Year 2015

VCE

Mathematical Methods

Trial Examination 1

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Victorian Certificate of Education
2015

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 15 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. If $y = \dfrac{\log_e(3x)}{2x}$, find $\dfrac{dy}{dx}$. 2 marks

b. Let $f(x) = x\sqrt{4x^2+9}$. Find $f'(-2)$. 2 marks

Question 2 (2 marks)

Solve the equation $3\times 9^x - 28\times 3^x + 9 = 0$ for x.

Question 3 (3 marks)

Consider the linear simultaneous equations

$$\begin{aligned} k\,x + 8y &= k-4 \\ 3x + (k-2)\,y &= 1 \end{aligned}$$ where k is a real constant.

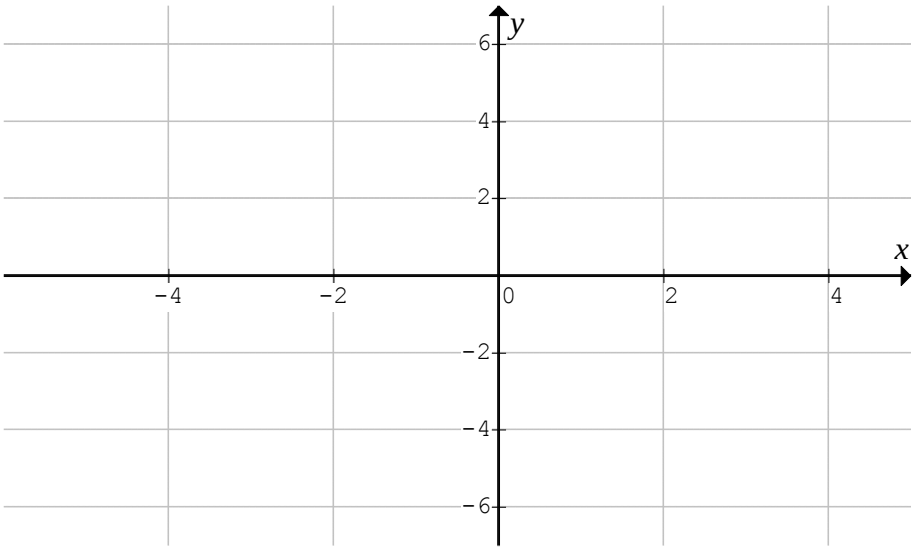
Find the value(s) of k, for which there is a unique solution.

Find the value(s) of k, for which there is infinitely many solutions.

Question 4 (7 marks)

a. Sketch the graph of $y = \frac{3}{x+2} - 1$ on the axes below, clearly indicating all axial intercepts and the equation of any asymptotes.

2 marks



b. Find the area bounded by the graph of $y = \frac{3}{x+2} - 1$ the x-axis and the line $x = 4$.

2 marks

c. Describe in words, giving scale factors, the transformations in a suitable order, required to sketch the graph of $y = \frac{3}{x+2} - 1$ from the graph of $y = \frac{1}{x}$.

1 mark

d. Given the function with the rule $f(x) = \frac{3}{x+2} - 1$, find the inverse function.

2 marks

Question 5 (2 marks)

If $f'(x) = 4e^{-2x} - 1$ and $f(0) = 1$, find $f(x)$.

Question 6 (7 marks)

a. State the range, period and amplitude of the function $f : R \to R$, $f(x) = 2 - 4\sin\left(\frac{\pi x}{6}\right)$

1 mark

b. Find the general solution of the equation $2 - 4\sin\left(\frac{\pi x}{6}\right) = 0$

2 marks

c. Solve the equation $2 - 4\sin\left(\frac{\pi x}{6}\right) = 0$ for $x \in [0,13]$ 1 mark

d. Sketch the graph of the function $g:[0,13] \to R$, $g(x) = \left|2 - 4\sin\left(\frac{\pi x}{6}\right)\right|$ on the axes below, clearly labelling the scale and stating the coordinates of all axial intercepts, turning points and end-points.

2 marks

e. State the domain of the function $g'(x)$. 1 mark

Question 7 (4 marks)

a. Raymond likes to use the BBQ for cooking dinner. If the weather is pleasant the probability that Raymond uses the BBQ for dinner is $\frac{3}{5}$. If the weather is unpleasant the probability that Raymond uses the BBQ for dinner is $\frac{1}{4}$. In November the probability of pleasant weather on any one day is $\frac{2}{3}$. Find the probability that the weather was unpleasant in November, given that Raymond did not use the BBQ for dinner.

2 marks

b. A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X = x)$	$\log_8(k-1)$	$\log_8(k-3)$

Find the value(s) of k.

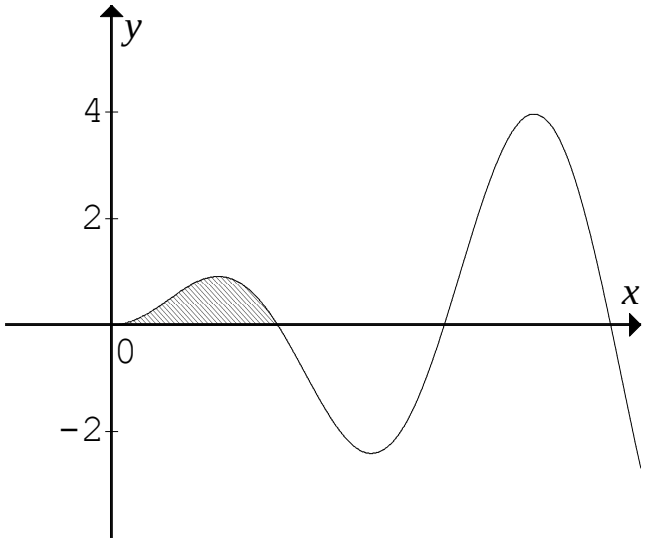
2 marks

Question 8 (4 marks)

a. Differentiate $x\cos(2x)$ with respect to x. 1 mark

b. The diagram shows part of the graph of $f:[0,\infty)\to R$, $f(x)=x\sin(2x)$.
Use your answer to **a.** to determine the area of the shaded region.

3 marks



Question 9 (3 marks)

Find the value of c if $4y - x + c = 0$ is a normal to the curve $y = 2e^{-2x} + 1$.

Question 10 (4 marks)

A piece of string of length 36 cm, is cut into two pieces. One piece is used to form a square and the other piece is used to form an equilateral triangle.

a. If the length of each side of the square is x cm, show that the total area A in square cm of the square and equilateral triangle is given by $A(x) = \left(\frac{4\sqrt{3}}{9} + 1\right)x^2 - 8\sqrt{3}x + 36\sqrt{3}$

2 marks

b. Find the values of x for which the total area A is a maximum and a minimum.

2 marks

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c \ , \ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A') \qquad \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\,f(x)\,dx$