

YEAR 12 Trial Exam Paper

2015

MATHEMATICAL METHODS (CAS)

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the questions

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SECTION 1

Question 1

Answer is E.

Worked solution

The function $y = 1 + \sqrt{a-x}$ is defined for $a-x \geq 0$, so $x \leq a$.

Question 2

Answer is D.

Worked solution

$$f(1) = -2$$

$$f(-2) = 10$$

$$f'(1) = -3$$

$$\text{and } f'(3) = -36.$$

In equation form, this can be written as:

$$a + b + c + d = -2$$

$$-8a + 4b - 2c + d = 10$$

$$3a + 2b + c = -3$$

$$27a + 6b + c = -36$$

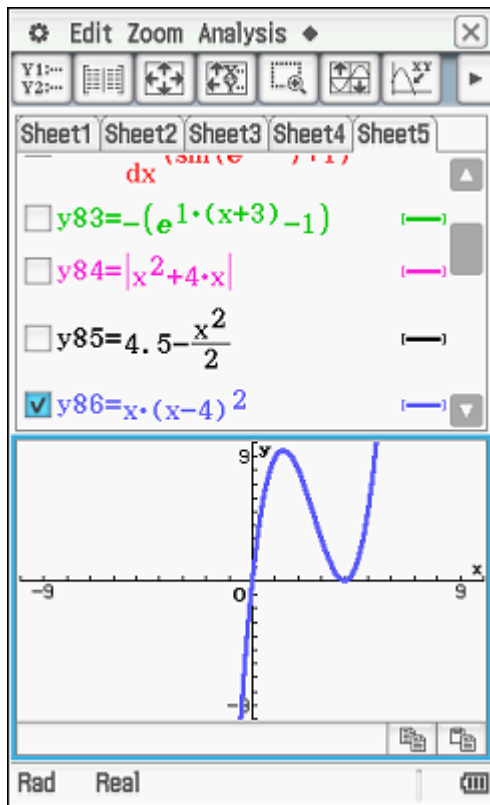
$$\text{which gives } \begin{bmatrix} 1 & 1 & 1 & 1 \\ -8 & 4 & -2 & 1 \\ 3 & 2 & 1 & 0 \\ 27 & 6 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -2 \\ 10 \\ -3 \\ -36 \end{bmatrix}.$$

Question 3

Answer is E.

Worked solution

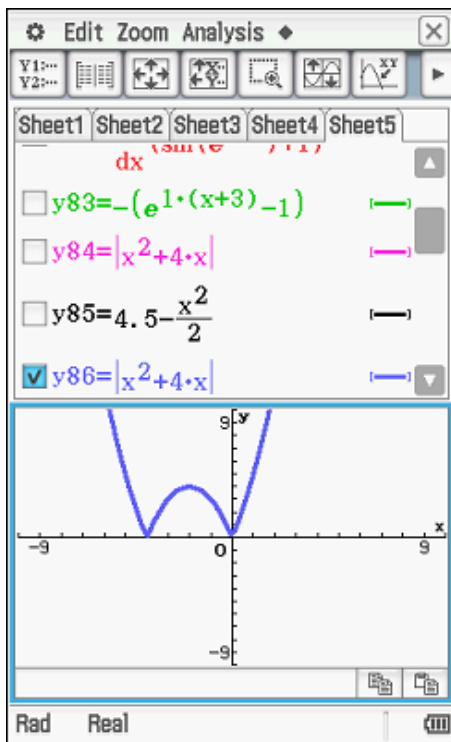
A sample graph with $a = -4$ shows



Hence, for $x < 0$, $f'(x) > 0$.

Question 4*Answer is D.***Worked solution**

The graph $f(x) = |x^2 + ax|$ has x -intercepts at $(0,0)$ and $(-a,0)$ and a turning point at $x = -\frac{a}{2}$, as shown below.



The gradient is positive for $x \in \left(-a, -\frac{a}{2}\right) \cup (0, \infty)$.

Question 5*Answer is B.***Worked solution**

$$\begin{aligned}
 \int_2^4 (2 + 5f(x)) dx &= \int_2^4 2 dx + 5 \int_2^4 f(x) dx \\
 &= [2x]_2^4 + 5 \times 3 \\
 &= (8 - 4) + 15 \\
 &= 19
 \end{aligned}$$

Question 6*Answer is A.***Worked solution**

$$\begin{aligned}
 \int_0^1 \frac{2x^2}{k} dx &= \frac{1}{k} \int_0^1 2x^2 dx \\
 &= \frac{1}{k} \left[\frac{2x^3}{3} \right]_0^1 \\
 &= \frac{2}{3k}
 \end{aligned}$$

$$\text{So, } \frac{2}{3k} = 1$$

$$k = \frac{2}{3}$$

Question 7*Answer is C.***Worked solution**

Owing to the symmetry of the distribution, the mean, median and mode occur concurrently at $x = 3$.

Question 8*Answer is C.***Worked solution**

$$\Pr(X < 2.5 | X < 3) = \frac{\Pr(X < 2.5)}{\Pr(X < 3)}$$

TI-84 Plus calculator screen showing the calculation of the probability $\Pr(X < 2.5 | X < 3)$. The screen displays the integral of $-0.75 \cdot (x-4) \cdot (x-2)$ from 2 to 2.5, resulting in 0.15625. Then it shows the integral from 2 to 3, resulting in 0.5. Finally, it shows the division $0.15625 / 0.5$, resulting in 0.3125.

TI-84 Plus calculator screen showing the calculation of the probability $\Pr(X < 2.5 | X < 3)$. The screen displays the integral of $-0.75 \cdot (x-4) \cdot (x-2)$ from 2 to 2.5, resulting in 0.15625. Then it shows the integral from 2 to 3, resulting in 0.5. Finally, it shows the division $0.15625 / 0.5$, resulting in 0.3125.

Question 9*Answer is A.***Worked solution**

Expanding the matrix gives

$$\left. \begin{array}{l} x' = -3x - \pi \\ y' = 2y - 2 \end{array} \right\} \Rightarrow \begin{array}{l} x = \frac{x' + \pi}{-3} \\ y = \frac{y' + 2}{2} \end{array}$$

So $y = \sin(3x)$ becomes

$$\frac{y+2}{2} = \sin(-x - \pi)$$

$$y = 2 \sin(-x - \pi) - 2$$

Question 10*Answer is D.***Worked solution**

The graph of $y = 5x - 3$ undergoes the transformation $1 + f(x + 2)$, so

$$\begin{aligned} y &= (5(x+2) - 3) + 1 \\ &= 5x + 8 \end{aligned}$$

Question 11*Answer is E.***Worked solution**

Using the quotient rule gives $g(x) = \frac{f(x)}{e^x}$, so $g'(x) = \frac{e^x f'(x) - e^x f(x)}{(e^x)^2} = \frac{f'(x) - f(x)}{e^x}$.

