## Section A: Multiple-choice questions

**Specific instructions to students**

- A correct answer scores 1 mark, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one letter is shaded on the answer sheet.
- Choose the alternative that most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 A B C D — **USE PENCIL ONLY**

- Use pencil only.
- Working space is provided under those questions that require working out.

### QUESTION 1

If $y = 3\log_e\left(\sqrt{x}\right)$, $\dfrac{dy}{dx} =$

**A** $\dfrac{3}{2x}$

**B** $\dfrac{3}{2\sqrt{x}}$

**C** $\dfrac{3}{x}$

**D** $3\sqrt{x}\log_e\left(\sqrt{x}\right)$

**E** $\dfrac{3}{2}$

$y = 3\log_e\left(\sqrt{x}\right) = 3\log_e\left(x^{\frac{1}{2}}\right) = \dfrac{3}{2}\log_e(x)$

$\dfrac{dy}{dx} = \dfrac{3}{2x}$

$\dfrac{d}{dx}(3\cdot\ln(\sqrt{x}))$ → $\dfrac{3}{2\cdot x}$

### QUESTION 2

Find $f'(x)$ if $f(x) = e^x - 3e^{-x}$

**A** $f'(x) = e^x - 3e^{-x}$

**B** $f'(x) = e^x + 3e^{-x}$

**C** $f'(x) = e^x + 3$

**D** $f'(x) = -2e^{-x}$

**E** $f'(x) = 6e^{2x}$

$f(x) = e^x - 3e^{-x}$
$f'(x) = e^x + 3e^{-x}$

### QUESTION 3

Find $f'(\pi)$ if $f(x) = 4x^3 \sin(2x)$

**A** $4x^2(3\sin(2x) + 2x\cos(2x))$

**B** $4\pi^2(3\pi + 2\pi^2)$

**C** $0$

**D** $4\pi^2$

**E** $8\pi^3$

$f(x) = 4x^3\sin(2x)$
$f'(\pi) = 8\pi^3$

diff$(4\cdot x^3\cdot\sin(2\cdot x), x, 1, \pi)$ → $8\cdot\pi^3$

### QUESTION 4

If $f'(x) = 3 - e^{2x-2}$, the anti-derivative function $f(x)$ is equal to

**A** $3x - 2e^{2x-2} + c$

**B** $3x - \dfrac{1}{2}e^{2x-2}$

**C** $3x - \dfrac{1}{2}e^{2x-2} + c$

**D** $\dfrac{1}{2}e^{2x-2} + c$

**E** $-2e^{2x-2} + c$

$f'(x) = 3 - e^{2x-2}$
$f(x) = 3x - \dfrac{1}{2}e^{2x-2} + c$

$\int 3 - e^{2\cdot x - 2}\,dx$ → $\dfrac{-e^{2\cdot x-2}}{2} + 3\cdot x$

### QUESTION 5

For $f(x) = \begin{cases} x+1, & x \ge 0 \\ e^x, & x < 0 \end{cases}$ $\quad f'(x) =$

**A** $\begin{cases} x, & x \ge 0 \\ e^x, & x < 0 \end{cases}$

**B** $\begin{cases} 1, & x \ge 0 \\ e^x, & x < 0 \end{cases}$


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**C** $\begin{cases} x, & x > 0 \\ e^x, & x < 0 \end{cases}$

**D** $\begin{cases} 1, & x > 0 \\ e^x, & x < 0 \end{cases}$

**E** $\begin{cases} 1, & e^x > 0 \\ e^x, & e^x < 0 \end{cases}$

$f(x) = \begin{cases} x+1, & x \geq 0 \\ e^x, & x < 0 \end{cases}$

$f'(x) = \begin{cases} 1, & x > 0 \\ e^x, & x < 0 \end{cases}$

## QUESTION 6

For $f(x) = \begin{cases} x+1, & x \geq 0 \\ e^x, & x < 0 \end{cases}$ $\quad f'(-1) =$

**A** $\begin{cases} 0, & x \geq 0 \\ e^{-1}, & x < 0 \end{cases}$

**B** $\begin{cases} 1, & x \geq 0 \\ e^{-1}, & x < 0 \end{cases}$

**C** $e^{-1}$

**D** 0

**E** 1

$f(x) = \begin{cases} x+1, & x \geq 0 \\ e^x, & x < 0 \end{cases}$

$x = -1$ is in the domain $x < 0$.

