TEST 7

Functions, graphs, algebra and calculus
Technology-free end-of-year examination
Total marks: 30
Suggested writing time: 45 minutes

Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 1

For the function $f(x) = e^x \sin(2x)$, find $f'(x)$.

$f(x) = e^x \sin(2x)$

Using the product rule,

$f'(x) = e^x \sin(2x) + 2e^x \cos(2x)$

2 marks

QUESTION 2

a For the function $f(x) = x^2 \log_e(x)$, find $f'(x)$.

$f(x) = x^2 \log_e(x)$

Using the product rule,

$$f'(x) = 2x \log_e(x) + x^2 \times \frac{1}{x}$$

$$= 2x \log_e(x) + x$$

2 marks

b Hence find $f'(2)$.

$f'(x) = 2x \log_e(x) + x$

$f'(2) = 4 \log_e(2) + 2$

1 mark
(Total: 3 marks)

QUESTION 3

a For the function $f(x) = \dfrac{\log_e(x^2+1)}{e^{3x}}$, find $f'(x)$.

$$f(x) = \frac{\log_e(x^2+1)}{e^{3x}}$$

Using the quotient and chain rules,

$$f'(x) = \frac{e^{3x} \times \frac{2x}{x^2+1} - \log_e(x^2+1) \times 3e^{3x}}{e^{6x}}$$

$$= \frac{\frac{2x}{x^2+1} - 3\log_e(x^2+1)}{e^{3x}}$$

3 marks

b Hence find $f'(1)$.

$$f'(x) = \frac{\frac{2x}{x^2+1} - 3\log_e(x^2+1)}{e^{3x}}$$

$$f'(1) = \frac{1 - 3\log_e(2)}{e^3}$$

1 mark
(Total: 4 marks)

QUESTION 4

a If $\dfrac{dy}{dx} = e^{3x}$, find an expression for y.

$$\frac{dy}{dx} = e^{3x}$$

$$y = \int e^{3x}\,dx = \frac{1}{3}e^{3x} + c$$

2 marks

b If $\dfrac{dy}{dx} = e^{3x}$ and $y = 1$ when $x = 0$, find y.

$$y = \frac{1}{3}e^{3x} + c$$

$x = 0, y = 1$ gives $1 = \frac{1}{3}e^0 + c$

$$c = \frac{2}{3}$$

$$y = \frac{1}{3}e^{3x} + \frac{2}{3}$$

2 marks
(Total: 4 marks)

ISBN 9780170354134

QUESTION 5

Evaluate $\int_0^1 x^2 dx$.

$$\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1$$
$$= \frac{1}{3}$$

2 marks

QUESTION 6

Evaluate $\int_0^{\frac{\pi}{6}} 2\cos(3x)dx$.

$$\int_0^{\frac{\pi}{6}} 2\cos(3x)dx = \left[\frac{2}{3}\sin(3x)\right]_0^{\frac{\pi}{6}}$$
$$= \frac{2}{3}\sin\left(\frac{\pi}{2}\right)$$
$$= \frac{2}{3}$$

2 marks

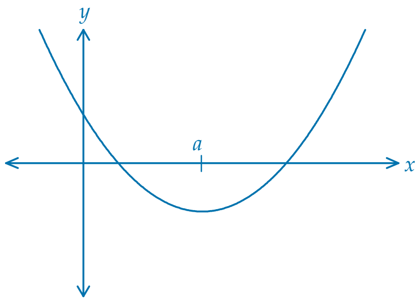
QUESTION 7

a What type of stationary point is found at $x = a$ using the information

$x < a, f'(x) < 0$

$x = a, f'(x) = 0$

$x > a, f'(x) > 0$?



A local minimum at $x = a$.

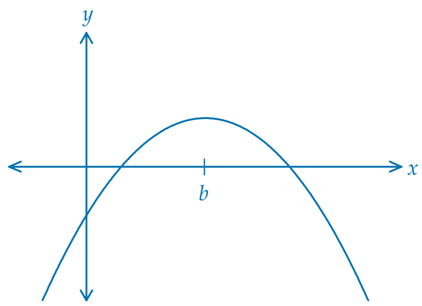
1 mark

b What type of stationary point is found at $x = b$ using the information

$x < b, f'(x) > 0$

$x = b, f'(x) = 0$

$x > b, f'(x) < 0$?



A local maximum at $x = b$.

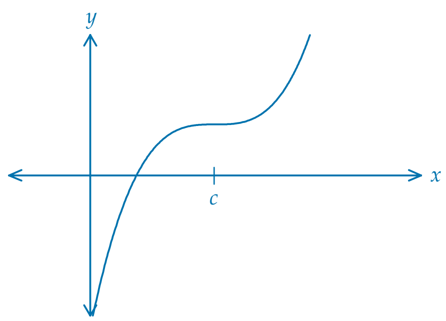
1 mark

c What type of stationary point is found at $x = c$ using the information

$x < c, f'(x) > 0$

$x = c, f'(x) = 0$

$x > c, f'(x) > 0$?



A stationary point of inflection at $x = c$.

1 mark
(Total: 3 marks)

QUESTION 8

a Find the x-coordinates for the stationary points of the graph of $f(x) = \frac{1}{2}x^4 - x^2 + 3$.

$f(x) = \frac{1}{2}x^4 - x^2 + 3$

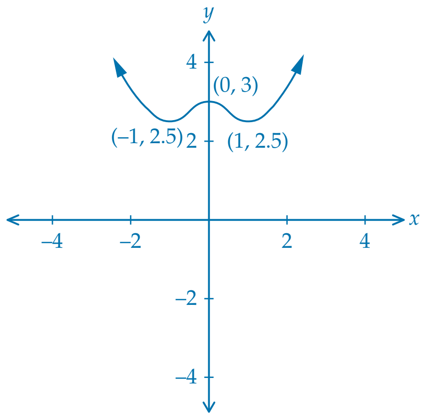
$f'(x) = 2x^3 - 2x = 0$ for stationary points.

$\Rightarrow 2x(x^2 - 1) = 0$

This gives $x = 0$, $x = 1$, $x = -1$

2 marks

b Given that there are no x-intercepts on the graph, sketch the graph of $f(x)$, labelling the coordinates of the stationary points.



2 marks

c State the intervals over which the graph is strictly increasing.

$x \in [-1, 0] \cup [1, \infty)$

1 mark

d State the intervals over which the graph is strictly decreasing.

$x \in (-\infty, -1] \cup [0, 1]$

1 mark
(Total: 6 marks)

QUESTION 9

It is known that $\int_1^3 f(x)dx = 1$

a Evaluate $\int_1^3 (f(x) + 1)dx$.

$$\int_1^3 (f(x)+1)dx = \int_1^3 f(x)dx + \int_1^3 1dx$$
$$= 1 + [x]_1^3$$
$$= 1 + 3 - 1$$
$$= 3$$

2 marks

b Evaluate $\int_3^1 (-2f(x) + 3)dx$.

$$\int_3^1 (-2f(x)+3)dx = -2\int_3^1 f(x)dx + \int_3^1 3dx$$
$$= (-2 \times (-1)) + [3x]_3^1$$
$$= 2 + 3 - 9$$
$$= -4$$

2 marks
(Total: 4 marks)

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