Question 12*Answer is E.***Worked solution**

$$X \sim Bi(n = 10, p = 0.3)$$

$$\Pr(X > 2) =$$

The screenshot shows a TI-84 Plus calculator interface. The display shows the following sequence of operations and results:

- Initial display: $.156257.5$
- Input: 0.3125
- Operation: `simplify(5(x+2)-3+1)`
- Result: $5 \cdot x + 8$
- Operation: `binomialPDF(2, 10, 0.3)`
- Result: 0.2334744405
- Operation: `binomialCDF(3, 10, 10, 0.3)` (highlighted in blue)
- Result: 0.6172172136

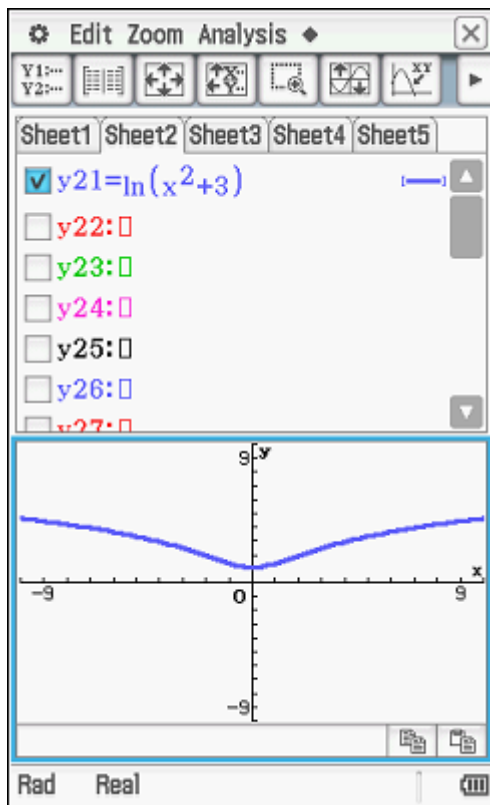
The calculator interface includes a menu bar with options like 'Edit', 'Action', and 'Interactive'. Below the display is a keypad with various mathematical functions categorized by Math1, Math2, Math3, Trig, Var, and abc. At the bottom, there are mode selection buttons for Alg, Decimal, Real, and Rad, along with a calculator icon.

Question 13*Answer is D.***Worked solution**

The graph has been reflected in both the x - and y -axes. The matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ produces reflections in both axes.

Question 14*Answer is D.***Worked solution**

The graph of $g(f(x))$ shows that the lowest point occurs at $x = 0$, $y = \log_e 3$.



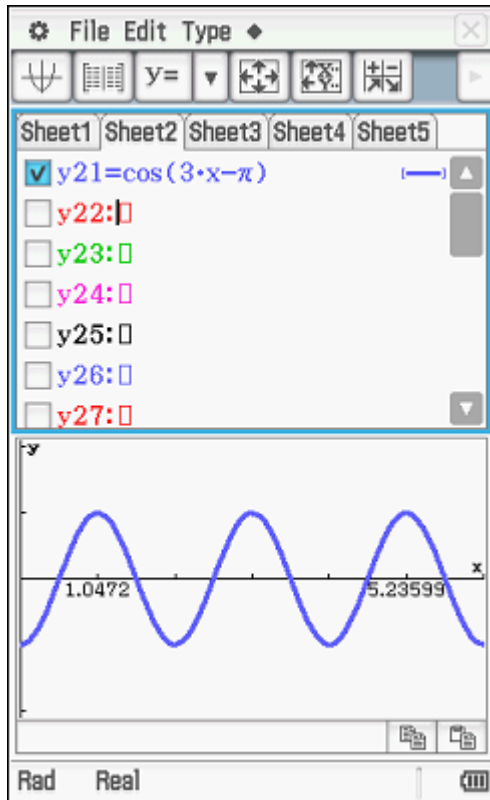
The range is $[\log_e 3, \infty)$.

Question 15*Answer is C.***Worked solution**

The rate of change for this function is $\begin{cases} -2 & \text{for } x < \frac{-1}{2} \\ 2 & \text{for } x > \frac{1}{2} \end{cases}$.

Question 16*Answer is D.***Worked solution**

For the inverse function to exist, the function must be one-to-one. The graph shows that the function is one-to-one for $x \in [0, \frac{\pi}{3}]$.



Hence, $a = \frac{\pi}{3}$.

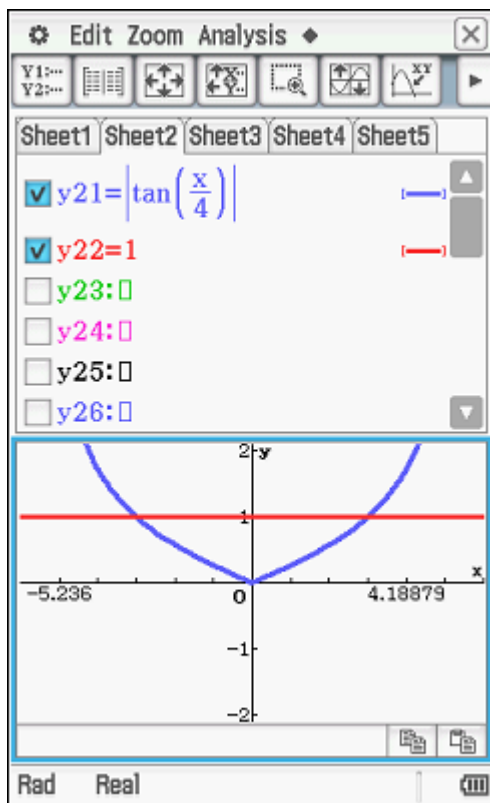
Question 17*Answer is C.***Worked solution**

For $4.2^{\frac{3}{2}}$, $x = 4$, $h = 0.2$, $f(x) = x^{\frac{3}{2}}$ and $f'(x) = \frac{3}{2}\sqrt{x}$.

$$\begin{aligned} \text{So, } f(4+h) &\approx f(4) + hf'(4) \\ &= 4^{\frac{3}{2}} + 0.2 \times \frac{3}{2} \times \sqrt{4} \\ &= 8 + 0.6 \\ &= 8.6 \end{aligned}$$

Question 18*Answer is C.***Worked solution**

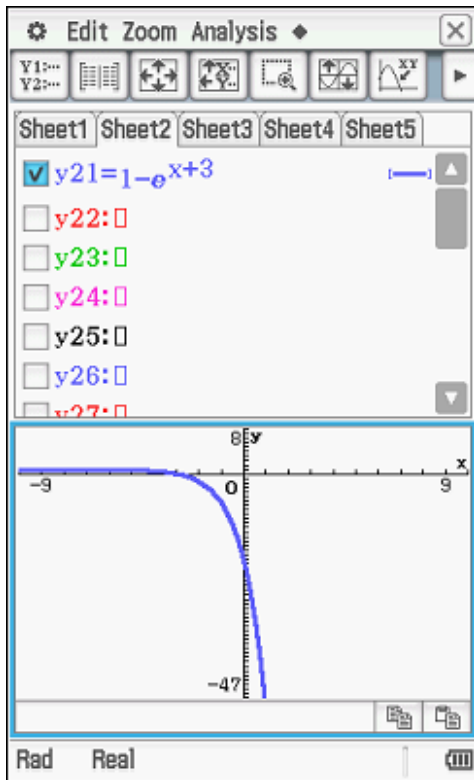
The graph shows that there are two points where it intersects with the line $y = 1$.

