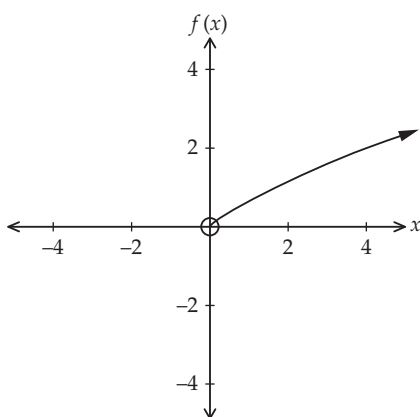


### Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

### QUESTION 1

The graph of  $f: \mathbb{R}^+ \rightarrow \mathbb{R}, f(x) = \sqrt{x}$  is shown below.



- a The graph of  $y = \sqrt{x}$  is dilated by a factor of 2 from the  $y$ -axis to become the graph of  $g$ . Write down the rule for  $g$ .

$$g(x) = \sqrt{\frac{x}{2}}$$

1 mark

- b The graph of  $y = \sqrt{x}$  is dilated by a factor of 3 from the  $x$ -axis to become the graph of  $h$ . Write down the rule for  $h$ .

$$h(x) = 3\sqrt{x}$$

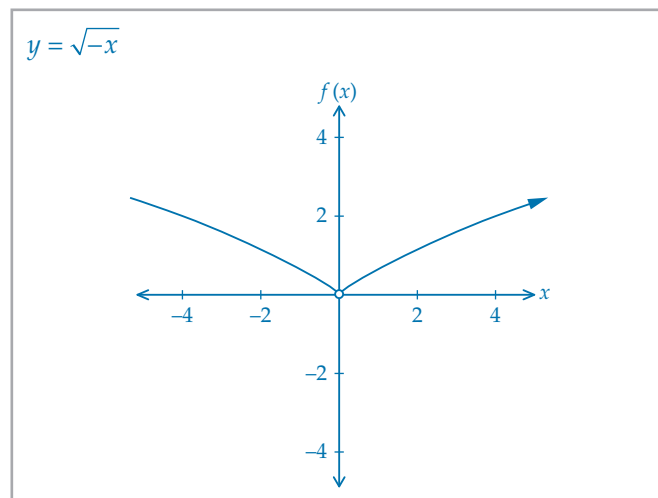
1 mark

- c The graph of  $y = \sqrt{x}$  is translated by 4 units in the positive direction of the  $x$ -axis to become the graph of  $j$ . Write down the rule for  $j$ .

$$j(x) = \sqrt{x-4}$$

1 mark

- d Give the equation of  $f$ , and add to the diagram given, a sketch of  $y = \sqrt{-x}$  if it is reflected over the  $y$ -axis.



2 marks  
(Total: 5 marks)

### QUESTION 2

- a For the function  $f(x) = x^4 + x^3 - 7x^2 - x + 6, f(1) = 0$  and  $f(-1) = 0$ . Find the fully factorised form of  $f(x)$ .

Using synthetic division

$$\begin{array}{r|rrrrr} -1 & 1 & 1 & -7 & -1 & 6 \\ & & -1 & 0 & 7 & -6 \\ \hline & 1 & 0 & -7 & 6 & 0 \end{array} \rightarrow \text{remainder}$$

and again

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array} \rightarrow \text{remainder}$$

This gives the quotient  $x^2 + x - 6$ .

$$\text{So } x^4 + x^3 - 7x^2 - x + 6 = (x-1)(x+1)(x^2 + x - 6)$$

$$f(x) = (x-1)(x+1)(x+3)(x-2)$$

2 marks

- b If  $g(x)$  is the graph of  $f(x)$  translated 1 unit to the right and dilated by a factor of 3 from the  $x$ -axis, write down the equation for  $g(x)$  in factorised form.

$$f(x) = (x-1)(x+1)(x+3)(x-2)$$

$$g(x) = 3f(x-1) = 3(x-1-1)(x+1-1)(x+3-1)(x-2-1)$$

$$g(x) = 3(x-2)(x)(x+2)(x-3)$$

$$g(x) = 3x(x-2)(x+2)(x-3)$$

2 marks  
(Total: 4 marks)

**QUESTION 3**

If  $x^2 + 4x + 1 = (x - a)^2 + b$ , find the values of  $a$  and  $b$ .

$$x^2 + 4x + 1 = (x^2 - 2ax + a^2) + b = x^2 - 2ax + a^2 + b$$

Equating coefficients,

$$4 = -2a \quad (\text{coefficients of } x^1)$$

$$1 = a^2 + b \quad (\text{coefficient of } x^0)$$

$$\text{Hence } a = -2, b = -3$$

2 marks

**QUESTION 4**

If  $f(x) = kx$ , where  $k \in \mathbf{R}$ , show that  $f(x) - f(y) = f(x - y)$ .

$$\text{LHS} = f(x) - f(y)$$

$$= kx - ky$$

$$= k(x - y)$$

$$\text{RHS} = k(x - y) = \text{LHS}$$

2 marks

**QUESTION 5**

Simplify the expression  $5 \log_2(5) \times \log_5(2)$ .