$f'(x) = e^x$

$f'(-1) = e^{-1}$

## QUESTION 7

$P(x) = ax^3 + bx^2 - cx - 1$ is a cubic polynomial and has factors of $(x - 1)$ and $(x + 3)$. The function $f(x) = ax^3 + bx^2 - cx - 1$ has a stationary point at $x = 0$. The values of $a$, $b$ and $c$ are

**A** $a = 0, b = \frac{5}{9}, c = \frac{7}{9}$

**B** $a = \frac{2}{9}, b = \frac{7}{9}, c = 0$

**C** $a = \frac{2}{9}, b = \frac{7}{9}, c = c$

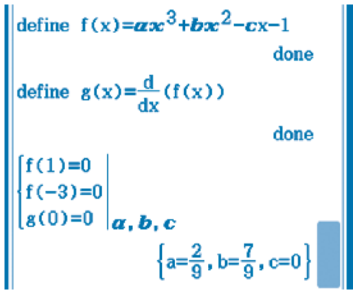
**D** no solution possible

**E** $a = 1, b = -2, c = 0$

$P(x) = ax^3 + bx^2 - cx - 1$ has factors of $(x - 1)$ and $(x + 3)$, and a stationary point at $x = 0$.

$\Rightarrow f(1) = 0, f(-3) = 0$ and $f'(0) = 0$.

Solving simultaneously,




$a = \frac{2}{9}, b = \frac{7}{9}, c = 0$

## QUESTION 8

$f(x) = 3\cos(x)$ and $g(x) = 2\log_e(x)$ for maximal domains. The function $f(g(x))$ can be defined as

**A** $f(g(x)) = 3\cos(2\log_e(x)), x \in R$

**B** $f(g(x)) = 2\log_e(3\cos(x)), x \in R$

**C** $f(g(x)) = 3\cos(2\log_e(x)), x \in (0, \infty)$

**D** $f(g(x)) = 2\log_e(\cos(x)) + 2\log_e(3), x \in [-1, 1]$

**E** $f(g(x)) = 2\log_e(\cos(x)) + 2\log_e(3), x \in [0, \infty)$

$f(x) = 3\cos(x)$ and $g(x) = 2\log_e(x)$

To find dom $f(g(x))$,

test ran $(g) \subseteq$ dom $(f)$

$\boldsymbol{R} \subseteq \boldsymbol{R}$

dom $f(g(x))$ = dom $g(x) = (0, \infty)$

Rule: $f(g(x)) = 3\cos(2\log_e(x))$ with domain $x \in (0, \infty)$.

## QUESTION 9

The graph of $f: (1, \infty) \to \boldsymbol{R}, f(x) = \frac{1}{(x-1)^2}$ has its inverse $f^{-1}(x)$ reflected over the $y$-axis. The image of $f^{-1}(x)$ after this transformation is

**A** $y = \frac{1}{(x-1)^2}$

**B** $y = \frac{1}{\sqrt{x}} + 1$

**C** $y = -\frac{1}{\sqrt{x}} + 1$

**D** $y = \frac{1}{\sqrt{-x}} + 1$

**E** $y = -\frac{1}{\sqrt{-x}} + 1$

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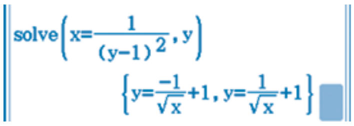
$f: (1, \infty) \to R, f(x) = \dfrac{1}{(x-1)^2}$

The inverse is found by swapping $x$ and $y$ and selecting the +ve solution for the domain given.




$f^{-1}(x) = \dfrac{1}{\sqrt{x}} + 1$

The image of $f^{-1}(x)$ after being reflected over the $y$-axis is

$y = \dfrac{1}{\sqrt{-x}} + 1$

## QUESTION 10

If $T: R^2 \to R^2$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is applied to the function $y = \log_e(x)$, the image function can be expressed as

**A** $y = 2\log_e(x - 1) - 1$

**B** $y = 2\log_e(x - 2) + 1$

**C** $y = 2\log_e(x - 2) - 1$

**D** $y = 0.5\log_e(1 - x) + 1$

**E** $y = 2\log_e(1 - x) - 1$

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ to $y = \log_e(x)$

$\begin{bmatrix} -x+1 \\ 2y-1 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

