use $h(4) = 0$ and $h(-14) = 2.88$

Solve simultaneously to get $a = 8$ and $b = -\frac{2}{25}$.

3 marks

b Sketch the function $h(x) = a + b(x + 6)^2$ that models the path of the water in the air, with your values of a and b. Use the domain $x \in [-14, 4]$.



2 marks

c The neighbours are sick of the children next door and have built a fence that is 5 metres high. Will the water go over the fence? Why/why not?

y-intercept (0, 5.12)

$5.12 > 5$

Yes, the water will go over the fence.

2 marks

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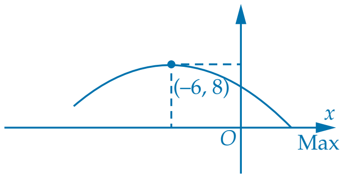
d There are power lines above the children's garden that are 10 metres off the ground. Will the water hit the power lines?

Maximum point is (–6, 8).

$8 < 10$

Water will not hit the power lines.



1 mark

e The children's dog likes to jump up and lap at the water in the air. The dog, Scruffy, can jump 2 metres in the air to reach the water. Where does Scruffy manage to drink the water?

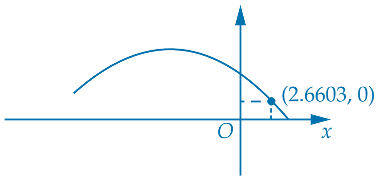
Solve $h(x) = 2$

This gives $x = -6 \pm 5\sqrt{3}$

The only solution in the domain $x \in [-14, 4]$ is

$x = -6 + 5\sqrt{3} \approx 2.66$

Scruffy will reach the water after the maximum point and in the neighbour's garden, approximately 2.66 metres beyond the fence and until it reaches the ground.



2 marks

(Total: 10 marks)

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