Using a change of base for both terms,

$$\log_2(5) = \frac{\log_e(5)}{\log_e(2)} \quad \text{and} \quad \log_5(2) = \frac{\log_e(2)}{\log_e(5)}$$

$$5 \log_2(5) \times \log_5(2) = \frac{5 \log_e(5)}{\log_e(2)} \times \frac{\log_e(2)}{\log_e(5)} = 5$$

2 marks

**QUESTION 6**

Evaluate

a  $\sin\left(\frac{3\pi}{2}\right)$

$$\sin\left(\frac{3\pi}{2}\right) = -1$$

1 mark

b  $\cos\left(\frac{7\pi}{2}\right)$

$$\cos\left(\frac{7\pi}{2}\right) = 0$$

1 mark

c  $\tan\left(\frac{5\pi}{3}\right)$

$$\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$$

1 mark

d  $\cos(-9\pi)$

$$\cos(-9\pi) = -1$$

1 mark

(Total: 4 marks)

**QUESTION 7**

Solve for  $x$  in the equation  $64^{x-2} \times 4^x = 16$

$$4^{3(x-2)} \times 4^x = 4^2$$

$$4^{3x-6} \times 4^x = 4^2$$

$$4^{4x-6} = 4^2$$

Equating indices,

$$4x - 6 = 2$$

$$4x = 8$$

$$x = 2$$

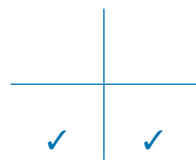
2 marks

**QUESTION 8**

Solve for  $x$  in the equation  $2 \sin(2x) = -1$ , where  $x \in [0, 2\pi]$ .

$$\sin(2x) = -\frac{1}{2}$$

$$\text{Reference angle: } \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$



$$2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 3\pi + \frac{\pi}{6}, 4\pi - \frac{\pi}{6}$$

$$2x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

3 marks

**QUESTION 9**

- a Find the general solution to the equation  $\cos(3x) = \frac{\sqrt{2}}{2}$ .

$$3x = 2n\pi \pm \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$3x = 2n\pi \pm \frac{\pi}{4}$$

$$x = \frac{1}{3}\left(2n\pi \pm \frac{\pi}{4}\right)$$

$$x = \frac{2n\pi}{3} \pm \frac{\pi}{12} \text{ where } n \in \mathbf{Z}$$

2 marks

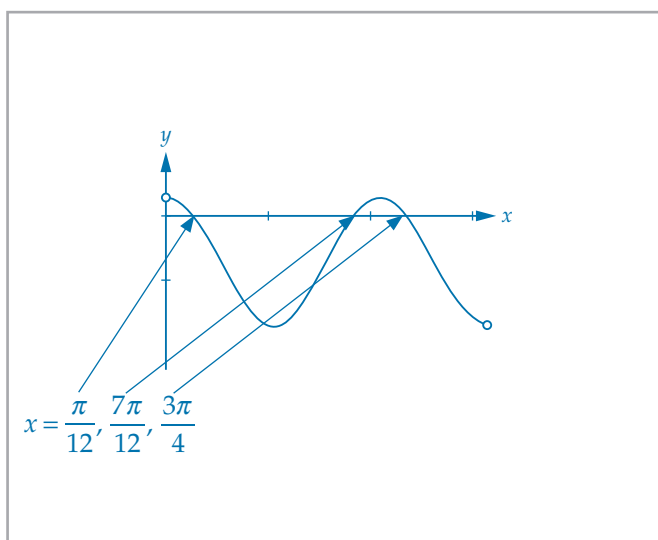
- b Hence, find all the exact solutions in the interval  $(0, \pi)$ .

$n$	0	1
$\frac{2n\pi}{3} + \frac{\pi}{12}$	$\frac{\pi}{12}$	$\frac{9\pi}{12} = \frac{3\pi}{4}$
$\frac{2n\pi}{3} - \frac{\pi}{12}$	$-\frac{\pi}{12}$	$\frac{7\pi}{12}$

**Answer:**  $x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{7\pi}{12}$

2 marks

- c Hence sketch the graph of  $y = \cos(3x) - \frac{\sqrt{2}}{2}$ , labelling axial intercepts.



2 marks

(Total: 6 marks)