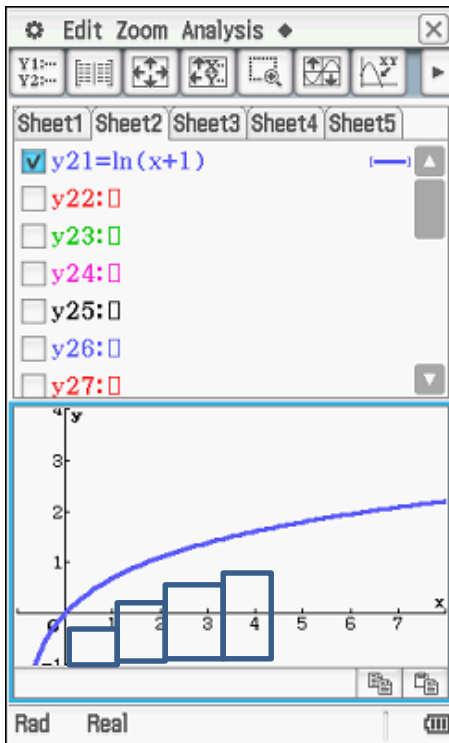


Question 19*Answer is A.***Worked solution**

The graph of $y = 1 - e^{(x+3)}$ shows the region is below the x -axis.

$$\text{So the area is } -\int_{-3}^0 (1 - e^{x+3}) dx = \int_{-3}^0 (e^{x+3} - 1) dx.$$



Question 20*Answer is D.***Worked solution**

The right rectangles have heights of:

$$\log_e(1+1), \log_e(2+1), \log_e(3+1), \log_e(4+1)$$

i.e. $\log_e(2), \log_e(3), \log_e(4), \log_e(5)$.

So the area is $\log_e(2) + \log_e(3) + \log_e(4) + \log_e(5) = \log_e(120)$.

Question 21*Answer is B.***Worked solution**

The distance is the area under the velocity–time graph. There is an x -intercept at $x = 3$, so the area is calculated in two parts as:

$$\int_0^3 \left(4.5 - \frac{1}{2}t^2\right) dt - \int_3^6 \left(4.5 - \frac{1}{2}t^2\right) dt$$

$$= 18 + 9 = 27$$

Question 22*Answer is E.***Worked solution**

$$f^{-1}(x) = e^x \neq \frac{1}{\log_e(x)}$$

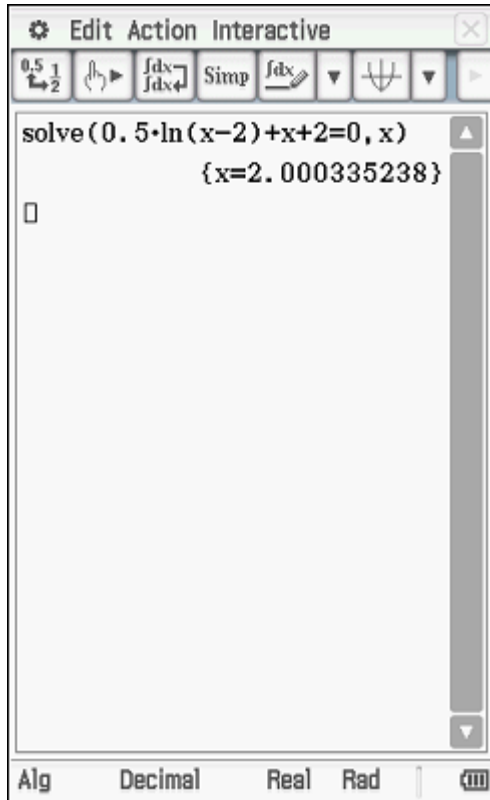
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SECTION 2

Question 1a.

Worked solution

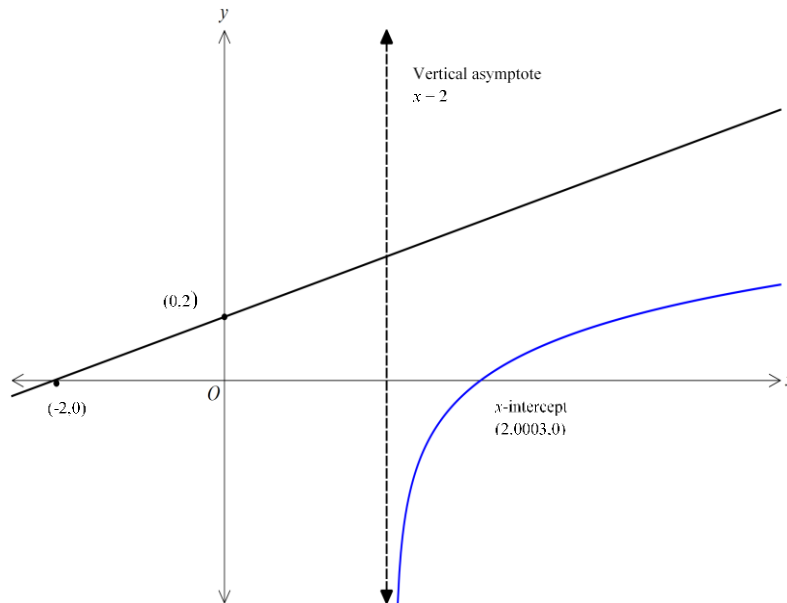
Using CAS:

So, $x = 2.0003$.**Mark allocation: 1 mark**

- 1 mark for 2.0003

**Tip**

- *Be sure to answer to the correct number of decimal places.*

Question 1b.**Worked solution****Mark allocation: 2 marks**

- 1 mark for the drawing the shape and asymptote correctly
- 1 mark for labelling the x -intercept correctly

**Tip**

- *When sketching a graph, always state the equation of any asymptotes and the coordinates of any intercepts.*

Question 1c.**Worked solution**

The domain of h is $(2, \infty)$.

