2015 VCAA Math Methods CAS Exam 1 Solutions

Q1a $y=(5x+1)^7$, $\frac{dy}{dx}=7\times5\times(5x+1)^6=35(5x+1)^6$

Q1bi $f(x)=\frac{\log_e(x)}{x^2}$,

$f'(x)=\frac{(x^2)(\frac{1}{x})-(2x)(\log_e(x))}{(x^2)^2}=\frac{x(1-2\log_e(x))}{x^4}=\frac{1-2\log_e(x)}{x^3}$

Q1bii $f'(1)=\frac{1-2\log_e(1)}{1^3}=1$

Q2 $f'(x)=1-\frac{3}{x}$, $f(x)=\int\left(1-\frac{3}{x}\right)dx=x-3\log_e|x|+c$

$f(e)=e-3\log_e|e|+c=-2$, .: $c=1-e$

.: $f(x)=x-3\log_e|x|+1-e$

Q3 $\int_1^4\left(\frac{1}{\sqrt{x}}\right)dx=\int_1^4 x^{-\frac{1}{2}}\,dx=\left[2x^{\frac{1}{2}}\right]_1^4=2\sqrt{4}-2\sqrt{1}=2$

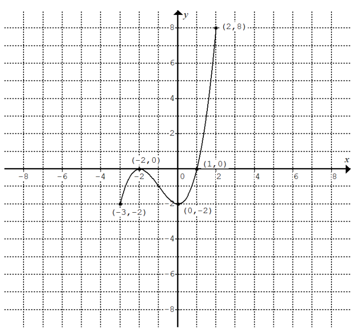
Q4a $f(x)=\frac{1}{2}(x^3+3x^2-4)$, $f'(x)=\frac{1}{2}(3x^2+6x)$

Let $f'(x)=0$, $\frac{1}{2}(3x^2+6x)=\frac{3}{2}x(x+2)=0$

.: $x=-2, 0$ and and the corresponding ordinates are $y=0,-2$

The stationary points are: $(-2,0)$ and $(0,-2)$.

Q4b



Q4c $\text{Av. value}=\frac{\int_0^2\frac{1}{2}(x^3+3x^2-4)dx}{2-0}=\frac{1}{4}\left[\frac{x^4}{4}+x^3-4x\right]_0^2=1$

Q5a $h(t)=14+8\sin\left(\frac{\pi t}{12}\right)$, minimum depth $=14-8=6$ m

Q5b $14+8\sin\left(\frac{\pi t}{12}\right)=10$ where $0\le t\le 24$

$\sin\left(\frac{\pi t}{12}\right)=-\frac{1}{2}$, $\frac{\pi t}{12}=\frac{7\pi}{6},\frac{11\pi}{6}$.: $t=14,\ 22$

Q6a $\Pr(X>3.1)=\Pr\left(Z>\frac{3.1-2.5}{0.3}\right)=\Pr(Z>2)=\Pr(Z<b)$, $b=-2$

Q6b $\Pr(Z<-1)=\Pr(Z>1)=0.16$, .: $\Pr(Z<1)=0.84$

.: $\Pr(0<Z<1)=0.84-0.5=0.34$

$\Pr(X<2.8\mid X>2.5)=\Pr(Z<1\mid Z>0)=\frac{\Pr(0<Z<1)}{\Pr(Z>0)}=\frac{0.34}{0.5}=0.68$

Q7a $\log_2(6-x)-\log_2(4-x)=2$, $\log_2\frac{(6-x)}{(4-x)}=2$, $\frac{6-x}{(4-x)}=2^2$

$6-x=16-4x$, $x=\frac{10}{3}$

Q7b $3e^t=5+8e^{-t}$, $3e^t-5-8e^{-t}=0$, $(3e^t-5-8e^{-t})e^t=0$

$3(e^t)^2-5e^t-8=0$.: $(3e^t-8)(e^t+1)=0$

Since $e^t+1>0$, .: $3e^t-8=0$, $e^t=\frac{8}{3}$, $t=\log_e\left(\frac{8}{3}\right)$

Q8a $\Pr(A\cap B)=\Pr(A\mid B)\Pr(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$

Q8b $\Pr(A\cap B)+\Pr(A'\cap B)=\Pr(B)$

.: $\Pr(A'\cap B)=\Pr(B)-\Pr(A\cap B)=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$

Q8c If A and B are independent, $\Pr(A)=\Pr(A\mid B)=\frac{3}{4}$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)=\frac{3}{4}+\frac{1}{3}-\frac{1}{4}=\frac{5}{6}$

Q9a $\Pr(\text{white})=\frac{3}{5}p+\frac{1}{5}(1-p)=\frac{1}{5}(2p+1)$

Q9bi $\Pr(B\mid\text{white})=\frac{\Pr(B\cap\text{white})}{\Pr(\text{white})}=\frac{\frac{1}{5}(1-p)}{\frac{1}{5}(2p+1)}=\frac{1-p}{1+2p}$

Q9bii $\frac{1-p}{1+2p}=0.3$, $p=\frac{7}{16}$

Q10a $T(2+2\cos\theta, 2\sin\theta)$

Q10b Gradient of $TC=\tan\theta$

.: gradient of tangent $=-\frac{1}{m_{\text{N}}}=-\frac{1}{\tan\theta}$

Alternatively, gradient of tangent $=\tan\angle TXx=\tan\left(\frac{\pi}{2}+\theta\right)$

Q10ci $B(2,b)$ is on the line $x\cos\theta+y\sin\theta=2+2\cos\theta$

.: $2\cos\theta+b\sin\theta=2+2\cos\theta$, .: $b=\frac{2}{\sin\theta}$

Q10cii $D(4,d)$ is on the line $x\cos\theta+y\sin\theta=2+2\cos\theta$

.: $4\cos\theta+d\sin\theta=2+2\cos\theta$, .: $d=\frac{2-2\cos\theta}{\sin\theta}$

Q10d Area $A=\frac{1}{2}(d+b)\times2=d+b=\frac{4-2\cos\theta}{\sin\theta}$

$\frac{dA}{d\theta}=\frac{\sin\theta(2\sin\theta)-(4-2\cos\theta)\cos\theta}{\sin^2\theta}$

$=\frac{2(\sin^2\theta+\cos^2\theta)-4\cos\theta}{\sin^2\theta}=\frac{2-4\cos\theta}{\sin^2\theta}$

Let $\frac{dA}{d\theta}=0$, $2-4\cos\theta=0$, $\cos\theta=\frac{1}{2}$, $\theta=\frac{\pi}{3}$

.: minimum $A=\frac{4-1}{\frac{\sqrt{3}}{2}}=2\sqrt{3}$ square units

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