So

$x = 1 - x_1$

$y = 0.5(y_1 + 1)$

So $y = \log_e(x) \Rightarrow 0.5(y_1 + 1) = \log_e(1 - x_1)$

$y_1 + 1 = 2\log_e(1 - x_1)$

$y = 2\log_e(1 - x) - 1$

The image is $y = 2\log_e(1 - x) - 1$

## QUESTION 11

$\int_0^1 (3x^2 + 2x - 2)^2 dx =$

**A** 2.1333

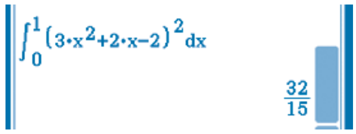
**B** $\dfrac{32}{15}$

**C** 0

**D** $\dfrac{1112}{15}$

**E** 1.262

$\int_0^1 (3x^2 + 2x - 2)^2 dx = \dfrac{32}{15}$




## QUESTION 12

If $f(x) = -\sqrt{x+1} + 2$ and $g(x) = e^x$, both for maximal domains, then the domain of the composite function $g(f(x))$ is

**A** $[-1, \infty)$

**B** $R$

**C** $(0, \infty)$

**D** $[2, \infty)$

**E** $(-\infty, 2]$

$f(x) = -\sqrt{x+1} + 2$ and $g(x) = e^x$

To find the domain of $g(f(x))$,

test ran $(f) \subseteq$ dom $(g)$

$(-\infty, 2] \subseteq R$

dom $g(f(x))$ = dom $f(x) = [-1, \infty)$

## QUESTION 13

The functionality equation $f(x) + f(y) = f(xy)$ is satisfied by

**A** $f(x) = 2e^x$

**B** $f(x) = \dfrac{3}{x}$

**C** $f(x) = 2\log_e(x)$

**D** $f(x) = x$

**E** $f(x) = 2x^2$

$f(x) + f(y) = f(xy)$ looks like it suits a log equation.

Test C: $f(x) = 2\log_e(x)$

LHS $= f(x) + f(y)$

$= 2\log_e(x) + 2\log_e(y)$

$= 2(\log_e(x) + \log_e(y))$

$= 2\log_e(xy)$

RHS $= f(xy) = 2\log_e(xy) =$ LHS

So the rule is $f(x) = 2\log_e(x)$

## QUESTION 14

The average rate of change between the points (0, –1) and (2, –5) is

**A** 2

**B** 1

**C** –2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D $-1$

E $-3$

Average rate of change between the points $(0, -1)$ and $(2, -5)$

$= \frac{-5+1}{2-0} = \frac{-4}{2} = -2$

## QUESTION 15

The average rate of change between the points $x = -\pi$ and $x = 2\pi$ on the curve with the equation $y = \cos^2(x)$ is

A $\frac{1}{3\pi}$

B $-\frac{1}{3\pi}$

C $-\frac{\pi}{2}$

D $\frac{1}{\pi}$

E 0

$y = \cos^2(x)$ has points $(-\pi, 1)$ and $(2\pi, 1)$

Average rate of change $= \frac{1-1}{2\pi+\pi} = 0$

## QUESTION 16

The gradient of the chord $PQ$ for the function $f(x) = x^2 + 3x$ from the $x$-coordinates of $P$, where $x = 1$ and $Q$ where $x = 1 + h$, can be expressed as

A $\lim\limits_{h\to 0}\frac{f(1+h)-f(1)}{h}$

B $\frac{f(1)-f(1+h)}{h}$

C $\lim\limits_{x\to h}\frac{f(1+h)-f(1)}{h}$

D $\frac{(1+h)^2+3(1+h)}{h}$

E $5 + h$

Gradient of the chord $PQ$, for $f(x) = x^2 + 3x$ from $x = 1$ to $x = 1 + h$ is

$= \frac{f(1+h)-f(1)}{h}$

$= \frac{(1+h)^2+3(1+h)-(1+3)}{h}$

$= \frac{2h+h^2+3h}{h}$

$= 5 + h$

**ONE ANSWER PER LINE** **USE PENCIL ONLY**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ■ | B | C | D | E | 9 | A | B | C | ■ | E |
| 2 | A | ■ | C | D | E | 10 | A | B | C | D | ■ |
| 3 | A | B | C | D | ■ | 11 | A | ■ | C | D | E |
| 4 | A | B | ■ | D | E | 12 | ■ | B | C | D | E |
| 5 | A | B | C | ■ | E | 13 | A | B | ■ | D | E |
| 6 | A | B | ■ | D | E | 14 | A | B | ■ | D | E |
| 7 | A | ■ | C | D | E | 15 | A | B | C | D | ■ |
| 8 | A | B | ■ | D | E | 16 | A | B | C | D | ■ |