Mark allocation: 1 mark

- 1 mark for $(2, \infty)$

**Tip**

- *The domain of the added function is the domain of $g \cap f$. The location of the vertical asymptote is maintained so the domain is strictly greater than 2.*

Question 1d.i.**Worked solution**

$$\frac{d}{dx} \left(x + 2 - \frac{1}{2} \log_e(x-2) \right) = \frac{(2x-5)}{2(x-2)}, \text{ as determined from CAS.}$$

The screenshot shows a CAS calculator window titled "Edit Action Interactive". The main display area shows the derivative of the expression $(x+2 - .5 \ln(x-2))$ with respect to x . The result is $\frac{0.5 \cdot (2 \cdot x - 5)}{x - 2}$. Below the display is a keypad with various mathematical functions and symbols, including Math1, Math2, Math3, Trig, Var, abc, and a numeric keypad. The bottom of the window shows the mode settings: Alg, Decimal, Real, Rad, and a display mode icon.

Mark allocation: 1 mark

- 1 mark for the correct answer

**Tip**

- Remember to use CAS – the question is worth only 1 mark and does not require working to be shown.

Question 1d.ii.**Worked solution**

The screenshot shows a TI-84 Plus calculator interface. The main display shows the derivative of $(x+2 - 0.5 \ln(x-2))$ with respect to x , resulting in $\frac{0.5 \cdot (2 \cdot x - 5)}{x - 2}$. Below this, the equation $\text{solve}\left(\frac{0.5 \cdot (2 \cdot x - 5)}{x - 2} = 0, x\right)$ is entered, and the solution $\{x=2.5\}$ is displayed. The calculator interface includes a menu bar (Edit, Action, Interactive), a toolbar with various mathematical symbols and functions, and a keypad with rows for Math1, Math2, Math3, Trig, Var, and abc, along with a numeric keypad and function keys like ans and EXE.

$$\frac{d}{dx}(x+2 - \log_e(x-2)) = \frac{(2x-5)}{2(x-2)}$$

Let $h'(x) = 0$:

$$2x - 5 = 0$$

$$x = 2.5$$

Mark allocation: 2 marks

- 1 mark for getting $2x - 5 = 0$
- 1 mark for the correct answer

**Tip**

- *This question is worth 2 marks so a 'thinking step' needs to be shown. It is enough to simply state $h'(x) = 0$ and then give the answer.*

Question 1e.**Worked solution**

The screenshot shows a TI-Nspire calculator interface. The input field contains the command `solve(x-2=0, x)`. The output area displays the solution set $\{x=2.5\}$, the function $x+2-.5\ln(x-2)$ evaluated at $x=2.5$, the numerical result 4.84657359 , and the exact answer $\frac{\ln(2)}{2} + \frac{9}{2}$. The calculator interface includes a toolbar with various mathematical symbols and a keypad with categories like Math1, Math2, Math3, Trig, Var, abc, and Alg.

$$\text{Min distance} = \frac{1}{2} \log_e(2) + \frac{9}{2}$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2a.i.**Worked solution**

$$h(x) = \sin(x) + 1$$

$$g(x) = e^{\frac{x}{20}}$$

Mark allocation: 2 marks

- 1 mark for each correct answer

Question 2a.ii.**Worked solution**

For $g(h(x))$ to exist:

Range of $h(x) \subseteq$ domain of $g(x)$

Range of $h(x) = [0, 2]$

Domain of $g(x) = R$

So, range of $h(x) \subseteq$ domain of $g(x)$.

$\therefore g(h(x))$ exists.

Mark allocation: 1 mark

- 1 mark for the correct answer with the correct reasons

Question 2b.**Worked solution**

Using CAS:

The screenshot shows a CAS interface with the following elements:

- Top bar: Edit, Action, Interactive
- Toolbar: $\frac{1}{2}$, $\frac{1}{x}$, $\frac{d}{dx}$, $\frac{d}{dx}$, $\frac{d}{dx}$, $\frac{d}{dx}$, $\frac{d}{dx}$, $\frac{d}{dx}$, $\frac{d}{dx}$, $\frac{d}{dx}$
- Input field: $\frac{d}{dx} \left(\sin \left(e^{\frac{x}{20}} + 1 \right) \right)$
- Output field: $\frac{\cos \left(e^{\frac{x}{20}} \right) \cdot e^{\frac{x}{20}}}{20}$
- Bottom panel: Math1 (Line, $\frac{1}{x}$, $\sqrt{\quad}$, π , \Rightarrow), Math2 (\square^{\square} , e^{\square} , \ln , i , ∞), Math3 ($\frac{d}{dx}$, $\frac{d}{dx}$, \int , \lim), Trig (\sin , \cos , \tan , θ , t), Var (\square , \square , \square , Σ , \square), abc (\leftarrow , \square , \square , ans, EXE)
- Bottom status bar: Alg, Standard, Real, Rad, \square

$$\frac{d}{dx} (\sin(e^{\frac{x}{20}}) + 1) = \frac{e^{\frac{x}{20}} \cos(e^{\frac{x}{20}})}{20}$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2c.**Worked solution**

Using CAS:

The screenshot shows a CAS interface with the following elements:

- Toolbar: Contains icons for fractions, square roots, integrals, simplification, and other mathematical operations.
- Input area: Displays the expression $\frac{\cos\left(e^{\frac{x}{20}}\right) \cdot e^{\frac{x}{20}}}{20}$ and the command `simplify(fMin(sin(e^(x/20))+1, x, {MinValue=0, x=20*ln(3*pi/2)}))`.
- Keypad: Organized into rows:
 - Math1: Line, square root, pi, arrow.
 - Math2: square, e, ln, i, infinity.
 - Math3: absolute value, d/dx, integral, limit.
 - Trig: sin, cos, tan, theta, t.
 - Var: sin, cos, tan, theta, t.
 - abc: sin, cos, tan, theta, t.
- Bottom bar: Modes (Alg, Standard, Real, Rad) and a calculator icon.

The first point on the base level occurs at $x = 20 \log_e \left(\frac{3\pi}{2} \right)$.

Mark allocation: 1 mark

- 1 mark for the correct answer

**Tip**

- *To find the coordinates of the first point, specify the restriction on the domain. It is apparent from the graph given that the first minimum occurs in the domain $[30, 35]$, so request CAS to find the point in this domain using $|30 \leq x \leq 35|$ in the solve command.*

Question 2d.i.**Worked solution**

(fMin(sin($e^{\frac{x}{20}}$))+1, x, 0, 35)
 {MinValue=0, x=20 \cdot ln($\frac{3\cdot\pi}{2}$)}
 simplify (fMax(sin($e^{\frac{x}{20}}$))+1, x,
 {MaxValue=2, x=20 \cdot ln($\frac{\pi}{2}$), x=:

The maximum height is 2 metres.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2d.ii.

