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Free Exam
for 2006-15 VCE study design

Engage Education Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$\int 4(2x-5)^3 dx = \frac{4(2x-5)^4}{4 \text{ x } 2} + c'$ [1]

$=\frac{1}{4}(2x-5)^4 + c$ [1] (the c is not required as asked for and an anti-derivative, c can be any value)

Question 1b

$\frac{d}{dx}\left(\frac{2\sin(x)}{x}\right) = \frac{x \text{ x } \frac{d}{dx}(2\sin(x)) - 2\sin(x) \text{ x } \frac{d}{dx}(x)}{(x)^2}$ [1]

$= \frac{2x\cos(x) - 2\sin(x)}{(x)^2}$ [1]

$f'(\pi) = \frac{2(\pi)\cos(\pi) - 2\sin(\pi)}{(\pi)^2} = \frac{2(\pi)(1)}{(\pi)^2} = \frac{2}{\pi}$ [1]

Question 2a

$\mu = 15, \sigma = 4$

$\Pr(X > 19) = \Pr(X > \mu + \sigma)$

$\Pr(\mu - \sigma > X > \mu + \sigma) = 0.68$

$\Pr(X > 19) = \frac{1 - \Pr(\mu - \sigma > X > \mu + \sigma)}{2} = \frac{1 - 0.68}{2} = 0.16$ [1]

Question 2b

$Z = \frac{x-\mu}{\sigma} = \frac{10-15}{4} = -\frac{5}{4}$

$\Pr(X < 10) = \Pr(Z < -\frac{5}{4})$ [1]

$\Pr(Z < -\frac{5}{4}) = \Pr(Z > \frac{5}{4})$ (dues to symmetry around Z=0)

$\Pr(X < 10) = \Pr(Z > \frac{5}{4})$, $c = \frac{5}{4}$ or $c = 1.25$ [1]

Question 3

$\begin{bmatrix} x^1 \\ y^1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ [1]

$x^1 = \frac{1}{2}x - 2 \Rightarrow x = 2(x^1 + 2)$

$y^1 = -3y + 1 \Rightarrow y = -\frac{1}{3}(y^1 - 1)$ [1]

$y = \frac{1}{x}$ sub in for $-\frac{1}{3}(y^1 - 1) = \frac{1}{2(x^1+2)}$ [1]

$f(x) = 1 - \frac{3}{2(x^1+2)}$ [1]

Question 4

$log_e(x^2) - log_e(x+4) = log_e(2)$

$log_e(\frac{x^2}{x+4}) = log_e(2)$ [1]

$\frac{x^2}{x+4} = 2$ [1]

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$\Rightarrow x = 4, x = -2$[1]

$Since\ x\ must\ be\ positive, the\ answer\ is:$

$x = 4$[1]

Question 5

Pr(different colours | first blue)= $\frac{\Pr(\text{different colours} \cap \text{first blue})}{\Pr(\text{first blue})}$ [1]

Pr(different colours ∩ first blue) = Pr(BR) + Pr(BG) = $\frac{1}{3}\text{x}\frac{1}{2}+\frac{1}{3}\text{x}\frac{1}{6}=\frac{2}{9}$[1]

Pr(first blue) = $\frac{1}{3}$

Pr(different colours | first blue)= $\frac{2/9}{1/3}=\frac{2}{3}$[1]

Question 6a

$\frac{\sin(2x)}{\cos(2x)}=\sqrt{3}\frac{\cos(2x)}{\cos(2x)}$ [1]

$\frac{\sin(2x)}{\cos(2x)}=\sqrt{3}$

$\tan(2x)=\sqrt{3}$

$2x=-\frac{2\pi}{3},\frac{\pi}{3}$

$x=-\frac{\pi}{3},\frac{\pi}{6}$ [1]

Question 6b

Range of $f(x)=[1-2,1+2]=[-1,3]$ [1]

End-points: $y=2\cos\left((0)-\frac{\pi}{3}\right)+1$

$y=2\left(\frac{1}{2}\right)+1=2$

$y=2\cos\left((2\pi)-\frac{\pi}{3}\right)+1$

$y=2\left(\frac{1}{2}\right)+1=2$

Endpoints: (0,2),$(2\pi,2)$ [1]

Question 7

$\int_0^a ax-x^2dx=\frac{9}{2}$ [1]

$\left[\frac{1}{2}ax^2-\frac{1}{3}x^3\right]_0^a=(\frac{1}{2}a^3-\frac{1}{3}a^3)-(\frac{1}{2}(0)^3-\frac{1}{3}(0)^3)=\frac{9}{2}$ [1]

$\Rightarrow\frac{1}{6}a^3=\frac{9}{2}\Rightarrow a^3=27$

$\Rightarrow a=3$ [1]

Question 8

$\frac{dU}{dt}=2\ \text{m/s}$

$L=\sqrt{25-U^2}$[1]

$\Rightarrow\frac{dL}{dU}=\frac{-U}{\sqrt{25-U^2}}=\frac{3}{4}\text{m/m when } U=3\text{ cm}$ [1]

$\frac{dL}{dt}=\frac{dL}{dU}\frac{dU}{dt}$ [1]

$=\frac{3}{4}\frac{2}{1}=\frac{3}{2}\ m/s$ [1]

Question 9

Shape of $\boldsymbol{f}$ and $\boldsymbol{f^{-1}}$ correct ($\boldsymbol{f^{-1}}$ is given by $\boldsymbol{f}$ reflected in both axes) [1]

For inverse let $f(x)=y$ and swap x and , $x=4e^{y-2}+1$ [1]

$f^{-1}(x)=y=log_e\left(\frac{x-1}{4}\right)+2$ [1]

Intercept for f at $\left(0,\frac{4}{e^2}+1\right)$ and intercept for f^{-1} at $\left(\frac{4}{e^2}+1,0\right)$ [1]

Question 10a

$\frac{dy}{dx} = m = 3x^2 - 2x - 1$

At pt (0,1) $m = 3(0)^2 - 2(0) - 1 = -1$[1]

Gradient of normal $= -\frac{1}{m} = -\frac{1}{(-1)} = 1$[1]

$y = x + c$ pt $(0,1) \Rightarrow 1 = c$

$y = x + 1$ [1]

Question 10b

$x + 1 = x^3 - x^2 - x + 1$ [1]

$x^3 - x^2 - 2x = x(x-2)(x+1) = 0$

$x = -1, 0, 2$ sub into $y = x + 1$ [1]

(-1,0), (0,1), (2,3) [1]