# Section B: Extended-response questions

**Specific instructions to students**

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

## QUESTION 17

**a** Find $\frac{dy}{dx}$ if $y = 3x\cos(x)$.

$y = 3x\cos(x)$

$\frac{dy}{dx} = 3\cos(x) - 3x\sin(x)$

**1 mark**

**b** Hence find an expression for $\int x\sin(x)\,dx$.

From part **a**,

$\int 3\cos(x) - 3x\sin(x)\,dx = 3x\cos(x)$

This gives

$3\int x\sin(x)dx = 3\int\cos(x)dx - 3x\cos(x)$

$\int x\sin(x)dx = \int\cos(x)dx - x\cos(x)$

$= \sin(x) - x\cos(x)$

**3 marks**

**c** Evaluate $\int_0^{\pi} x\sin(x)dx$.

$\int x\sin(x)dx = \sin(x) - x\cos(x)$

$\int_0^{\pi} x\sin(x)dx = [\sin(x) - x\cos(x)]_0^{\pi}$

$= (\sin(\pi) - \pi\cos(\pi)) - (0 - 0)$

$= \pi$

**1 mark**

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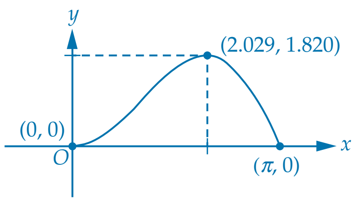
Consider the curve of $f(x) = x \sin(x)$.

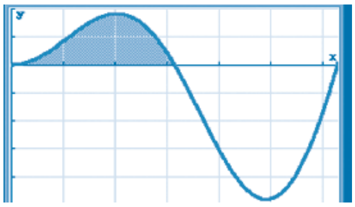
**d** Sketch the graph of $f: [0, \pi] \to \boldsymbol{R}, f(x) = x \sin(x)$, labelling the coordinates of the endpoints and the local maximum point, correct to 3 decimal places.




**2 marks**

**e** Find the area under the curve $f(x) = x \sin(x)$ for $x \in [0, \pi]$.

Graph the curve $f(x) = x \sin(x)$ for the domain $x \in [0, \pi]$.



The area required is all above the $x$-axis, so area = $\pi$.

**1 mark**

**f** Solve the equation $f'(x) = 0$ for $x \in [0, \pi]$, giving your answer correct to 3 decimal places.

$f(x) = x \sin(x)$

$f'(x) = \sin(x) + x \cos(x) = 0$

gives


Within the domain, turning points are at $x = 0$, $x = 2.029$

So the local maximum is at $x = 2.029$, matching the graph in part **d**.

**2 marks**

**(Total: 10 marks)**

## QUESTION 18

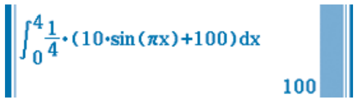
A particular rock concert emits sound (measured in decibels) that ranges from 90 dB to quite a high level. The function that models the sound at the concert follows the function $f(t) = 10 \sin(\pi t) + 100$, where $t \geq 0$, $t$ hours after the start of the concert at 7 p.m.

**a** What is the average noise level, in dB, at the rock concert from 7 p.m. to 11 p.m.?

$f(t) = 10 \sin(\pi t) + 100$

$t = 0$ to $t = 4$

Average value $= \dfrac{1}{4-0}\int_0^4 (10\sin(\pi t) + 100)\,dt = 100$ dB



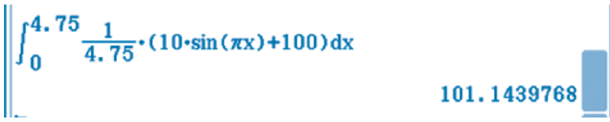

**1 mark**

**b** The concert is so good that the band returns for encores, and the concert finishes at 11.45 p.m. What is the average sound, in dB, correct to 2 decimal places, at the rock concert from 7 p.m. to 11.45 p.m.?

$t = 0$ to $t = 4.75$

Average value $= \dfrac{1}{4.75-0}\int_0^{4.75} (10\sin(\pi t) + 100)\,dt$

$= 101.14$ dB




**1 mark**

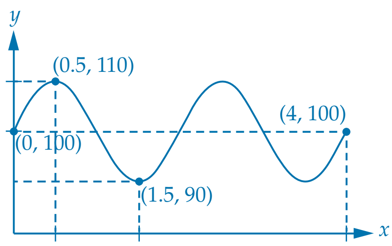
**c** For $f(t) = 10 \sin(\pi t) + 100$, what is the amplitude and period of the function?

Amplitude = 10

Period $= \dfrac{2\pi}{\pi} = 2$

**2 marks**

**d** Sketch the graph of $f(t) = 10 \sin(\pi t) + 100$ for the domain $x \in [0, 4]$, showing the coordinates of the endpoints and the maximum and minimum turning points.