Worked solution

Edit Action Interactive
 (fMin(sin(e²⁰)+1, x, 0, 35))
 {MinValue=0, x=20·ln($\frac{3\pi}{2}$)}
 simplify (fMax(sin(e ^{$\frac{x}{20}$})+1, x,
 MaxValue=2, x=20·ln($\frac{\pi}{2}$), x=20)

Edit Action Interactive
 (fMin(sin(e²⁰)+1, x, 0, 35))
 {MinValue=0, x=20·ln($\frac{3\pi}{2}$)}
 simplify (fMax(sin(e ^{$\frac{x}{20}$})+1, x,
 ln($\frac{\pi}{2}$), x=20·ln($\frac{5\pi}{2}$), x=20)

Edit Action Interactive
 (fMin(sin(e²⁰)+1, x, 0, 35))
 {MinValue=0, x=20·ln($\frac{3\pi}{2}$)}
 simplify (fMax(sin(e ^{$\frac{x}{20}$})+1, x,
 x=20·ln($\frac{5\pi}{2}$), x=20·ln($\frac{9\pi}{2}$))

So, this maximum height of 2 m occurs at

$$x = 20 \log_e \left(\frac{\pi}{2} \right), x = 20 \log_e \left(\frac{5\pi}{2} \right), x = 20 \log_e \left(\frac{9\pi}{2} \right).$$

Mark allocation: 2 marks

- 1 mark awarded for giving only one correct value
- 2 marks awarded for all three correct values

**Tip**

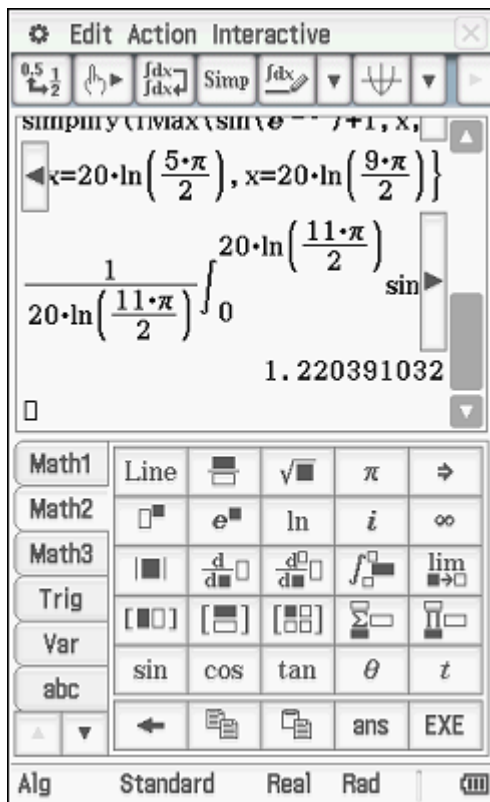
- Remember: an exact value is required here! Be sure to be in standard mode and to help with getting a simplified answer, put the simplify command in your CAS calculation.

Question 2e.**Worked solution**

This horizontal cross-section height is the average value of the function.

$$\text{Average height} = \frac{1}{20 \log_e \left(\frac{11\pi}{2} \right)} \int_0^{20 \log_e \left(\frac{11\pi}{2} \right)} \left(\sin \left(e^{\frac{x}{20}} \right) + 1 \right) dx$$

Using CAS, this is:



So, the average height is 1.22 metres.

Mark allocation: 2 marks

- 1 mark for writing the average value $\frac{1}{20 \log_e \left(\frac{11\pi}{2} \right)} \int_0^{20 \log_e \left(\frac{11\pi}{2} \right)} \left(\sin \left(e^{\frac{x}{20}} \right) + 1 \right) dx$
- 1 mark for the correct answer of 1.22

Question 2f.**Worked solution**

The gradient of the pole is -10 , which means the gradient of the curve is $\frac{1}{10}$.

So, we need to find $\left\{ x : f'(x) = \frac{1}{10}, \text{ for } x \in [30, 40] \right\}$.

Using CAS gives:

The screenshot shows a CAS calculator interface with the following content:

- Top bar: Edit Action Interactive
- Toolbar: $\frac{0.5}{2}$, $\frac{1}{2}$, $\frac{f dx}{f dx}$, $\frac{f dx}{f dx}$, $\frac{f dx}{f dx}$, $\frac{f dx}{f dx}$, $\frac{f dx}{f dx}$, $\frac{f dx}{f dx}$, $\frac{f dx}{f dx}$
- Input area: $\frac{1}{20 \cdot \ln\left(\frac{11 \cdot \pi}{2}\right)} \int_0^{\sin} \sin$
- Output area: 1.220391032
- Command: $\text{solve}\left(\frac{d}{dx}\left(\sin\left(e^{\frac{x}{20}}\right)+1\right)=0.1,\right)$
- Result: $\{x=32.64035003, x=40.5293\}$
- Bottom panel: Math1 (Line, $\frac{1}{x}$, $\sqrt{\quad}$, π , \rightarrow), Math2 (\square^{\square} , e^{\square} , \ln , i , ∞), Math3 ($|\square|$, $\frac{d}{d\square}\square$, $\frac{d^2}{d^2\square}\square$, $\int \square$, $\lim_{\square \rightarrow \square}$), Trig ($[\square\square]$, $[\frac{\square}{\square}]$, $[\frac{\square}{\square}]$, $\sum \square$, $\prod \square$), Var ($[\square\square]$, $[\frac{\square}{\square}]$, $[\frac{\square}{\square}]$, $\sum \square$, $\prod \square$), abc (sin, cos, tan, θ , t), \leftarrow , \rightarrow , $\frac{f dx}{f dx}$, $\frac{f dx}{f dx}$, ans, EXE
- Bottom bar: Alg, Standard, Real, Rad, $\frac{1}{x}$

The pole is inserted at $x = 32.64035$ metres.

Using CAS, we must find the equation of the normal to the curve at this point.

Equation of normal is $y = -10x + 326.48314$.

The pole reaches the base level at $y = 0$.

So, $-10x + 326.48314 = 0$

$$x = 32.6483$$

$$\therefore x = 32.648 \text{ m}$$

Mark allocation: 3 marks

- 1 mark for setting $f'(x) = \frac{1}{10}$
- 1 mark for finding the equation of the normal
- 1 mark for the correct answer

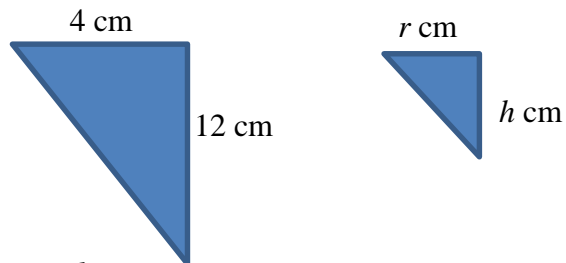


Tip

- *Remember: when asked to write an answer correct to 3 decimal places, always work to more decimal places and then round at the last step.*

Question 3a.i**Worked solution**

Using similar triangles:



$$\begin{aligned} \text{So, } \frac{h}{12} &= \frac{r}{4} \\ h &= \frac{12r}{4} \\ &= 3r \end{aligned}$$

Mark allocation: 1 mark

- 1 mark for using similar triangles method

Question 3a.ii.**Worked solution**

$$\begin{aligned} V_{\text{cone}} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi \frac{h^2}{9} h \\ V &= \frac{\pi h^3}{27} \end{aligned}$$

Mark allocation: 1 mark

- 1 mark for substituting correctly into the formula to get $V = \frac{\pi h^3}{27}$

Question 3b.**Worked solution**

$$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$$

$$\frac{dV}{dt} = \frac{4\pi}{9}$$

$$V = \frac{\pi h^3}{27} \Rightarrow \frac{dV}{dh} = \frac{\pi h^2}{9}$$

$$\therefore \frac{dh}{dV} = \frac{9}{\pi h^2}$$

$$\begin{aligned} \frac{dh}{dt} &= \frac{dV}{dt} \times \frac{dh}{dV} \\ &= \frac{4\pi}{9} \times \frac{9}{\pi h^2} = \frac{4}{h^2} \end{aligned}$$

Mark allocation: 2 marks

- 1 mark for the rate equation $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$
- 1 mark for finding $\frac{dh}{dV} = \frac{9}{\pi h^2}$

Question 3c.i.**Worked solution**

When $h = 2$, $\frac{dh}{dt} = \frac{4}{(2)^2} = 1$ cm/min.

Mark allocation: 1 mark

- 1 mark for the correct answer

**Tip**

- Remember to give the units.

Question 3c.ii.**Worked solution**

$$\text{Let } \frac{dh}{dt} = \frac{1}{2}.$$

$$\frac{4}{h^2} = \frac{1}{2}$$

$$h^2 = 8$$

$$\therefore h = 2\sqrt{2} \text{ cm}$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3d.i.**Worked solution**

$$\frac{dh}{dt} = \frac{4}{h^2}$$

$$\therefore \frac{dt}{dh} = \frac{h^2}{4}$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3d.ii.**Worked solution**

$$t = \int \frac{h^2}{4} dh$$

$$t = \frac{h^3}{12} + c$$

When $t = 0$, $h = 0$, so $c = 0$.

$$t = \frac{h^3}{12}$$

$$\therefore h = \sqrt[3]{12t}$$

Mark allocation: 1 mark

- 1 mark for the correct answer, which must include $+ c$

Question 3e.i.**Worked solution**

$$d = 36 - 1.5t$$

Mark allocation: 1 mark

- 1 mark for the correct expression

Question 3e.ii.**Worked solution**

Let $h = d$, giving:

$$\sqrt[3]{12t} = 36 - 1.5t$$

Using CAS to solve for t , we get:

The screenshot shows a CAS interface with the following content:

Edit Action Interactive
 solve $\left((12 \cdot t)^{\frac{1}{3}} = 36 - 1.5 \cdot t, t \right)$
 $\{t=19.86627747\}$
 $(12 \cdot t)^{\frac{1}{3}} | t=19.866277$
 6.200583747
 $(12 \cdot t)^{\frac{1}{3}} | t=19.866277$

Math1: Line, $\sqrt{\square}$, π , \rightarrow
 Math2: Define, f, g, i, ∞
 Math3: solve(, dSlv, ', $\left\{ \begin{matrix} \square \\ \square \end{matrix} \right\}$, |
 Trig: <, >, (), { }, []
 Var: \leq , \geq , =, \neq , \angle
 abc: \leftarrow , \rightarrow , ans, EXE
 Alg, Decimal, Real, Rad, $\left(\frac{\square}{\square} \right)$

$t = 19.87$ min; i.e. 9:20 a.m.

Mark allocation: 2 marks

- 1 mark for equating the two equations correctly
- 1 mark for the correct answer

Question 4a.**Worked solution**

The function f_a is strictly increasing for $f'(x) \geq 0$.

Use CAS to find $f'(x) = 0$.

The screenshot shows a CAS window titled "Edit Action Interactive". The input field contains the expression $\text{simplify}\left(\frac{d}{dx}\left(\sqrt{x} + \frac{a}{x} - 3\right)\right)$. The output shows the derivative $\frac{1}{2\sqrt{x}} - \frac{a}{x^2}$. Below this, the user has entered $\text{solve}\left(\frac{1}{2\sqrt{x}} - \frac{a}{x^2} = 0, a\right)$ and the result is $\left\{ \frac{3}{2} \right\}$. The interface includes a toolbar with icons for differentiation, simplification, and solving, and a keypad with mathematical symbols and functions.

The screenshot shows the same CAS window. The input field now contains $\text{solve}\left(\frac{1}{2\sqrt{x}} - \frac{a}{x^2} = 0, x\right)$. The output shows the solution $\left\{ x = 4 \frac{1}{3} \cdot a \frac{2}{3} \right\}$. The interface is identical to the previous screenshot, showing the same toolbar and keypad.

So, the function is increasing for $x \in \left[4^{\frac{1}{3}} a^{\frac{2}{3}}, \infty\right)$.

Mark allocation: 2 marks

- 1 mark for letting $f'(x) = 0$
- 1 mark for the correct answer

Question 4b.**Worked solution**

The minimum occurs when $f'(x) = 0$.

The screenshot shows a TI-84 Plus calculator in the 'Edit Action Interactive' mode. The display shows the following steps:

$$\left\{ x = 4^{\frac{1}{3}} \cdot a^{\frac{2}{3}} \right\}$$

simplify $(\sqrt{x} + \frac{a}{x} - 3 | x = 4^{\frac{1}{3}} \cdot a^{\frac{2}{3}})$

$$(2 \cdot |a|)^{\frac{1}{3}} + \frac{(2 \cdot a)^{\frac{1}{3}}}{2} - 3$$

The calculator interface includes a toolbar with various mathematical symbols and functions, and a keypad with rows for Math1, Math2, Math3, Trig, Var, abc, and a bottom row with Alg, Standard, Real, Rad, and a grid icon.

So, when $x = 4^{\frac{1}{3}} a^{\frac{2}{3}}$, $f(x) = \frac{3(2a)^{\frac{1}{3}}}{2} - 3$.