**3 marks**

The level of sound, in decibels, at which a person may sustain hearing loss is 95 dB.

**e** For what period of time during the 4-hour concert will the decibel level be dangerous for patrons?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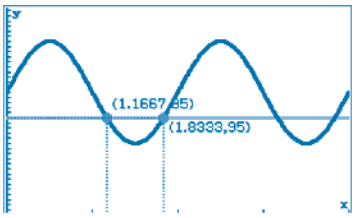
Dangerous ≥ 95

Solve $f(t) \geq 95$




Dangerous for $0 \leq t \leq 1\frac{1}{6}$ and $1\frac{5}{6} \leq t \leq 3\frac{1}{6}$ and $3\frac{5}{6} \leq t \leq 4$.

Dangerous from 7 p.m. to 8:10 p.m., 8:50 p.m. to 10:10 p.m., and 10:50 p.m. to 11 p.m.

**2 marks**

Normal conversation is held between 60 and 65 dB.

**f** Give a reason as to why normal conversation cannot be heard during the 4-hour concert.

The minimum noise level of the concert is 90 dB.

A conversation between 60 and 65 dB cannot be heard.

**1 mark**

**g** Find $\frac{d}{dt}(10\sin(\pi t) + 100)$.

$\frac{d}{dt}(10\sin(\pi t) + 100) = 10\pi\cos(\pi t)$

**1 mark**

**h** Hence solve the equation $f'(t) = 0$.

General solution

$10\pi\cos(\pi t) = 0$

$\Rightarrow \cos(\pi t) = 0$

$\Rightarrow \pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$

$t = \frac{1}{2} + n, n \in \mathbf{Z}$

**2 marks**

The loudest recommended exposure with hearing protection is 140 dB. Even short-term exposure can cause permanent hearing loss at this level.

**i** Using your information from part **h**, explain why there is no possibility of permanent hearing loss for the concert patrons.

Stationary points at $t = \frac{1}{2} + n, n \in \mathbf{Z}$

1st stationary point is a maximum at $t = \frac{1}{2}, f\left(\frac{1}{2}\right) = 110$

The 3rd stationary point is a maximum at $t = \frac{1}{2} + 2 = \frac{5}{2}, f\left(\frac{5}{2}\right) = 110$, etc.

So the maximum noise level during the concert is 110, which is < 140.

So there is no hearing loss.

**1 mark**

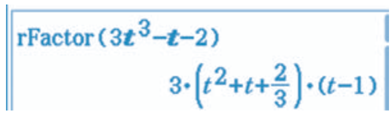
**(Total: 14 marks)**

**QUESTION 19**

The velocity of a particle is described by the function $v(t) = 3t^3 - t - 2$, where $v(t)$ is in m/s and $t$ is in seconds, $t \geq 0$.

**a** By finding the discriminant of a quadratic factor in $v(t)$, show that its graph has only one $t$-intercept.

$v(t) = 3t^3 - t - 2$




$v(t) = (3t^2 + 3t + 2)(t - 1) = 0$ for $t$-intercepts.

A quadratic factor is $3t^2 + 3t + 2$

Discriminant $= 9 - 4 \times 3 \times 2 = -15$, so there are no real factors.

The only $t$-intercept is at $t = 1$.

**2 marks**

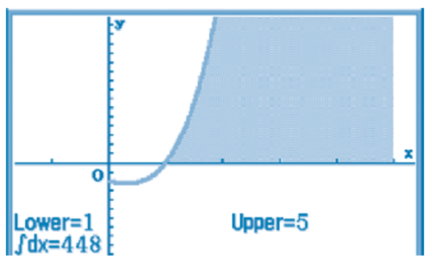
**b** Find when the particle momentarily stops in its journey.

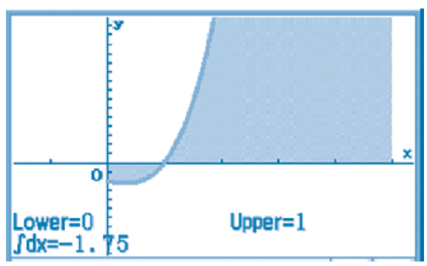
$v'(t) = 9t^2 - 1 = 0$ gives $t = \pm\frac{1}{3}$

Since $t \geq 0$, $t = \frac{1}{3}$.

**1 mark**

**c** What distance does the particle travel in the first 5 seconds?




Total distance = 1.75 + 448 = 449.75 m

**3 marks**

**(Total: 6 marks)**