Mark allocation: 2 marks

- 1 mark for $x = 4^{\frac{1}{3}} a^{\frac{2}{3}}$
- 1 mark for $f(x) = \frac{3(2a)^{\frac{1}{3}}}{2} - 3$

Question 4c.**Worked solution**

For $f_a(x) = \frac{a}{x} + \sqrt{x} - 3$ and $f'(x) = \frac{1}{2\sqrt{x}} - \frac{a}{x^2}$ at $x = 4$,

$$f(4) = \frac{a}{4} - 1 \quad \text{and} \quad f'(4) = \frac{1}{4} - \frac{a}{16}.$$

So, the equation of the tangent is:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \left(\frac{a}{4} - 1\right) &= \left(\frac{1}{4} - \frac{a}{16}\right)(x - 4) \\ y &= \left(-\frac{a}{16} + \frac{1}{4}\right)x - 1 + \frac{a}{4} + \frac{a}{4} - 1 \\ y &= \left(-\frac{a}{16} + \frac{1}{4}\right)x + \frac{a}{2} - 2 \\ y &= -\left(\frac{a}{16} - \frac{1}{4}\right)x + \frac{a}{2} - 2 \\ y &= -\frac{(a-4)}{16}x + \frac{(a-4)}{2} \end{aligned}$$

Mark allocation: 3 marks

- 1 mark for $x = 4$, $f(4) = \frac{a}{4} - 1$
- 1 mark for $f'(4) = \frac{1}{4} - \frac{a}{16}$
- 1 mark for the correct tangent line equation

Question 4d.**Worked solution**

The y-intercept of $f_a(x)$ is $y = \frac{a-4}{2}$.

The y-intercept of $g_a(x) = f_a(x) + b$ is $b + \frac{a-4}{2}$.

Let the y-intercept equal zero, giving:

$$b + \frac{a-4}{2} = 0$$

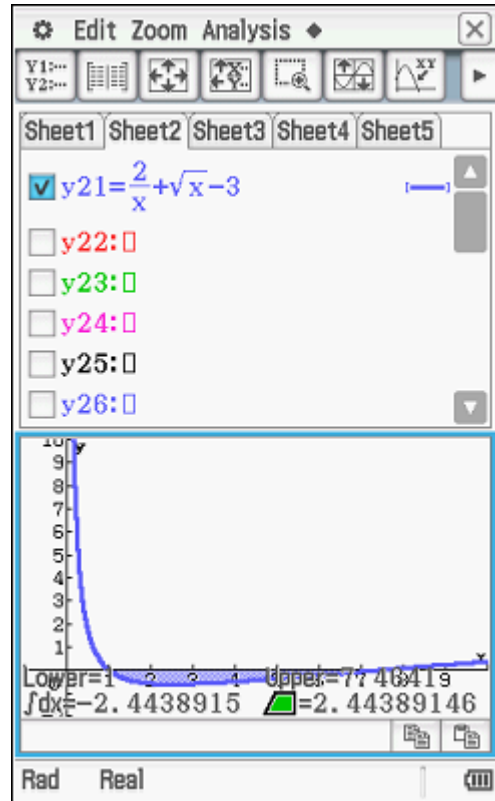
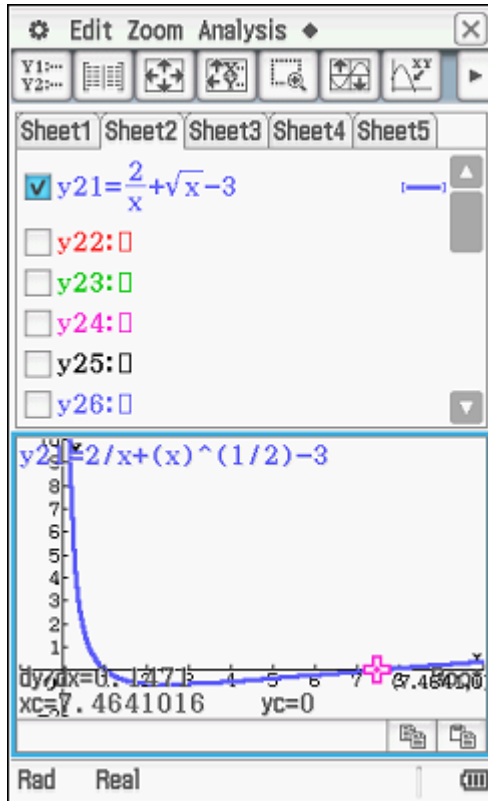
$$b = \frac{4-a}{2}$$

Mark allocation: 2 marks

- 1 mark for finding the y-intercept
- 1 mark for $b = \frac{4-a}{2}$

Question 4e.i.**Worked solution**

Using CAS, the x -intercepts occur at $x = 1$ and $x = 7.4641$.



So, the area is equal to $\left| \int_1^{7.4641} \left(\frac{2}{x} + \sqrt{x} - 3 \right) dx \right|$.

Using CAS, this is 2.444.

Mark allocation: 2 marks

- 1 mark for writing area is equal to $\left| \int_1^{7.4641} \left(\frac{2}{x} + \sqrt{x} - 3 \right) dx \right|$
- 1 mark for the correct answer

**Tip**

- *This question is worth 2 marks, so be careful to show a 'thinking step'. In this case, include the integral that allows the area to be calculated.*

Question 4e.ii.**Worked solution**

Given that $h_a(x) = f_a(2x)$, the graph of $y = h_a(x)$ is formed by a dilation of factor 0.5 in the x -direction. This means that the area formed is half the size; i.e. the area that is bounded by the x -axis and the graph of $y = h_a(x)$ for $a = 2$ is

$$0.5 \times 2.44389 = 1.2219 = 1.222$$

Mark allocation: 2 marks

- 1 mark for 0.5×2.44389
- 1 mark for the correct answer

**Tip**

- *This is a 'hence' question and therefore you must show that you have used the previous result.*

Question 5a.i.**Worked solution**

$$0.7^4 = 0.2401$$

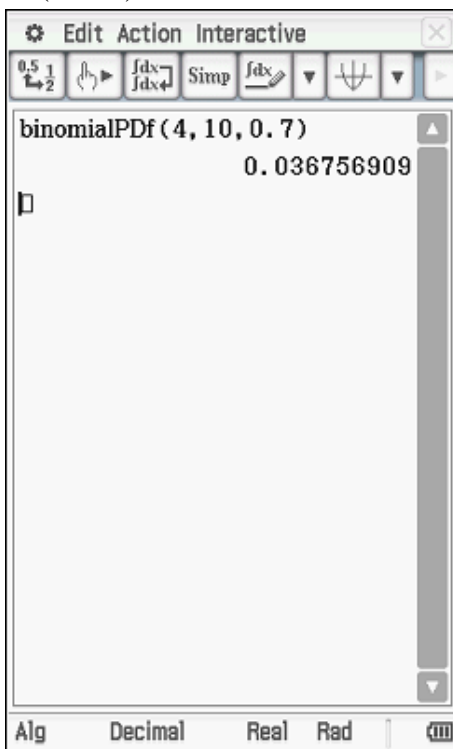
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 5a.ii.**Worked solution**

$$X \sim Bi(n=10, p=0.7)$$

$$\Pr(X = 4) = 0.0368$$

**Mark allocation: 1 mark**

- 1 mark for the correct answer

Question 5a.iii**Worked solution**

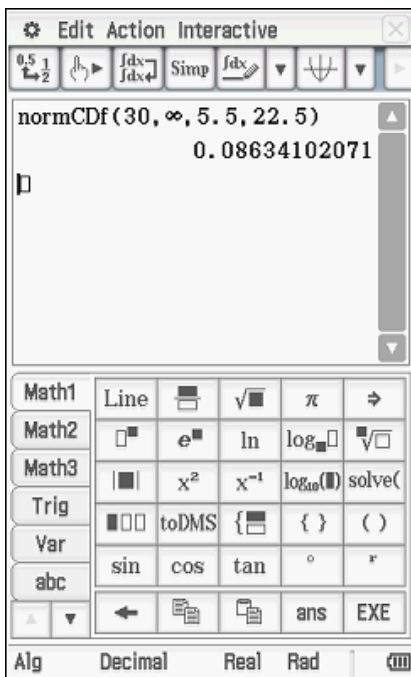
$$\begin{aligned} & \Pr(GGGGMMMMMM | X = 4) \\ &= \frac{\Pr(GGGGMMMMMM)}{\Pr(X = 4)} \\ &= \frac{0.7^4 0.3^6}{0.036756909} \\ &= 0.0048 \end{aligned}$$

Mark allocation: 2 marks

- 1 mark for $\Pr(GGGGMMMMMM | X = 4)$
- 1 mark for the correct answer

Question 5b.**Worked solution**

Using CAS:



$$X \sim N(\mu = 22.5, \sigma = 5.5)$$

$$\Pr(X > 30) = 0.0863$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 5c.i.**Worked solution**

normCdf(30, ∞, 5.5, 22.5)
0.08634102071
invNormCdf("R", 0.08, 1, 0)
1.40507156
invNormCdf("L", 0.04, 1, 0)
-1.750686071

$$X_A \sim N(\mu, \sigma)$$

$$\Pr(X_A > 35) = 0.08 \text{ and } \Pr(X_A < 5) = 0.04$$

$$\Pr\left(Z > \frac{35 - \mu}{\sigma}\right) = 0.08 \text{ and } \Pr\left(Z < \frac{5 - \mu}{\sigma}\right) = 0.04$$

Using the inverse normal distribution:

$$\frac{35 - \mu}{\sigma} = 1.4051 \text{ and } \frac{5 - \mu}{\sigma} = -1.7507$$

$$\mu + 1.4051\sigma = 35 \text{ and } \mu - 1.7507\sigma = 5$$

Mark allocation: 2 marks

- 1 mark for $\Pr(X_A > 35) = 0.08$ and $\Pr(X_A < 5) = 0.04$
- 1 mark for leading to the result $\frac{35 - \mu}{\sigma} = 1.4051$ and $\frac{5 - \mu}{\sigma} = -1.7507$

Question 5c.ii.**Worked solution**

$$\mu = 21.643 \text{ and } \sigma = 9.506.$$

The screenshot shows a TI-84 Plus calculator interface. The display area contains the following text:

```

INVNORMCDF(L, 0.04, 1, 0)
-1.750686071
binomialCDF(7, 20, 20, 0.08)
6.375493933E-4
{x+1.4051y=35 |
{x-1.7507y=5 | x,y
{x=21.64268965, y=9.50630

```

The calculator interface includes a toolbar with various mathematical functions and a keypad with rows for Math1, Math2, Math3, Trig, Var, abc, and a bottom row with navigation and execution keys.

Mark allocation: 2 marks

- 1 mark for finding the mean
- 1 mark for finding the standard deviation

Question 5d.**Worked solution**

$$X \sim Bi(n = 20, p = 0.08)$$

$$\Pr(X \geq 7) = 0.00064$$

The screenshot shows a TI-84 Plus calculator interface. The display shows the following sequence of operations and results:

- 0.08634102071
- invNormCdf("R", 0.08, 1, 0)
- 1.40507156
- invNormCdf("L", 0.04, 1, 0)
- 1.750686071
- binomialCdf(7, 20, 20, 0.08)
- 6.375493933E-4

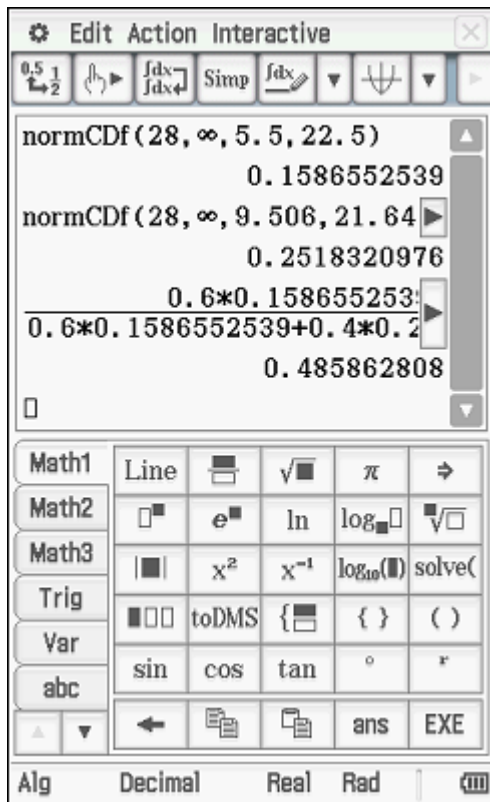
The calculator interface includes a menu bar (Edit, Action, Interactive), a toolbar with various mathematical functions, and a keypad with rows for Math1, Math2, Math3, Trig, Var, abc, and a bottom row with navigation and execution keys. The mode is set to Alg.

Mark allocation: 2 marks

- 1 mark for identifying the binomial distribution $X \sim Bi(n = 20, p = 0.08)$
- 1 mark for the answer $\Pr(X \geq 7) = 0.00064$

Question 5e.**Worked solution**

$$\begin{aligned}
 \Pr(\text{home goal} \mid \text{goal is more than 28 m}) &= \frac{\Pr(\text{home goal and goal is more than 28 m})}{\Pr(\text{goal is more than 28 m})} \\
 &= \frac{0.6 \Pr(X_H > 28)}{0.6 \Pr(X_H > 28) + 0.4 \Pr(X_A > 28)} \\
 &= \frac{0.6 \times 0.158655}{0.6 \times 0.158655 + 0.4 \times 0.251832} \\
 &= 0.4859
 \end{aligned}$$

**Mark allocation: 2 marks**

- 1 mark for understanding the conditional probability

$$\Pr(\text{home goal} \mid \text{goal is more than 28 m}) = \frac{\Pr(\text{home goal and goal is more than 28 m})}{\Pr(\text{goal is more than 28 m})}$$

- 1 mark for the correct answer